Optimal Reserve Accumulation with Capital Controls

Bada Han* Minwoo Park†

Very Priliminary

July 2018

Abstract

In this paper, we first build a new model of international reserves as a policy tool against sudden stop crisis where we introduce two distinctive features. First, we assume that reserves can work as a collateral so as to provide Emerging Market Economies (EMEs below) more leverage. This assumption not only provides an incentive to hold costly reserves, but also explains why EMEs were hesitant to deplete their reserves during the Global Financial Crisis and the market turmoil in 2013. Second, following recent papers of foreign exchange market interventions, we assume that no arbitrage condition does not perfectly hold in foreign exchange markets in EMEs, which allows reserves accumulation to raise the Net Foreign Assets of the EME. Based on the newly constructed model, we explore the relation between reserve accumulation and capital controls. As usual in the literature, we derive the optimal tax on foreign borrowings. Like the preceding works, we find that one of roles of the optimal control on the foreign borrowing is to eliminate the moral hazard. However, the magnitude of the moral hazard is lower in our model since the reserve accumulation results in higher borrowing rates, thereby punishing the borrowers. Along with the taxes on the borrowers, we study the optimal taxation on international investors who are the supplier of capitals in our model. While one of roles of the taxation on international investors is, like the taxation on the borrowers, to control the overborrowing, another role is to adjust frictions in the capital flows so that households can smooth their consumptions in a more efficient way. While it is not clear whether the amount of the optimal reserves holding will increase or decrease according to the introduction of the optimal taxations, the optimal policy mix of reserves management and capital controls should be much more powerful and efficient than relying on the either of the two policy tools solely.

Keywords: International Reserves, Capital Controls, Sudden Stop Crisis

JEL Classification: E60, E61, F30, F38, G01

*Department of Economics, University of Southern California, badahan@usc.edu
†Department of Economics, University of Southern California, minwoopa@usc.edu

1 Introduction

One of the fastest growing literature in international macroeconomics for the last decade is the literature of sudden stop crisis and policies to cope with it. Sudden stop crisis in the literature is well-defined: a sudden stop of capital inflow to an Emerging Market Economy (EME), and the subsequent currency depreciation, which results in a serious balance sheet deterioration in the country. Along with the interest in macro-prudential policy to prevent financial crises after the Global Financial Crisis (GFC below), policy tools to suppress excessive foreign borrowing, capital controls mostly, have been extensively studied in the literature. While most academic professionals and policy makers agree upon the negative externality of foreign borrowing in EMEs, the focus of policy makers is slightly different from the academic circle; many papers in the literature explore the Pigouvian tax as a capital control and suggest it as an effective policy tool, but it seems that policymakers prefer to accumulate reserves rather than to impose capital controls as a precautionary measure against sudden stops (see e.g. IMF, 2013).

The large accumulation of international reserves in EMEs has attracted much attention from academics, and also sparked a debate about the motivation for building such a large amount of international reserves. Some papers insist that the main motivation of holding such costly international reserves lies in a growth strategy in the EMEs, so called export promotion strategy or mercantilism view, while another view argues that EMEs have built a large stock of reserves as a buffer against disruptions in international financial markets. In the reality, the large stock of reserves should be explained by the mixture of the two different views, and the relative importance of one against another varies by regions and countries. Despite the ongoing debate over its deriving forces, amid the recent experience of EMEs after the Global Financial Crisis, now it is widely believed among many economists and almost all practitioners that international reserves help EMEs with weathering a tough time of a massive capital outflow. However, many questions regarding international reserves as a precautionary policy tool remain unanswered. How exactly does international reserve work in dealing with a sudden stop crisis? What is the adequate level of international reserves on a precautionary purpose? And furthermore, how does a policy maker use international reserve during a crisis?

Our first contribution is to suggest a model of international reserves that explain why EMEs hold a massive amount of international reserves and external debts simultaneously, how international reserves are helpful for preventing and going through a sudden stop crisis, and moreover why EMEs were hesitant to deplete their reserves during the GFC and similar global financial market turbulence events, but seems just having international reserves was somehow effective in the tough time for EMEs. Some readers may think that aforementioned questions were already answered in the literature. Of course, our understanding of international reserve holding by EMEs have been deepened for the last two decades, thanks to the contributions from some important papers such as Aizenman and Lee (2007), Romain and Jeanne (2011), Bocola and Lorenzoni (2018), and Bianchi et al. (2018). However, as it will be shown in the next section, existing models have a difficulty in capturing some important stylized facts regarding international reserve. The first contribution in this paper is to build a model explaining all the related facts. Furthermore, we constructed the model in the framework of Fisherian Deflation, which was developed and have been progressed by Bianchi, Jeanne, Mendoza, and Korinek. We intentionally choose their framework because it is not only very tractable, but also it is most widely used in the literature, to our knowledge. Other advantage of working with Fisherian Deflation type model is that it makes it possible to conduct a reasonably precise quantitative analysis, and it provides an environment where we can study reserve accumulation policy with macro-prudential capital control.

To model international reserves as an effective policy tool, we adopt two distinctive specifications from recent papers. First, following the few recent papers, for example Cavallino (2018) and Fanelli and Straub (2018), that analyzed the effectiveness of foreign exchange market intervention with the idea of infrequent portfolio adjustment in foreign exchange market, we assume that international financial markets are imperfect and the exchange rate is determined by capital flows rather than stocks. That is, due to a fixed transaction cost or exogenous position limits, arbitrage of international or domestic investors is limited, which in turn derives interest rate spreads or UIP spreads in foreign exchange market. In other words, capital supply from international investors to an EME is not perfectly elastic to UIP spreads. In such an environment, foreign exchange market intervention by a central bank, equivalently more demand for external capital from the central bank, temporarily derives up the interest rate on the foreign borrowing and depreciates their domestic currency. This suppresses the demand from private agents for foreign borrowing, and thereby increases net foreign asset (NFA below) of the EME. Hence, as widely believed among practitioners, in our model the foreign exchange market intervention to accumulate foreign reserve tend to raise current account surplus so as to increase the NFA as well. Second, we assume international reserves can be used as a collateral as it is suggested in Shousha (2017). While some financial assets, mostly US treasury bond, are largely used as a collateral in interbank market, in particular in repo market, the idea that international reserves, which are mostly composed of US treasury bonds or bonds issued by governments in other advanced economics, can be adopted as a collateral was rejected because of the absence of an explicit contract and the legal structure of international reserves. However, letting international reserves be used as a collateral in our model gives few decisive advantages. It will be explained in much detail later, but to give an essence of the specification, international reserves as a collateral can explain why EMEs or central banks were often reluctant to offset massive capital outflows using their reserves during the GFC and other times of market turmoil. In addition, it seems the assumption corresponds to the behavior of market participants; often whether an EME has adequate level of international reserves is in the check-list of investors before they finalize their decision. At the side of theory, some forefront researches provide a microfoundation for the use of financial assets as collateral (see Gottardi, Maurin and Monnet, 2016; and Parlatore 2018). In a broader sense, our assumption that international reserves are used as a collateral is related with some papers in which agents sell their assets or use them as a collateral to obtain more liquidity (Bigio, 2015).

The two new specifications jointly provide realistic descriptions of accumulating and using international reserves. Once EMEs realize[1] that they are fragile to a sudden stop of capital inflow, they desire to build a stock of international reserves, but the process is gradual since too much market intervention to accumulate reserve within a short time causes temporary higher yields on the foreign borrowing. The EME holds lots of external liabilities and reserves simultaneously during and after the process because holding reserves provide enough liquidity during the time of capital drain. With respect to the use of reserves, there are two options to use international reserves during a crisis. First, reserves can be used to pay back their debts, offsetting the capital outflow. In the other way, the central bank of EME can hold its reserves to use the reserves as a

[1]The rally of Asian countries to build a stock of reserves after the East Asian crisis in 1997, which was followed by Latin American countries to some extent, can be understood a result that these countries began becoming aware of the risk. More formally, this can be thought as an update of the countries in a Bayesian fashion. For more details of this point, we refer reader to Hur and Kondo (2016).

collateral. We found that EMEs prefer to use reserves as collateral when 1) they have a hunger for liquidity, i.e., the credit constraint on the country is more severely binding, or 2) it is predicted the ongoing crisis is more persistent with higher uncertainty. So, our model replicates one important interesting fact about the use of international reserves during a crisis; contrary to the results of many existing models, most EMEs had a small and short-lived international reserves depletion during the GFC, rebuilding their stocks very quickly after that. Aizenman and Sun (2012) documented this phenomenon with the remark "fear of losing reserves seems to play a key role in shaping the actual use of international reserves by emerging markets"[2].

The second contribution of this paper is to find relationship between macro-prudential capital control and international reserves, so as to provide a policy implication about the mix of two policy tools. One interesting question about macro-prudential capital control is how the ex-ante preventive policy is related with ex-ante intervention policy. Surely, this is important for understanding the nature of policies in EMEs as well as designing the best combination of the two policy tools. While there have been few excellent papers on this issue in a more general sense, so for the combination of an ex-ante policy and ex-post policy (e.g., Jeanne and Korinek, 2017), surprisingly the mixture of macro-prudential policy and international reserves remains almost unexplored . In the side of empirics, there are few papers documenting how macro-prudential policies are used in conjunction with international reserves (e.g., Aizenman et al., 2018). But, in the theory side, the only paper that analyzed the issue of mixture between macro-prudential policy and international reserves, to the best of our knowledge), is Acharya and Krishnamurthy (2018). Their main conclusion is that reserve accumulation and macroprudential capital control are compliments rather than substitutes with each other; that means the with the capital controls, an EME might hold more reserves than otherwise. In their model, because of the insurance provided by reserves, private agents borrow more in the presence of significant amount of international reserves (Moral Hazard). The concern over such side effects make governments to hold less reserves. One of the functions of capital control is to eliminate these distortion of holding reserves, and therefore EMEs should hold more reserves with capital controls, in their model. While our model also have similar side effect of reserves (Moral Hazard of private agents), we also found that international reserves and the capital controls might be quantitively substitutes with each other; adequate use of capital control could decrease the amount of reserve holding by EMEs. This is because in our model, foreign exchange market interventions to accumulate reserves has a function of punishing the over-borrowing by itself; the intervention raises the interest rate on the foreign borrowing and depreciates the domestic currency, which makes the foreign borrowing more expensive. Meanwhile, the capital controls suppress the foreign borrowings, so as to reduce the chance and severity of a sudden stop crisis, and accordingly the demands for reserves as well. Moreover, the capital controls extend the wedge in foreign exchange market, by which makes it possible for an EME to manipulate its terms of trade with less FX market intervention.

Related Literature Our paper is related with several strands of literature in international macroeconomics. First and foremost, our paper is a part of the literature that studied the nature of sudden stop crisis and policies to deal with it. The literature has a really long history, which date backs to at least the later 1990's. Among numerous

[2]The fact about "fear of losing reserve" is replicated in the model in Shousha (2017), from which we borrowed the specification of reserves as a collateral. However, we did it with more plausible values of some parameters of the liquidity from holding reserves. Of course, our model explains few more important facts. This will be discussed in a later section.

papers, our paper borrows many results and insights from the works that investigated the role of pecuniary externalities of foreign borrowings in the presence of occasionally borrowing constraint in a country level. One of the earliest paper in this tradition is Caballero and Krishnamurthy (2001). Inheriting the idea from the paper, Bianchi, Jeanne, Korinek, and Mendoza have made a significant progress in the analysis of sudden stop crisis in EMEs, while sharing a common framework among them. All their works are important, but the papers worth mentioning here are Bianchi (2011), Mendoza (2010), Korinek and Sandri (2016), Jeanne and Korinek (2010, 2017), and Korinek (2018). The strength of their framework is its realistic description of sudden stop crisis in a simple set up so that various quantitative analyses are possible in the environment; private agents don't consider the effect of their consumption on the borrowing constraints in the future, which includes the real exchange rate, and accordingly once a crisis happens – the collateral constraint binds – the real exchange rate drops, making the borrowing constraint even tighter[3]. However, the focus of their works in terms of policy is on ex-ante preventive Pigouvian tax although some papers such as Jeanne and Korinek (2017) and Korinek (2018) paid some attention to ex ante intervention policy or international reserves. Our contribution to this literature is to suggest a modification to their model, which accumulation of international reserves does act in a way to prevent a sudden stop crisis and lessen the magnitude of the crisis.

Secondly, our paper is related with the literature of optimal reserve accumulation. The main objective in the literature is to find why EMEs hold a huge amount of costly international reserves. As it was explained above, some articles argued that international reserves accumulation has a mercantilism purpose, while many other articles interpret the reserves as a buffer against sudden stops (precautionary view). Early works in the literature of the mercantilism view includes Dooley et al. (2004 a,b), and there have been few noticeable recent papers of similar ideas (Korinek and Serven, 2016; Beningo and Fornaro, 2012, Choi and Taylor, 2017; and Alfaro et al, 2018). We clearly state that we do admit that to a certain degree the reserve accumulation has been driven by the mercantilism view. The tremendous amount of reserves in China and few other east Asian countries are hard to be justified by the precautionary view only. Obviously, China has an almost closed financial market, which leaves almost zero concern over a sudden stop. As it is reported in Jeanne and Ranciere (2011), if we consider the reserve holdings of east Asian countries as a sole behavior of precautionary motive, the high level of reserves in the data can only be explained with extremely high cost of sudden stop crisis or an extreme risk averse preference. However, although the motivation to accumulate much international reserves lies in a mercantilism view, it is not deniable that the precautionary view is also an important driver of the reserve accumulation, and moreover, for some EMEs, it might be the main motivation. Historical evidence supportive for the precautionary view is the fact that most EMEs began building their stocks of reserves after experiencing financial crises, in particular after the East Asian crisis in 1997. While agreeing both of the views are valid, through this paper, we will assume the purpose for EMEs to hold reserves lies only in the precautionary motivation rather than prove it. The papers in the literature of international reserves that mainly follow the precautionary view will be discussed later in much detail. Relatively early works worth listing are Aizenman and Lee (2007) and Jeanne and Ranciere (2011). More recently, Bocola and Lorenzoni (2018), Bianchi et al. (2018), Shousha (2017) and Basu et al. (2017) are noticeable. Closest to us is Shousha (2017) in which international reserves work as another collateral in credit constraint of an EME. We revised the model

[3]For the readers who are not familiar with the literature, the intrinsic idea of the mechanism is almost identical to the famous paper, Kiyotaki and Moore (1997).

in Shousha (2017) by adding a few elements to explain the stylized facts in the next section, to make reserves workable with more reasonable values of parameters, and to conduct various policy experiments in a similar environment.

The third literature related to ours is the nascent literature that studied the effectiveness of foreign exchange market interventions in FX markets where arbitrages of market participants are significantly limited. It is Gabaix and Maggiori (2015), which started the literature by resurrecting the old idea of the forgotten portfolio balance theory of FX market. In a simple model, Gabaix and Maggiori showed how limits to the arbitrage in global asset markets can explain some important puzzles in international finance such as the forward premium puzzle. As an extension, they also showed interventions in FX market should be effective and can be welfare-improving. Cavallino (2018) built a New Keynesian general equilibrium model that analyzed foreign market intervention, using almost same specifications with Gabaix and Maggiori (2015), but in a continuous time. From a slightly different microfoundation, Fanelli and Straub (2017) derived general principles of foreign exchange market interventions. To model limited arbitrage in a foreign exchange market, we mostly followed Fanelli and Straub (2017), although we abstract from some important features in their mode, according to our purpose. The difference of our model from theirs is that the main purposes in our model are to accumulate reserves against sudden stops in the future and use the reserve (by just holding or depleting) when the crisis comes, while in Fanelli and Straub (2017), the main purpose is to provide a guideline of managing UIP spreads to avoid further speculations and lessen unnecessary fluctuations in foreign exchange markets.

Our paper is also related with the ongoing debate about "Global Financial Cycle". To summarize the intense debate shortly, the most important questions in the literature is "How much discretion does an EME have facing Global Financial shocks from center economies?" To give a full answer to the question is beyond the scope of this paper[4]. But, the conclusion in our paper hints that though EMEs are not fully free from the financial conditions of center economies, but the countries can insure themselves in part, using adequate combination of policy tools.

Layout The rest of the paper is organized as follow. Section 2 illustrates some facts regarding international reserves in EMEs. We also discuss models in the preceding papers based on the facts we listed. Section 3 introduces a three periods model. We focus on deriving analytic "pen and paper" results so that readers can see the underlying mechanisms behind our results. In Section 4, we extend the model in section to a more general infinite horizon model and conduct quantitative analyses with different parameter values, and alternative assumptions of policy makers' preference and capital flows. Section 5 concludes and discuss avenues for future researches.

2 Background on International Reserves in EMEs

According to a guide by International Monetary Fund (IMF), a country's international reserves refer to *"...those external assets that are readily available to and controlled by monetary authorities for meeting balance of payments financing needs, for intervention in exchange markets to affect the currency exchange rate, and for other related purposes (such as maintaining confidence in the currency and the economy, and serving as a basis for foreign borrowing)"* (IMF BPM6, paragraph 6.64). In short, international reserves

[4] For more details on the debate, we refer readers to Rey (2013), Cerutti (2017), Aizenman (2018).

Figure 1

Figure 2

are the external assets (perhaps, very liquid and safe assets) held by a central bank or a government. It is well known that international reserves are mainly composed of key currencies such as US dollar, Euro or Japanese Yen. For most countries, the exact composition of international reserves is classified, but the official statistics from IMF show that at least, half of the reserves are the US dollars, of which is mainly composed of the US treasury bonds. For more details of the compositions of international reserves, see the Appendix.

In this section, we document certain important facts on international reserves. The purpose of it is to show the facts that we want to capture in our model. After reviewing the facts, we will discuss to what extent the literature is capable of capturing those facts, but also show the existing models in the literature is still short of explaining the facts, although all the facts we will show are not new.

2.1 Related Facts

Fact 1: Rapid, but steady increase of international reserves after mid 1990's
As we noted in the beginning, EMEs began accumulating a massive amount of international reserves since the mid 1990's. How much reserve a EME holds, and how rapidly a EME has accumulated vary along with a region and a country, but nevertheless it seems that EMEs have built a "war chest" of reserves to insure against sudden stops since the 1990s[5].

As in the figure 1, the level of reserves in the 80's and early 90's was around 5% of GDP for most EMEs. It is clear that the reserves-GDP ratio started skyrocketing after 97 crisis in the East Asian countries. As of 2017, many east Asian EMEs are holding reserves at the amount of 20~30% of their GDP, but for some countries (e.g., Malaysia) the ratio is approximately 40%. Some east European countries have reserves by the amount of around 20 % of GDP. In Latin America, the reserves-GDP ratio is 10~15% for most of the countries.

Another related fact that has received less attention is that EMEs have gradually accumulated reserves. We haven't checked all the countries, but for selected countries, the trend is very clear. In addition, EMEs tend to accumulate reserves when the economies' current accounts are surplus; in other words, their central banks there absorb the surpluses through foreign exchange market interventions. It is evident in the figure 2.

Fact 2: EMEs are holding a lot of reserves and liabilities simultaneously
Given the fact that many EMEs are sitting on a huge pile of reserves, one fact that the literature has struggled to explain is that the EMEs are simultaneously holding large amounts of external liabilities and reserves. The figure 2 below exhibits a regression

[5]One related interesting question is "Why do EME's suddenly begin building a wall of reserves?" The history may imply that policy makers in EMEs realized they are vulnerable to sudden reversals of cross-border capital flows. Hur and Kondo (2016) presents an interesting model in which agents in the model learn the risk of sudden stops through realized events. Though it is not our main interest, we will discuss about this point at the end of this paper.

Figure 3

Figure 4

line showing seriously positive correlation between the level of reserves and the level of external liabilities. Rodrick (2006) insists that this is puzzling because it should be better for the EMEs to pay back their debts using their reserves rather than holing the two together; obviously the yields on reserves are much lower than the liabilities of EMEs. According to the estimation in Rodrick (2006), the income loss due to the yield difference amounts to close to 1 percent of GDP.

By looking at the line in the figure 3, one might notice that the slope of the line is "to some extent" less than 45 degrees. Of course, we shouldn't take the slope of the line seriously, but nevertheless it may hint that foreign exchange market interventions have effects of improving the current accounts, thereby raising Net Foreign Assets. We can see it more clearly in the figure 4.

Although many previous empirical studies failed in finding evidences of the effectiveness of foreign exchange market interventions, we should be careful in accepting the results in the papers because of the concern about simultaneity; interventions influence exchange rates but also respond to shocks to exchange rates. Few careful studies (e.g., Aizenman et al., 2012) found the effectiveness of the interventions, and Kearns and Rigobon (2015) provides a similar evidence from the natural experiments in Japan and Australia. It will be discussed below, but some of existing models of reserves predict the slope of the line in figure 3 is slightly steeper than 45 degrees line (slight negative slope in figure 4).

Fact 3: the EMEs with more reserves weathered GFC in a better shape Because of such enormous costs of holding reserves, some academics (e.g., Alfaro and Kanczuk, 2009) explicitly casted doubts on its effectiveness as a policy tool on a precautionary purpose, while many policy makers in EMEs believe that holding enough reserves would help protect their countries against sudden stop crises. Then the following crisis and a subsequent market turmoil – GFC and the news of QE tapering in 2013 – provided opportunities to test the beliefs of the policy makers. Definitely, judging whether reserves helped EMEs with suffering less from the huge negative shock from center economies poses tough empirical challenges. However, despite the challenges and difficulties, there are a few empirical studies reporting reserves were helpful for EMEs during the tough times (See Dominquez et al., 2011; Bussiere et al., 2015; Frankel and Saravelos, 2012; and Aizenman et al., 2015).

These papers evaluated the performances of different EMEs during GFC and the market turmoil in 2013, and checked how the performance had been changed according to the amounts of reserves holding. To be more specific, Dominquez et al. (2011) showed relatively higher growths of EMEs with higher reserves and Bussiere et al. (2015) documented similar results. Aizenman et al. (2015) reported that EMEs with insufficient reserves tended to experience more severe exchange rate depreciations after the announcement of QE tapering.

Fact 4: EMEs were hesitant to deplete reserves during a crisis Perhaps, the most interesting and puzzling fact regarding international reserves is that EMEs

Figure 5

were hesitant to deplete reserves when the chance to use accumulated reserves finally came during the GFC. Many papers documented that EMEs had a small and short-lived international reserves depletion during the GFC, and then rebuilt their stocks quickly after the short depletion. A few papers documented it, but such hesitance to deplete reserves is particularly well-documented in Aizenman and Sun (2012) in which the authors stated that during the GFC the "Fear of Losing Reserves seems to play a key role in shaping the actual use of international reserves by emerging markets". Recently, Basu et al. (2016) summarized such behaviors of EMEs from the GFC to the recent market turmoil in China in 2015. Figure 4 is from their paper[6].

Seemingly, this phenomenon should sound puzzling . EMEs had built huge stocks of reserves since mid the 90's, incurring nonnegligible costs of holding reserves. When the chance to use reserves came, they didn't use it aggressively as one had expected. But, from the fact 3, we know reserves were effective through some channels. Now what we know is that it was effective even though it wasn't used much. A natural conclusion from these facts is that there exist some benefits of just having reserves, and hence the fear of losing reserves is the "fear of losing the benefit". We argue that the benefit is liquidities from just holding reserves: international reserves work as a collateral.

Fact 5: The relation between capital controls and the effectiveness of international reserves Because of technical toughness in such empirical works, there are not many other evidences or counter-evidences on the relation between capital count openness and the effectiveness of reserves management on a precautionary purpose. Nevertheless, one interesting empirical finding in Bussiï¿œere et al., (2015) is that during the GFC, international reserves were more helpful for the EMES whose capital accounts are more closed. This finding is related with our second question in this paper; how are macroprudential capital controls related with international reserves management? Our theoretical prediction is in the same line with a finding in Bussiere et al., (2015).

2.2 Discussion of existing models of international reserves

Here, we discuss about the approaches in the models in the preceding works, and point out how the models miss some of the facts we listed above. Some relatively early papers in the literature before the GFC presumed that the purpose of holding reserves is to use reserves in international trades after a default or in any other extreme situations. Alfaro and Kanczuk (2009) is one of the well-known papers in this fashion. They argued that in the precautionary purpose, the optimal reserve holding is zero since higher reserve holding increases the incentive to default; the EME can sustain longer relying on reserves after the default. Now, it seems that their underlying assumption is incorrect. Their claim is just opposite to the fact 3; the EMEs with hither reserves performed better during the GFC. What is more serious counterfactual in the claim by Alfaro and Kanczuk is that we can't see many defaults in the worlds where EMEs hold a lot more reserves than before.

[6]We like to note for readers that the tendency of the hesitance was pervasive among EMEs, the magnitude was surely much heterogenous across EMEs. For the details of how different the usage of reserves during an event of capital outflows is across EMEs, we refer readers to Basu et al. (2016).

Another main criticism on reserve holding on the precautionary purpose comes from Ricardian Equivalence Theorem (RET below). That is, rational private agents recognize that the governmental budget constraint is ultimately part of their own budget constraint, and raise more borrowing responding to more reserves, which will be given to them in the future. (See Korinek, 2018). To see this more clearly, let's assume that private agents in an EME can raise foreign borrowings at a given interest rate as much as they want, and there is no credit constraint for the agents. In addition, we assume the yield on reserves is same as the borrowing rate of the private agents. Then, it is straightforward that private agents exactly offset the reserve accumulations of the government; they increase borrowings by the same amount of increased reserves. As hinted in this rather extreme example, the RET arises because reserves are nearly perfect substitutes to borrowings. Hence, to make reserves accumulation effective, we need to make reserves non-substitutable to foreign borrowings.

One way to make reserves non-substitutable is to build a Diamond-Dybvig style model, which assumes borrowers invest money in long-term projects. In this type of model, the amounts of borrowings significantly depend on the availability of profitable projects. Acharya and Krishnamuthy (2018) belongs to this category. Although it is a wise and efficient modelling strategy, one can question whether the reason EMEs often rely on external liabilities is to invest in long-term projects. Furthermore, these models are silent about the forth fact — the fear of losing reserves.

Another way is to allow the EME in the model to issue long term bonds. Bianchi et al. (2018), influenced by Arellano (2008), constructed a model that EMEs can issue long-term bonds. In this environment, international reserves give the EMEs little liquidities, so that the chance of defaults become lower, and risk-premiums on the EMEs fall accordingly. Interestingly, by construction, if all the debts are short-term debts, the optimal level of reserves is zero. Surely, their model is innovating, and provided a new benefit of adequate reserve holding; reserves gift lower risk premiums. However, a question of the validity of the assumptions in the model naturally follows; aren't many EMEs relying on short-term debts? The data shows at least we need to be cautious at the assumption in Bianchi et al. (2018). Furthermore, Bianchi et al. (2018) is still silent about the fact 4; fear of losing reserves.

A different way to model reserves as an effective policy tool against sudden stop crises is to appeal to the existence of multiple equilibria. Hernandez (2017) and Bocola and Lorenzoni (2018) followed this way. A very noticeable work is Bocola and Lorenzoni (2017), which suggests an elegant model, borrowing some features from Gertler and Kiyotaki (2009). In their paper, reserves prevent an undesirable equilibrium among the equilibria from being realized although reserves are not depleted. Therefore, their model captures the fact 4. However, their model explicitly assumes a significant dollarization, which might be a good description of many EMEs in Latin America and East Europe, but perhaps not in Asia.

Other papers worth mentioning here are Jeanne and Ranciere (2011) and Shousha (2017). Jeanne and Ranciere chose to model reserves as a sort of Arrow Debrew security to answer a question "What would be an adequate level of reserves". Another paper by Shousha, which is closely related to our paper, assumes that reserves work as a collateral, so that EMEs accumulate reserves to use as collaterals during sudden stop crises. The model by Shousha can explain the facts 1 through 4. However, there is no externality in his model, so we can't discuss other macroprudential policies in the presence of international reserves. Moreover, his model misses the facts; the gradual accumulation of reserves, and the weakly positive relation between NFA and the amount of reserves.

3 Three Periods Model

In this section, we present a relatively simple three periods model. The purpose of this simple model is to derive analytic 'pen and paper' results through which readers can see some intuition on how the mechanism works. Also, we discuss key assumptions in our model that gives us results that resemble the facts in the previous session. We will present numerical solutions of the model and an extension in the next session.

3.1 Model Setup

We consider a small open economy with three periods, t = 0, 1, 2. There are three agents in our model: domestic households, a domestic government, and international investors. Throughout this paper, we abstract from production. Hence, the economy in our model is an endowment economy. The small open economy faces a credit constraint only in period 1[7], which may or may not bind depending on the states. Thus, on a precautionary purpose, government wants to accumulate reserves in period 0 when there is no concern over the binding credit constraint

Households The overall utility of the representative agent i given by

$$U = u(c_0^T, c_0^N) + \beta u(c_1^T, c_1^N) + \beta^2 u(c_2^T, c_2^N) \tag{1}$$

where the utility function $u(c_t^T,\, c_t^N) = ln\left(\left(c_t^T\right)^{\alpha}\left(c_t^N\right)^{1-\alpha}\right)$. Following the tradition, α and $1-\alpha$ are the shares of tradable goods and non-tradable goods respectively, and β is the discount rate. For the simplicity, we use log-utility function.

The households enter period 0 with a stock of bond b0 and receive endowments of tradable goods yt,0 and non-tradable goods yn,0. For simplicity, we assume that the output stream is deterministic, that is, y_t^T and y_t^N are given without any uncertainty. We let $y_0^T < y_1^T < y_2^T$ while setting $y_0^N = y_1^N = y_2^N$. Hence, the representative household has a strong incentive to borrow against incomes in the future.

Besides, we denote the relative price of non-tradable good to tradable good in period t by p_t. Therefore, pt is a measure of the country's real exchange rate in period t. We let $b < 0$ correspond to borrowing. −b is the face value of the bond[8]. The government will impose lumpsum tax to accumulate reserves or grants the reserves to households. We denote it by T. Hence, the budget constraint of the representative household in period t is given by

$$c_t^T + p_t c_t^N + b_{t+1} + T_t = y_t^T + p_t y_t^N + b_t\left(1 + r_t\right) \tag{2}$$

Credit constraint Households face a credit constraint in period 1. That is,

$$-b_1\left(1 + r_1\right) \le \phi\left(y_{T,1} + \psi p_1 y_{N,1}\right) + \theta IR_1 \tag{3}$$

Without the second term in the right hand side θIR_1, the credit constraint is just same as the standard specification in sudden stop literature (e.g., Bianchi, 2011; Korinek, 2018).

[7]This is same as Korinek and Sandri (2016). We can put the credit consraint in period 0. But, this makes it harder to solve the model, without providing any extra insight.

[8]This is slightly different from the convention that b is the book value of the bond. We deviate from the convention because the yields on the bond will be determined in the foreign exchange market, and therefore letting b be face value is more convenient for us.

The idea of the standard specification of credit constraint in EMEs is that the borrowers in an EME may default on their external borrowing, but in such cases international investors can take some properties in the country, which actually prevents a default of the EME. Since the international investors can't fully utilize the properties in the EME, the international investors should discount the values of the collateral.

We add two distinctive assumptions to this standard form. First, holding reserves can relax the credit constraint by the amount of θIR_1. And furthermore, θ is slightly larger than 1, which implies reserves give a little leverage to the holders. This modification is originally suggested by Shousha (2017), which argues the main role of foreign reserves is to function as collateral during sudden stop events. Since it is one of the most important assumptions in this paper, we will separately discuss the details of the assumption. Second, in our model, ϕ is stochastic. It is frequently argued that an important driver of sudden stops is a change in the amount of funds that international investors are willing to provide for a given amount of collateral, i.e. changes in the leverage parameter ϕ. A few theoretical works in macro-finance, such as Geanakopolos (2008), and Brunnermeier and Pederson (2009) document such pro-cyclical leverage ratios as a general feature of financial markets. It isn't necessary, but for the convenience and tractability, we assume ϕ has a support of an interval, and further its cdf and pdf are both continuous. We discuss more details of these two new specifications in the next sub-session.

Foreign exchange market and international investors Unlike few recent papers (Gabaix and Maggiori, 2015; Cavallino, 2018) that pioneered foreign exchange market interventions in imperfect foreign exchange markets, we do not explicitly model foreign exchange market here. Instead, following Fanelli and Straub (2018), we will assume that the bond market clearing conditions will simultaneously determine the real exchange rate and interest rates on the foreign borrowing. In addition, to simply further from the specification in Fanelli and Straub (2018), we abstract from domestic intermediaries. The key feature we want to deploy here is finitely elastic foreign capital supplies to arbitrages. In the literature of macroprudentical capital controls, it is assumed that EM intermediaries can borrow as much as they want at a given interest rate as long as the borrowing doesn't hit borrowing constraints. The implicit assumption behind this is that once the arbitrages occur it will be quickly eliminated since investors will rush to enjoy the arbitrates. In contrast, in an environment where portfolio of investors can't be adjusted quickly because of some transaction costs, asymmetric information, or any others, the arbitrage remains. If it is so, then it would be natural to think that capital inflows from foreign investors would increase with the remaining arbitrage. In a more familiar expression, the capital supply curve from international investors shall be upward slopping in a quantity-price space[9].

Here we introduce a more formal microfoundation of the assumption, following Fanelli and Straub (2018)[10]. There exists a continuum of international investors, labeled by $j \in [0, \infty)$. Assume there are two important restrictions to the investors' investment decisions First, each intermediary is subject to a net open position limit $X > 0$. Second,

[9]Another way that makes foreign exchange market interventions effective is to fully model the interventions by central banks, so that the interventions can ease the financial constraints of domestic intermediaries in a different way from our paper. For more details of it, we refer readers to Chang (2018). There is an interesting point of foreign exchange market interventions about the limited capital of central banks that reminds the episode in Korea in 2007.

[10]Their microfoundation isn't fully developed by themselves. Mainly, they followed Alvarez, Atkeson and Kehoe (2009), and borrow some from Bachetta and Van Wincoop (2010).

each investor faces heterogeneous participation costs. In particular, each investor j is obliged to pay a participation cost of exactly j. Thus, an investor j optimally invests an amount $x_{j,t}$, solving

$$\max_{x_{j,t} \in [-X,X]} x_{j,t} \left(r_t - r_*\right) - 1_{\{x_{j,t} \neq 0\}} j$$

Investor j's present value of net profits conditional on investing is $X \mid r_t - r_* \mid$, so investing is optimal for all intermediaries $j \in [0, \xi)$ with the marginal intermediary ξ given by $\xi = X \mid r_t - r_* \mid$. This gives an aggregate investment volume of

$$-b_t = \xi X \cdot sign \left(r_t - r_*\right)$$

Defining $\Gamma = \left(X^2\right)^{-1}$ and substituting $\bar{j}$, we obtain

$$-b_t = \frac{1}{\Gamma} \left(r_t - r_*\right) \tag{4}$$

Government The government accumulates international reserves in period 0 when there is no concern over the binding credit constraint. To finance the accumulation of reserves, the government imposes lump sum taxes by the amount of (capital T) units of tradable goods. With the revenue from the tax, the government purchases foreign bonds in period 0, which will earn $1 + \bar{r}$ units of tradable goods in period 1 per one unit of the bond. Accordingly, the dynamics of international reserves holding is given by

$$T \left(1 + \bar{r}\right) = IR_0$$

Of course, T should be positive. If the parameter phi in period 1 is significantly high so that the credit constraint is slack, then the government will fully deplete its reserves; depleting reserves by gifting the reserves to the households, since there is no reason for the government to hold costly reserves. Unfortunately for the EME, if the realized phi is so low that the credit constraint binds, then government has to choose whether it holds reserves to ease the credit constraint or deplete reserves to subsidize households. We found that it is convenient to denote the proportion of reserves depletion in period 1 by μ. Hence, the reserves in the period 1 will be

$$IR_1 = IR_0 \left(1 - \mu\right)$$

In the last period, government deplete all the left reserves. Meanwhile, the budget constraint of a household in period 1 can be expressed as below.

$$c_1^T + p_1 c_1^N + b_2 = y_1^T + p_1 y_1^N + b_1 \left(1 + r_1\right) + \theta \left(1 - \mu\right) IR \tag{5}$$

From the equation (5), we denote IR_0 by just IR and accordingly $\theta \left(1 - \mu\right) IR$ for IR_1.

3.2 Discussion of assumptions

Reserves as a collateral One of the most important assumptions in this paper is that international reserves could work as a collateral so that holding reserves can ease credit constraints in Mes. The first benefit of this assumption is that reserves as a collateral can reconcile few seemingly puzzling facts on international reserves. As it was discussed in the last section, Memes are holding massive amounts of international reserves and external liabilities simultaneously. Considering the spreads between the yield on reserve assets and the yields on the external liabilities, it might worsen the external position of EMEs. However, as we also have seen in the last section, it seems that EMEs with more reserves weathered the GFC in a better shape. Moreover, it is also widely documented that EMEs are usually hesitant to use their reserves to stabilize FX markets during sudden stop crises. We will show how reserves as a collateral can explain these stylized facts.

Some readers may not be familiar with the setup that financial assets are used as a collateral. However, many financial assets, in particular the US treasury bonds, are widely accepted as collaterals in financial markets. Such trades are especially active in repo markets. If EMEs have tremendous amounts of the US treasury bonds, there is no reason that a similar mechanism can not be applied to the reserves. There are several excellent theoretical works shedding light on the mechanism of how liquid financial assets can works as a collateral (e.g., Gottardi et al., 2016; and Parlatore, 2018). The paper that is tightly linked to our assumption is Parlatore (2018) in which the author focuses on how liquid financial assets are used as a collateral rather than are sold to raise funds. The key insight in the paper is that borrowers value the financial assets more than lenders because of the borrowers' non-observable investment opportunities. In our paper, we don't introduce such a asymmetric investment opportunity, but the borrowers, EMEs, value their reserves more than international investors: the US dollar or other key currencies are the medium of cross-border financial transaction and international trades, and in our setup, EMEs don't have unlimited access to the key currencies. In other words, it is costly for EMEs to gather reserves under limited access to the liquidity of key currencies whereas international investors, who are doing every transactions using key currencies, have easy access to the liquidity. This asymmetric access to liquidity lead us to the assumption that international reserves allow their holders to raise a leverage: the parameter θ in front of reserves in credit constraints is larger than 1.

Furthermore, this assumption corresponds to some behaviors of investors and beliefs in markets. One of the conditions that major credit agencies check when they assess the country risk of an emerging market country is the amount of international reserves. And the amount of reserve holding is one of the conditions shown up typically in a catalogue to get customers to join a mutual fund of investing in EMEs[11].

One serious difficulty in the assumption of reserves collateral is the legal structure of international reserves. As it is noted Alfaro and Kanczuk (2009b), which rejected the idea that reserves can be used as collateral, under the Foreign Sovereign Immunities Act of 1976 of the United States and similar laws in other countries, central bank assets, including international reserves, are usually protected against attachment. This means that reserves can be accepted as collateral only if EME governments are willing to pledge as such. Our first answer to such a possible criticism is that it is very unrealistic for many EMEs to default, leaving much reserves. If the scenario that EM countries declare a default, keeping their reserves to enjoy it is a realistic case, then we should see more defaults in this world where EMEs have much more reserves than before. Perhaps, it is because it turns out for many countries that the economic and political costs of currency crises or defaults are much higher than thought previously, as mentioned in Benigno et al. (2016). Our second answer is that for some countries that are more prone to default, reserves may not work as collateral, or although it works, the parameter theta shall be less than for those countries, in which case it is meaningless to accumulate reserves to use them as collateral. Admittedly, in countries where the quality of institution is

[11] For example, in South Korea, one can sign up for the mutual fund to make an investment in Brazilian government bond markets, and in a catalogue of the mutual fund, it is clearly written "Such amount of international reserves in Brazil significantly reduce any risk in our investment"

low or the political cost of defaults isn't so high, reserves can't be used as a collateral. Argentina, which defaulted in 2001, is holding a low level of reserves comparing with other Latin American countries, while the county is also owing relatively low liabilities than the others[12].

No default Some readers might question if it is fair not to allow a default in a model of international reserves. Traditionally, the possibility of default in EMEs and the risk premium on the debts of EMEs have been an important issue in the literature of EM economies (e.g., Arellano, 2008). In this paper, we abstract from a default. Although almost all EMEs are not totally free from the possibility of default, most of events when EMEs had a tough time (e.g., the GFC, after the announcement of QE tapering in 2013, and the recent market turmoil in May 2018) didn't result in a default or substantial hair-cuts on the debts. Our view is that the holding international reserves on a precautionary purpose is mainly for handling capital drains during sudden stop crises. In this paper, we want to explain the facts in the countries where substantial reserves have been accumulated. That might imply that political costs of defaults or sudden stops are significantly high in the countries, which makes it hard for any default to be realized in those countries. Relatedly, risk premium is an also important factor for the sustainability of the external position in EMEs. Like the conclusion in Bianchi et al. (2018), another benefit of enough international reserves might be lower the sovereign debts risk premiums. Just for simplicity and tractability, we abstract from the potential effects of reserves on risk premiums. However, the quantitative analysis in Bianchi et al. (2018) may imply the lower risk premium is one of the benefits of having much reserves, but it might not be the most important motivation[13].

Stochastic Leverage Ratio The important parameter phi in credit constraints follows an exogenous stochastic process, hence the credit constraint in our paper is time-varying. Such description of the credit constraint isn't new in the literature (Acharya and Krishnamurthy, 2018; Korinek, 2018; Shousha, 2018). The exogenous change of the phi mostly reflects "global financial shocks" – changes in global financial market conditions. Like Shin (2012), and Bruno and Shin (2015), the change in global financial market conditions, which may stem from center economics, would cause changes in risk appetites of international investors. For example, when the conditions in global financial market become worsen, the total risk that the investors can take will be cut down, and therefore the investors will ask EMEs to provide more collaterals.

On the contrary with the phi, the parameter of reserves collateral is assumed to be invariant. It is well-known among practitioners and in the literature (e.g., Caballero and Krishnamurthy, 2008; Baele et al., 2013) that devastate market turmoil leads investors to demand safe assets more than during a tranquil time (Flight to Safety). In that sense, it is not odd to assume the value of reserves as collateral doesn't drop along with phi. Actually, it could be more realistic to assume that theta rises when phi drops; since reserves are mostly composed of safe assets in key currencies, the value would rise as a negative global financial shock occurs. However, to avoid further complexity, we keep theta as a constant through this paper.

[12] After the announcement in by Fed in May 2018 that it will gradually normalize its monetary policy, Argentina government asked IMF to bail-out the country.

[13] In their paper, the calibrated adequate level of reserves is 6% on average, which is much less than observed levels in most countries

3.3 Solving the Model

Now we solve the model. First, we solve the model on the assumption there isn't any capital control. Since it is a three periods model, we solve via backward induction. As usual in any paper that solves a Ramsey problem, we first solve the problem of private agents, and then solve for the solution of social planner. Next, we introduce the capital control on the borrowing, and then compare the solution with capital control to the former case – the solution without capital control.

3.3.1 Equilibrium without Capital control

Utility maximization of households By construction, there is no dynamic decision of households in the last period. Households consume all the endowments of non-tradable goods and remaining all the tradable goods after paying back all the debts and receiving remaining reserves from government (if it is given). That is,

$$c_2^T = y_2^T + b_2\left(1+r_2\right) + IR(1-\mu)\left(1+\bar{r}\right),\ c_2^N = y_2^N$$

In period 1, the households take the states of economy as given and solve the utility maximization problem. It is important to note that the states include ϕ. As we emphasized earlier, ϕ is a random variable whose value is determined at the beginning of the period 1. A difficult question is "what would be a good distribution of ϕ that resembles the reality?" To materialize an idea of a disaster, it seems a positively skewed distribution would be good, i.e., its pdf has a left-leaning curve. Here, we only assume that the distribution has a support on a interval of positive real numbers. That is, the support of ϕ is $\left[\underline{\phi}, \overline{\phi}\right]$ where $\underline{\phi} > 0$ and $\bar{\phi} < \infty$and . Also, let's suppose it is nicely well-defined, so that we don't have any trouble of using calculus. The utility maximization is formally defined as below.

$$\max_{c_t^T,\, c_t^N,\, b_t} u\left(c_0^T,\ c_0^N\right) + E\left[\beta u\left(c_1^T,\ c_1^N\right) + \beta^2 u\left(c_2^T,\ c_2^N\right)\right] \qquad \textit{subject to}$$

$$\begin{aligned}
c_0^T + p_0 c_0^N + b_1 + T &= y_0^T + p_0 y_0^N \\
c_1^T + p_1 c_1^N + b_2 &= y_1^T + p_1 y_1^N + b_1(1+r_1) + \mu IR \\
c_2^T + p_2 c_2^N &= y_2^T + p_1 y_2^N + b_2(1+r_2) + (1-\mu)IR(1+\bar{r}) \\
-b_2(1+r_2) &\leq \phi(\omega)(y_1^T + \psi p_1 y_1^N) + \theta(1-\mu)IR
\end{aligned}$$

where $u(c_t^T,\ c_t^N) = ln\left(\left(c_t^T\right)^\alpha \left(c_t^N\right)^{1-\alpha}\right)$ and $IR = T(1+\bar{r})$.

Market clearing conditions will be given by the pricing functions. +

$$p_t = \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{c_t^T}{c_t^N}\right) \quad \textit{and} \enspace r_t = -\Gamma b_t + r_*$$

Individual households in a decentralized equilibrium don't take the impact of their consumption on those prices, from which negative externalities of tradable goods consumption are generated. See the negative externality through the interest rate exists as long as the households are borrowers, whereas the externality through the real exchange rate p_1 does exist only if the credit constraint binds. We make it clearer as we progress.

If the credit constraint doesn't bind, i.e., the realized phi is high enough, then the household determines its consumption of tradable goods according to her Euler equation.

The amount of the borrowing in period will be determined by

$$-b_2 = \frac{-\beta\left(1+r_2\right)\left(y_1^T + b_1\left(1+r_1\right)\right) + y_2^T - \ \left(\beta\left(1+r_2\right)\mu - \left(1-\mu\right)\left(1+\overline{r}\right)\right)IR}{\left(1+r_2\right)\left(1+\beta\right)} \quad (6)$$

The interest rate r_2 is accordingly determined by $r_2 = -\Gamma b_2 + r_*$.

If the credit constraint binds, the consumption of tradable goods will be determined by the credit constraints. The constraint is $c_{T,1} = y_{T,1} + b_1(1+r_1) - b_2 + \mu IR$ and $b_1\left(1+r_1\right)$ were determined in the period 0. Plugging these into the credit constraint equation, we can derive[14]

$$-b_2 = \frac{-\left(1+r_*\right) + \phi\psi\left(\frac{1-\alpha}{\alpha}\right) + \sqrt{\left(1+r_* - \phi\psi\left(\frac{1-\alpha}{\alpha}\right)\right)^2 + 4\Gamma\kappa}}{2\Gamma} \quad (7)$$

$$r_2 = \frac{1+r_* + \phi\psi\left(\frac{1-\alpha}{\alpha}\right) + \sqrt{\left(1+r_* - \phi\psi\left(\frac{1-\alpha}{\alpha}\right)\right)^2 + 4\Gamma\kappa}}{2} \quad (8)$$

where $\kappa = \phi\left(y_1^T + \frac{1-\alpha}{\alpha}\psi\left(y_1^T + b_1\left(1+r_1\right) + \mu IR\right)\right) + \theta\left(1-\mu\right)IR$. For algebras, refer to the Appendix. Readers might be interested in economic intuitions on the results above. We discuss several economic intuitions as we move forward.

Since ϕ has a support of an interval, we can derive a formula of the cut-off of ϕ, below which the credit constraint binds, given other states and government policies. We can obtain

$$\phi^c = \frac{-\beta\left(1+r_2\right)\left(y_{T,1} + b_1\left(1+r_1\right)\right) + y_2^T - \ \left(\beta\left(1+r_2\right)\mu - \left(1-\mu\right)\left(\left(1+\overline{r}\right) + \theta\left(1+\beta\right)\right)\right)IR}{\left(1+\beta\right)\left(y_1^T + \psi\frac{1-\alpha}{\alpha}\left(y_1^T + b_1\left(1+r_1\right) + \mu IR\right) + \theta\left(1-\mu\right)IR\right)} \quad (9)$$

where r_2 is determined by (8).

We now turn to the period 0 optimization problem. Since there is no credit constraint, decentralized households optimally choose the consumption of tradable goods, so the amount of borrowing $-b_1$, while taking all the states, government policies, prices given and expecting the solutions of the households in the future as above. The solution is the Euler equation. By denoting the marginal utility of good x in period t as u_t^x ,

$$u_0^T\left(b_1,\ ;\right) = \beta\left(1+r_1\right)\left(\int_{\underline{\phi}}^{\phi^c} u_1^T\left(b_1,\ b_{2,c},\ ;\right)d\mathcal{F}_\phi + \int_{\phi^c}^{\overline{\phi}} u_1^T\left(b_1,\ b_{2,u},\ ;\right)d\mathcal{F}_\phi\right) \quad (10)$$

See the b_2 in the first term in the right hand side is determined by (7), while the b_2 in the second term is determined by (6).

We can easily notice the amount of borrowing in equation (10) isn't socially optimal. The borrowing b_1 will impact the interest rates r_1, r_1, and p_1. The pecuniary externalities of b_1 through the real exchange rate p_1 has been at the center of researches of pecuniary externalities in small open economies[15]. Additionally, there are other pecuniary externalities through the interest rates, of which individual households don't take

[14] To have a unique equilibrium, we need a condition $\phi\frac{1}{1+r_2}\left(\frac{1-\alpha}{\alpha}\right) < 1$. Here, it is easily satisfied since the credit constraint only binds for low values of ϕ

[15] The externality through real exchange rates is magnified by the financial amplification mechanism defined in the literature (see Korinek and Davila, 2018). When the interest rate is given, the financial amplification multiplier is nicely characterized by (Korinek, 2018, p10), abbreviating the derivative of the exchange rate function by p'. Because the interest rate is a function of the amount of borrowing in our model, we can't get such a nice characterization. However, in the states in which the credit constraint binds, the externality through real exchange rates is quantitatively dominating.

account: marginal consumption of tradable goods pushes up the interest rates on the foreign borrowing. We summarize this point below, with other findings.

Lemma 1. *1. Let* b_1^{priv} *be the solution of (10), and* b_1^{sp} *be the solution by a Social planner. Then* $b_1^{sp} > b_1^{priv}$ *as long as* $\phi^c > \underline{\phi}$ *or* $\Gamma > 0$.

2. Assume ϕ^c *is located in the left tail of the distribution of* ϕ*. Further we assume f'>0 for all* $\phi < \phi^c$*, where f is the pdf of* ϕ*. Then, we can show*

i) $\mid b_1^{sp} - b_1^{priv} \mid$ *decreases in the 1st moment of* ϕ.

ii) $\mid b_1^{sp} - b_1^{priv} \mid$ *is not monotone in the 2nd moment of* ϕ.

iii) $\mid b_1^{sp} - b_1^{priv} \mid$ *is not monotone in the 3nd moment of* ϕ.

Proof) See the Appendix.

The first claim in the lemma is well-known in the literature and intuitive. Once we describe ϕ as a random variable with a certain distribution, one following question is "How related are the characteristics of the distribution with market inefficiencies or corresponding policies?" The lemma above gives some hints to find answers to the question. The amount of overborrowing increases as the whole pdf of ϕ shifts left. This is intuitive: a low average and ϕ implies it is more likely to face sudden stop crises in the future, perhaps in a tougher way. The second and third claims in the lemma might look seemingly counter-intuitive. This is because, given the first moment of ϕ, the higher variance and the less left skewedness (more right skewedness) should lessen financial amplification effects, which increases in ϕ. In overall, the lower ϕ yields to a more severe crisis, but the effect will be offset by the lower negateve financial amplification effects. In short, the magnititude of the externality isn't necessarily monotone in ϕ. For more details, we refer readers to the proof of lemma 1 in the appendix.

Planning problem of the government Now we proceed to our main interest, planning problem of the government. Until now, we haven't introduced capital controls, hence the only policy tool for the government is to accumulate international reserves in period 0 and then use adequately in period 1. Like households' problem, we solve vis backward induction.

We formulate the government problem in period 1 as below.

$$\max_{\mu} u\left(c_1^T,\, c_1^N\right) + \beta u\left(c_2^T,\, c_2^N\right) \qquad \textit{subject to}$$

$$\begin{aligned}
-b_2 &= \begin{cases} \frac{-\beta(1+r_2)\left(y_1^T+b_1(1+r_1)\right)+y_2^T\ -\ (\beta(1+r_2)\mu-(1-\mu)(1+\overline{r}))IR}{(1+r_2)(1+\beta)} & if \quad \phi \in [\underline{\phi}, \phi^c] \\ \frac{-r_*+\phi\psi\left(\frac{1-\alpha}{\alpha}\right)+\sqrt{\left(r_*-\phi\psi\left(\frac{1-\alpha}{\alpha}\right)\right)^2+4\Gamma\kappa}}{2\Gamma} & if \quad \phi \in [\phi^c, \phi^c] \end{cases} \\
c_1^T &= y_1^T + b_1(1+r_1) - b_2 + \mu IR \\
c_2^T &= y_2^T + b_2(1+r_2) + (1-\mu)\, IR\, (1+\overline{r}) \\
c_t^N &= y_t^N \\
r_t &= -\Gamma b_t + r_*
\end{aligned}$$

As it is noted earlier, if phi is high enough so that the credit constraint does not bind, then there is no reason for the government to hold any reserve (We assumed

$y_0^T < y_1^T < y_2^T$). Hence, the states to which we pay attention are when phi is so low that the government needs to take some actions to ease the credit constraint. To make the problem interesting, we impose

ASSUMPTION 1 $\frac{\partial(-b_2)}{\partial\mu}\mid_{\phi\leq\phi^c}< 0$ and furthermore there exists $\phi \in \left(\underline{\phi}, \phi^c\right)$, say ϕ^d, such that $\frac{\partial(-b_2)}{\partial\mu}\mid_{\phi\leq\hat{\phi}^d}< -IR$

The first condition implies that the amount of borrowing in the states of binding credit constraints decreases with reserves depletion. With most of the sets of reasonable parameter values, assumption 1 is easily satisfied. The second condition is slightly more restrictive. The amount of the additional borrowing by using reserves as a collateral is more than the consumption that households can have by just depleting reserves. In other words, reserves give the EME a little leverage. We need this assumption to make sure that there will be a case in which the government holds reserves to have more liquidity, using reserves as collateral rather than depleting reserves to subsidize households directly. Holding assumption 1, by (4), this implies interest rates on the borrowing decrease with reserves depletion. More explicitly,

$$\frac{\partial{-r_2}}{\partial\mu}\mid_{\phi\leq\phi^c}< 0$$

In short, holding reserves as collateral gives more liquidity, but the cost of it is more interest payments in the future.

We also impose one more assumption to prevent a theoretically possible, but empirically unrealistic result. We assume government will never enlarge its reserves during a sudden stop crisis. That is, μ is a nonnegative real number. Because of political difficulties or market pressures, accumulating more reserve during a sudden crisis might not be a feasible option for governments. Moreover, with reasonable values of parameters, the chance on which governments will be forced to accumulate reserves in a crisis is very limited.

ASSUMPTION 2 $\mu \in [0, 1]$

Let $\hat{\phi}$ be the the parameter value of ϕ such that μ^* reaches its floor – zero. Now we present our first main finding.

Proposition 1. *1. As long as the credit constraint doesn't bind, $\mu^* = 1$. That is, the government fully depletes its reserves.*

2. The solution $\mu^*$is characterized as follows,

i) if $\phi \in [\underline{\phi}, \hat{\phi}]$, then $\mu^* = 0$ and the credit constraint binds.

ii) if $\phi \in [\hat{\phi}, \widetilde{\phi}]$, then $\mu^* \in [0, 1]$ and the credit constraint binds. Furthermore, μ^* increases in ϕ.

iii) if $\phi \in [\widetilde{\phi}, \phi^c]$, then $\mu^* = 1$ and the credit constraint binds.

iv) if $\phi \in [\phi^c, \overline{\phi}]$, then $\mu^* = 1$ and the credit constraint doesn't binds.

Proof) See the Appendix.

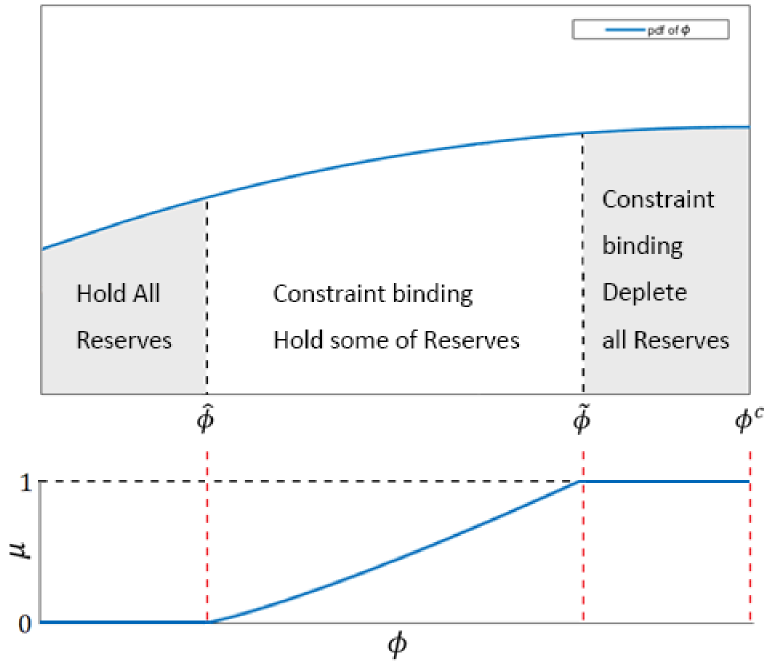

Figure 6: Partial Reserves Depletion during a Sudden Stop Crisis

Key implication from the proposition is very intuitive: the lower phi is so that households have tighter credit constraints, the more liquidity government needs to provide, which requires less reserves depletion (holding more reserves). Another interesting finding is that the government deplete reserves as long as the credit constraint doesn't bind. That is because the borrowing decision of households doesn't take account of its effect on the interest rate. If the credit constrain becomes unbinding thanks to the international reserves, then the government immediately deviates from the state by depleting more reserves. Since it is socially beneficial to suppress the over-consumption of tradable goods, the cut-off of ϕ, below which the credit constraint binds, in the equilibrium is lower than the cut off under the policy of holding reserves fully. The interval on which reserves are fully depleted, but the constraint binds emerges because the Assumption 2; facing a very exterme capital outflows, the EME needs to buy more reserves, but the only option is to hold all the reserves since it is impossible to tax households in a crisis.

The reserve depletion policy, choice of μ is decided in period 1. Then here arises an usual question in a similar problem. If the decision of μ can be pre-determined, is it going to be same with the decision in the decision made in period 1? In a more familiar expression, does time-inconsistency problem exist here? To see this point, take the first order condition of reserves depletion problem in period 1, while assuming the solution is an interior solution.

$$u_1^T\left(b_2\left(\mu,;\right),\mu,;\right)=\left[\beta\left(1+r_2\right)-\Gamma b_2\left(\mu,;\right)\right]u_2^T\left(b_2\left(\mu,;\right),\mu,;\right) \tag{11}$$

On the other hand, suppose the government has a commitment power, and the decision of mu is made in period 0. Abusing the notation μ, it would be

$$u_1^T\left(b_2\left(\mu,;\right),\mu,;\right)=\left[\beta\left(1+r_2\right)-\Gamma b_2\left(\mu,;\right)\right]u_2^T\left(b_2\left(\mu,;\right),\mu,;\right)+\frac{\partial\left(-b_1\right)}{\partial\mu}\frac{\partial V}{\partial\left(-b_1\right)} \tag{12}$$

where $V=u\left(c_0^T,\,y_0^N\right)+E_0\left[\beta u\left(c_1^T,\,y_1^N\right)+\beta^2u\left(c_2^T,\,y_2^N\right)\right]$. The second term in right hand side in equation (12) is negative since individual households make a decision according to her Euler equation and the tradable goods consumption in period 1 weakly decreases in the reserve depletion[16]. We summarize and formalize this idea in the following lemma.

Lemma 2. *(Time inconsistency of reserves depletion) Let μ^* be the reserves depletion determined in period 1, hence the solution of (11), while μ^{**} be the reserves depletion in period 0, hence the solution of (12). Then $\mu^* < \mu^{**}$*

Proof) See the discussion above. For a more formal proof, see the Appendix.

Now we head to our main interest: international reserves accumulation in period 0. We don't suppose any commitment power for the remaining in this paper. Hence, the government decide the amount of reserves accumulation with expectations of the states in the future, decisions of households, and her own decision in period 1. The problem can be formulated as below.

$$\max_{IR} u\left(c_0^T,\, y_0^N\right) + E_0\left[\beta u\left(c_1^T,\, y_1^N\right) + \beta^2 u\left(c_2^T,\, y_2^N\right)\right] \qquad \textit{subject to}$$

$$\begin{aligned}
c_0^T &= y_0^T + b_0(1+r_0) - b_1 - T\\
c_1^T &= y_1^T + b_1(1+r_1) - b_2 + \mu IR\\
c_2^T &= y_2^T + b_2(1+r_2) + (1-\mu)\, IR\,(1+\bar{r})\\
b_1 &= \textit{the solution of } (10)\\
-b_2 &= \begin{cases} \frac{-\beta(1+r_2)\left(y_1^T+b_1(1+r_1)\right)+y_2^T \,-\, (\beta(1+r_2)\mu-(1-\mu)(1+\bar{r}))IR}{(1+r_2)(1+\beta)} & if \quad \phi\in\left[\underline{\phi},\phi^c\right]\\ \frac{-r_*+\phi\psi\left(\frac{1-\alpha}{\alpha}\right)+\sqrt{\left(r_*-\phi\psi\left(\frac{1-\alpha}{\alpha}\right)\right)^2+4\Gamma\kappa}}{2\Gamma} & if \quad \phi\in\left[\phi^c,\overline{\phi}\right]\end{cases}\\
\mu &= \begin{cases}\textit{the solution of } (11) & if \quad \phi\in\left[\underline{\phi},\phi^c\right]\\ 1 & if \quad \phi\in\left[\phi^c,\overline{\phi}\right]\end{cases}\\
r_t &= -\Gamma b_t + r_*
\end{aligned}$$

where $u(c_t^T,\, c_t^N) = ln\left(\left(c_t^T\right)^{\alpha}\left(c_t^N\right)^{1-\alpha}\right)$ and $IR = T(1+\bar{r})$.

Define $V = u\left(c_0^T,\, y_0^N\right) + E_0\left[\beta u\left(c_1^T,\, y_1^N\right) + \beta^2 u\left(c_2^T,\, y_2^N\right)\right]$. Then deriving the first order condition and working at algebra yields

$$\beta\int_{\underline{\phi}}^{\phi^c}\frac{\partial\left(-b_2\right)}{\partial IR}\left(u_1^T-\beta\left(1+r_2\right)u_2^T\right)d\mathcal{F}_\phi+\beta IR\int_{\underline{\phi}}^{\phi^c}\frac{\partial\mu}{\partial IR}\left(u_1^T-\beta\left(1+\bar{r}\right)u_2^T\right)d\mathcal{F}_\phi-\beta^2\Gamma\int_{\underline{\phi}}^{\overline{\phi}}b_2\frac{\partial b_2}{\partial IR}u_2^T d\mathcal{F}_\phi$$
$$= u_0^T\left(\frac{1}{1+r_*}\right)-\beta\left(\int_{\underline{\phi}}^{\phi^c}u_1^T\mu d\mathcal{F}_\phi+\int_{\phi^c}^{\overline{\phi}}u_1^T d\mathcal{F}_\phi\right)-\beta^2\int_{\underline{\phi}}^{\phi^c}u_2^T\left(1-\mu\right)\left(1+\bar{r}\right)d\mathcal{F}_\phi-\frac{d\left(-b_1\right)}{dIR}\frac{dV}{d\left(-b_1\right)} \tag{13}$$

where $\frac{dV}{\partial d(-b_1)} = \beta\Gamma\int_{\underline{\phi}}^{\overline{\phi}} b_1 u_1^T + \beta b_2\frac{\partial b_2}{\partial b_1}u_2^T d\mathcal{F}_\phi + \beta\int_{\underline{\phi}}^{\phi^c}\frac{\partial b_2}{\partial b_1}\left(u_1^T-\beta\left(1+r_2\right)u_2^T\right)$

$+IR\frac{\partial\mu}{\partial(-b_1)}\left(u_1^T-\beta\left(1+\bar{r}\right)u_2^T\right)d\mathcal{F}_\phi$.

All the terms in the LHS are the costs of the reserves accumulation, while the benefits are in RHS. To explain briefly, the main costs of reserves accumulation are more

[16]Such time-inconsistency is very usual in a model of ex-ante intervention. For related discussion, see Bianchi and Mendoza (2018), and Jeanne and Korinek (2018).

borrowing due to the presence of international reserves holding by government (Moral Hazard) and income losses caused by low yields on reserves. The most important benefit of holding reserves is to provide liquidity during a crisis: government directly subsidizes households by gifting the reserves or letting them borrow more by using reserves as collaterals.

To give detailed interpretations to the equation, the first term in the LHS is the cost related with moral hazards that households borrow more as the government accumulates more reserves: more borrowing will induce interest rates to rise, while decreasing the amount of borrowing in period 1,– b2. In addition, the increased borrowing will suppress the reserves depletion during sudden stop crises, which the policy maker in period 1 can't take account of. That is the term of Time Inconsistency we classified in Lemma 2. The second term in RHS is the marginal effect of the saving by the reserves accumulation. Since the yield on reserves is lower than the borrowing rates to households, the term is always positive (positive cost).

In the benefit side, the first term shows a marginal benefit of holding reserves by lowering borrowing rates in period 1[17]. The second term is the most important benefit of reserves. Holding reserves during a crisis does provide liquidity $(\partial(-b_2)/\partial IR > 0)$. In the second term, notice that $u_1^T - \beta(1+r_2)u_2^T$ must be positive in the states since the binding credit constraints constrain the consumptions in period 1 so that marginal utilities can't decrease to the point where two marginal utilities are same, which actually happens only if the credit constraint unbinds. Also, notice that the distance of the marginal utilities measures the severity of the crisis in the state; the bigger difference means less consumptions in period 0, equivalently a tighter credit constraint. Naturally, the liquidity from reserves is more valuable when facing a tighter borrowing constraint. In the third term, one can see having more reserves can give more looms to deplete reserves during a sudden stop crisis.

We reinstate above first order condition and other key insights in our second proposition.

Proposition 2. *Denote the optimal reserves accumulation by IR^*. Then $IR^*$is characterized by (13). In addition, IR^*decreases in $\frac{d(-b_1)}{dIR}$.*

Proof) Along with the derivation of (13), see the Appendix.

The second statement of the proposition indicates that the concerns over the moral hazard and the time inconsistency suppress the demand for reserves as long as the borrowing in period 0 is increasing in reserves accumulation, which is a true. The concern over moral hazards from the bail-out using international reserves is well-discussed in Acharya and Krishnamurthy (2018), in which the presence of the moral hazard leads to the assertion that using capital controls will increase the optimal amount of reserves holding because the macroprudential capital controls can eliminate the moral hazard; in our model, it is equivalent to setting $\frac{d(-b_1)}{dIR}$ to be zero. We will return to this point later, but here we like to note that although the reserves accumulation would lead to more borrowing, the magnitude in our model is very likely to be much smaller than results in other papers such as Acharya and Krishnamurthy (2018) and Korinek (2018). This is because the foreign exchange market interventions to build a stock of reserves should raise interest rates on the foreign borrowing. In other words, the market intervention

[17] Of course, in the states of binding credit constraints, the sing of (partial b2/partial IR) is negative. However, with reasonable probabilities that credit constraint binds, which is 10% at maximum, the overall sign of the term should be positive.

has a function of punishing the overborrowing by itself; strictly positive Γ in our model implies more demands for external borrowing to replenish the lost tradable goods should cause a higher marginal cost of the borrowing. To summarize, our result is qualitatively somehow similar with the preceding works, but quantitively very different. Since it is an important point, particularly at the respect of policy implications, we make it as a separate remark

Remark 1. the sign of $\frac{d(-b_1)}{dIR}$ is positive. Hence, the concern over Moral Hazard exists in our model. However, $\frac{d(-b_1)}{dIR}$ decreases in Γ, which implies the reserves accumulation purnishes the overborrowing through higher interest rates ($\frac{dr_1}{dIR} > 0$). As a result. the Morea Hazard effects will be weakened substantially.

To see the marginal impact of the reserve accumulation on the borrowing in period 0 more explicitly, we can derive an analytical expression. Invoking the envelope theorem at the Euler equation of households at period 0 yields[18]

$$\frac{d(-b_1)}{dIR} = \frac{u_0^{''} - \beta(1+r_1)E_0\left[u_1^{''}\left(\mu(1+r_1) + \frac{\partial(-b_2)}{\partial IR} - \frac{\partial\mu}{\partial IR}IR\right)\right]}{u_0^{''} - \beta(1+r_1)E_0\left[u_1^{''}\left(1+r_1 - \Gamma b_1 - \frac{\partial b_2}{\partial b_1}\right)\right] - E_0\left[u_1^{'}\beta\Gamma\right]} \tag{14}$$

where $u_t^{''} = \frac{\partial u_t}{\partial c_t^T}$. Obviously, higher Γ suppresses the responsiveness of the borrowing to the reserves accumulation. Also, in (14), we can explicitly identify the source of the moral hazard; the additional liquidity provided by reserves in the possible future crises raises the borrowing today[19] . We illustrate the equation (14) in the following figure 7. In a simple undergraduate level graph analysis, one can easily see how higher gamma induce the less responsiveness of borrowing to more reserves[20].

As a last point in this section, we illustrate the equation in a different way so that readers can see the second mechanism of how international reserves become a policy took on a precautionary purpose. The first mechanism illustrated in Proposition 2 is extra liquidities from international reserves when it is used as a collateral. The second mechanism in the corllary below is related the remark above, and it lays out how reserves accumulation raises the Net Foreign Asset of the EME so as to lessen the concern over a possible shortage in liquidity.

Corollary 1. *Define* $NFA = IR - b_1(1+r_1)$ *and suppose* $\frac{dNFA}{dIR} > 0$

[18]If (gamma) and (theta) is zero and the yield on reserves is same as the borrowing rates, then we will have $\frac{d(-b_1)}{dIR} = 1$. That is, the perfect Ricardian equivalence holds since the reserves and bonds are perfect substitutes in this special case.

[19]See $E_0\left[u_1^{''}\left(\mu(1+r_1) + \frac{\partial(-b_2)}{\partial IR} - \frac{\partial\mu}{\partial IR}IR\right)\right] = \int_{\underline{\phi}}^{\phi^c} u_1^{''}\left(\mu(1+r_1) + \frac{\partial(-b_2)}{\partial IR} - \frac{\partial\mu}{\partial IR}IR\right)d\mathcal{F}_\phi$ $+ \int_{\phi^c}^{\overline{\phi}} u_1^{''}\left(\mu(1+r_1) + \frac{\partial(-b_2)}{\partial IR} - \frac{\partial\mu}{\partial IR}IR\right)d\mathcal{F}_\phi$. In the first term in the right hand side, $\frac{\partial(-b_2)}{\partial IR}$ is large because reserves are used as collaterals in the states of binding constraint.

[20]This is similar with Beningo et al. (2016). They showed if a planner can manipulate real exchange rates, hence the cost of borrowing, then, by the manipulation, the planner can achieve a same result with imposing optimal capital controls. The main insight provided by them is there are multiple solutions for the planning problem as usual for Ramsey problem.

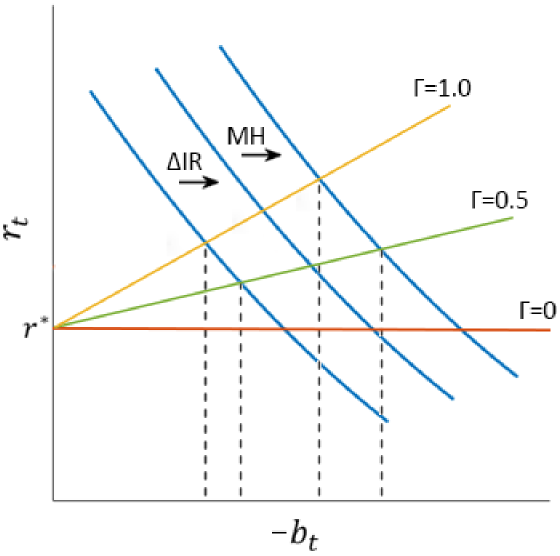

Figure 7: The impact of Γ on the magnititude on the moral hazard

The intervention by governments shifts right the demand curve. Obviously, higher Γ leads to less increase of the borrowing. If Γ is zero, then the borrowing will increase nearly as much as the size of the intervention.

2. Let $\theta = 0$. Then the first order condition in equation turns out to be

$$\beta \int_{\underline{\phi}}^{\phi^c} \frac{\partial (-b_2)}{\partial NFA} \frac{dNFA}{dIR} \left(u_1^T - \beta (1 + r_2) u_2^T \right) d\mathcal{F}_\phi - \beta^2 \int_{\underline{\phi}}^{\overline{\phi}} \Gamma b_2 \frac{\partial b_2}{\partial IR} u_2^T d\mathcal{F}_\phi =$$
$$u_0^T \left(\frac{1}{1 + r_*} \right) - \beta \int_{\underline{\phi}}^{\overline{\phi}} u_1^T d\mathcal{F}_\phi + \frac{d(-b_1)}{dIR} \beta \Gamma \left(\int_{\underline{\phi}}^{\overline{\phi}} b_1 u_1^T + \beta b_2 \frac{\partial b_2}{\partial b_1} u_2^T d\mathcal{F}_\phi \right) \quad (15)$$

For reasoable parameter values, IR^*>0 may solve the equation (15)

Proof) See the Appendix.

To explain the corollary 1, first of all, the market intervention by governments to accumulate reserves raises the saving of the whole nation by itself. That is, the EME has less net foreign liabilities in period 1. Obviously, then the chance onf a sudden stop crisis should be lower and the pain of a crisis should be less accordingly, given same amounts of capital outflows. Therefore, even if the reserves can't be used as a collateral, the optimal amount of reserves holding can be positive[21]. There may arise a question about the assumption $\frac{dNFA}{dIR} > 0$. Comparing with other models in which capital inflows are infinitely elastic to the interest rate, the assumption $\frac{dNFA}{dIR} > 0$ is easily satisfied in our model: actually, it is normally satisfied. with parameters at reasonable values.

To see why it is so, envoke the envelope theorem to the Euler equation of the households. Then we have

[21]One can notice the presence of overborrowing makes higher NFA socially desirable. Intuitively, the overborrowing implies that there remain too little resources for the future. This point is well-illustrated in the literature.

$$\frac{dNFA}{dIR} = \frac{u_0''\left(\frac{d(-b_1)}{dIR} - \frac{1}{1+\bar{r}}\right) - \beta\Gamma\frac{d(-b_1)}{dIR}E_0\left[u_1'\right]}{\beta\left(1+r_1\right)E_0\left[u_1''\left(\frac{d(-b_2)}{dNFA}+1\right)\right]} \tag{16}$$

The second term $-\beta\Gamma\frac{d(-b_1)}{dIR}E_0\left[u_1'\right]$ raises $\frac{dNFA}{dIR}$, and it appears because the borrowing rate in our model is linear increasing function of the amount of the borrowing in our model. The same intuition in the discussion of Remark 1 applies here; higher demands for borrowing is absorbed by the higher borrowing rates since the "supply curve" is upward slopping[22]

3.3.2 Equilibrium without Capital control

As we noted earlier, one of main interests in this paper is to find the relation between international reserves management and macroprudential capital controls. To be slightly more specific, are they complements or substitutes with each other? We answer this question in this section. Because of the assumption of limited arbitrage of international investors, which results in a finitely elastic supply of foreign capital, we introduce two different capital controls. First, we introduce the Pigouvian tax on foreign borrowings, which has been extensively explored in the literature. Second, we add capital controls in the literature of foreign exchange market interventions; governments can impose taxes (or subsidize) on purchases of domestic bonds by international investors. The easiest way to understand these two different policy tools is to think of the Pigouvian tax as a control of demands, while regarding the taxes on international investors as a control of supplies. Except for the introduction of the taxes, everything is same as the one in the previous section. Also, we assume government imposes taxes only in period 0. Although there is a need to impose the tax on the borrowing in period 1 in the states of unbinding credit constraint because the more borrowing leads to higher borrowing rates, we here discard it for the simplicity and it corresponds to the perception that the main purpose of capital controls is to prepare for the possibility of abrupt capital outflows (In our model, the credit constraint only exists in period 1)

Utility maximization of households Problems of households in period 1 and 2 are exactly same as before. In the presence of the tax on foreign borrowing, the Euler equation of the representative household in period 0 turns out to be

$$u_0^T\left(b_1,\ ;\right) = \beta\left(1+r_1+\tau_d\right)\left(\int_{\underline{\phi}}^{\phi^c} u_1^T\left(b_1,\ b_{2,c},\ ;\right)d\mathcal{F}_\phi + \int_{\phi^c}^{\overline{\phi}} u_1^T\left(b_1,\ b_{2,u},\ ;\right)d\mathcal{F}_\phi\right) \tag{17}$$

22

To provide a more reasoning, see $\frac{dNFA}{dIR} = -\left(\left(1+\bar{r}\right)\frac{d(-b_1)}{dIR} - 1\right) + 2\Gamma b_1\frac{d(-b_1)}{dIR}$ if we assume $\bar{r} = r_*$. This assumption isn't harmful. Rather, $r_* > \bar{r}$ strengthens our result. Suppose $\frac{dNFA}{dIR} < 0$. Since $\frac{\beta\Gamma\frac{d(-b_1)}{dIR}E_0\left[u_1'\right]}{\beta(1+r_1)E_0\left[u_1''\left(\frac{d(-b_2)}{dNFA}+1\right)\right]} > 0$ for ϕ^c low enough and $u'' < 0$, we must have $\frac{d(-b_1)}{dIR} - \frac{1}{1+\bar{r}} < 0$. Because $\frac{dNFA}{dIR} = -\left(\left(1+\bar{r}\right)\frac{d(-b_1)}{dIR} - 1\right) + 2\Gamma b_1\frac{d(-b_1)}{dIR}$, it implies $|\ 2\Gamma b_1\frac{d(-b_1)}{dIR}|$ is large enough to change the sign of $\frac{dNFA}{dIR}$. To have $\frac{d(-b_1)}{dIR} - \frac{1}{1+\bar{r}} < 0$ and a large$|\ 2\Gamma b_1\frac{d(-b_1)}{dIR}|$simultaneously, we need at least either of high Γ or high $-b_1$. However, the both of high Γ and high $-b_1$ increases $\beta\Gamma\frac{d(-b_1)}{dIR}E_0\left[u_1'\right]$ so that it can be hard to have $\frac{dNFA}{dIR} < 0$.

As usual in the literature, the collected taxes will be distributed to households as lump-sum taxes[23].

The Pigouvian taxation on the foreign borrowing First, we begin working at the first tax scheme–Pigouvian tax on the foreign borrowing. To avoid a possible confusion, readers should notice that this tax is imposed on households who issue bonds and see them to international investors. Therefore, we solve for the optimal tax in the planning problem. Then, we again solve the problem of the optimal reserves accumulation in Proposition 2 and Corollary 1, with the optimal Pigouvian taxation. We suggest the formula of the tax in the following lemma.

Lemma 3. *The optimal tax on the borrowing is characterized as below.*

$$\tau_d = \frac{1}{u_0^T}\beta[\int_{\underline{\phi}}^{\phi^c} \frac{d(-b_2)}{db_1}\left(u_1^T - \beta(1+r_2)u_2^T\right)d\mathcal{F}_\phi + \int_{\underline{\phi}}^{\overline{\phi}} -\Gamma b_1 u_1^T + \beta\Gamma b_2 \frac{d(-b_2)}{db_1}u_2^T d\mathcal{F}_\phi + \int_{\underline{\phi}}^{\phi^c} IR\frac{d\mu}{db_1}\left(u_1^T - \beta(1+\overline{r})u_2^T\right)d\mathcal{F}_\phi] \quad (18)$$

Let's denote the borrowing of the households under the optimal tax by $b_1^{priv}(\tau_d)$. Then $b_1^{priv}(\tau_d) = b_1^{sp}$.

Proof) See the Appendix.

The rough interpretation of the formula of the optimal tax is as follows: the tax rate increases in the cost of sudden stop crisis, which depends on the probability of a crisis $\int_{\underline{\phi}}^{\phi^c} d\mathcal{F}_\phi$ and the severity of a crisis $u_1^T - \beta(1+r_2)u_2^T$ as we explained earlier, while the rate decreases in marginal utilities in period 0 since higher marginal utilities imply more responsiveness of the consumptions to imposed taxes.

Now we present the forth proposition that characterizes the determination of reserves accumulation in the presence of optimal capital controls.

Proposition 3. *The optimal reserves accumulation under the optimal tax on the borrowing, say $\widehat{IR}$, is characterized by the equation below.*

$$\beta\int_{\underline{\phi}}^{\phi^c} \frac{\partial(-b_2)}{\partial IR}\left(u_1^T - \beta(1+r_2)u_2^T\right)d\mathcal{F}_\phi + \beta IR\int_{\underline{\phi}}^{\phi^c} \frac{\partial\mu}{\partial IR}\left(u_1^T - \beta(1+\overline{r})u_2^T\right)d\mathcal{F}_\phi = u_0^T\left(\frac{1}{1+\overline{r}}\right) - \beta\left(\int_{\underline{\phi}}^{\phi^c} u_1^T\mu d\mathcal{F}_\phi + \int_{\phi^c}^{\overline{\phi}} u_1^T d\mathcal{F}_\phi\right) - \beta^2\left(\int_{\underline{\phi}}^{\phi^c} u_2^T(1-\mu)(1+\overline{r})d\mathcal{F}_\phi\right) \quad (19)$$

$\widehat{IR}$ doesn't depend on $\frac{d(-b_1)}{dIR}$, and theorefore $\mu^* = \mu^{**}$.

For reasonable parameter values, $\widehat{IR} < IR^*$.

[23] An interesting case is that reserves accumulation is financed by the macroprudential taxes on the foreign borrowing. Jeanne (2015) deployed this idea. Jeanne and Korinek (2017) explored this issue more explicitly. But, they found financing bail-out policies through ex-ante policy measures might not be an optimum. Hence, we disregard it.

Proof) By lemma 3, we can treat b_1 as another choice variable. Then the first order condition gives the result immediately.

The most noticeable difference of the equation (18) from the equation (13) in Proposition 2 is that the moral hazard term, $\frac{db_1}{dIR}$ vanishes. This implies that an important function of macroprudential capital controls is to blow away concerns on using ex-ante bail out policies. Similar insights are provided in few preceding works; Jeanne and Korinek, 2017; Bianchi and Mendoza, 2017; and Acharya and Krishnamurthy, 2018. Clearly, eliminating the moral hazard and the along with time inconsistency will raise the demand for reserves since it lowers the costs of holding reserves.

However, on the other side, optimal capital controls should significantly weaken necessities of holding a large amount of reserves, as one might feel at first glance; naturally, the tax on borrowing will reduce the amount of borrowing and accordingly the concerns over an abrupt stop of capital inflows. Moreover, the second channel through reserves work in the last section disappears now, as it is explicitly in the second statement in Proposition 4. Intuitively, once it becomes possible to control the level of NFA by choosing the amount of borrowing through taxes, it is always to better to rely on this way rather than using FX market interventions; the yields on international reserves are always lower than the borrowing rates.

Considering these altogether, it is very likely that the optimal macroprudential capital controls lower the optimal level of reserves accumulation with reasonable values of parameters albeit no worries about moral hazards. In short, they are substitutes with each other. This is strikingly contrast with the theoretical finding in Acharya and Krishnamurthy (2018), but corresponds to the empirical finding in Aizenman et al. (2017) that hints many EMEs are substituting international reserves with other macroprudential policy tools. The difference comes from the ways in which reserves management works as a precautionary policy tool in our model. One of the two benefits from reserves accumulation in our model is to increase NFA (Net Foreign Assets), but this isn't necessary anymore after being able to do same manipulation through taxes on the borrowing.

The taxation on the profits of international investors Suppose henceforth governments set taxes (or subsidies) on the excessive returns of international investors. That is, noting the tax rate is τ_s, the revenue for governments from the tax scheme is $\tau_s(-b_1 (r_1 - r_*))$. We assume that the revenues from the taxes on international investors is distributed to households as a lump sum. In the case that the government subsidize the international investors, corresponding expenditures are taxed from households as a lump sum as well. Clearly, the net return for international investors is $(1-\tau_s)(-b_1 (r_1 - r_*))$, and the capital supply is as below accordingly.

$$-b_t = \frac{1-\tau_s}{\Gamma} (r_t - r_*) \tag{20}$$

Noticeably, the newly introduced tax has a same effect as controlling the parameter Γ for the households. This is a very well-known result in Gabaix and Maggiori (2015) in which capital controls boost the effectiveness of foreign market interventions by widening UIP wedges. The role of the taxes on international investors is almost identical to Gabaix and Maggiori (2015); putting a sack of sands in the wheel of international investors so that capital inflows with more frictions (taxes, tow > 0) or inserting more airs into the wheel to let the capitals inflow more smoothly (subsidizes, tow < 0). Without formulating the problem of the government to determine the optimal tax rates, we directly provide our result in the following proposition.

Proposition 4. *Suppose* Γ *isn't too low*[24], *then*

1. If $b_1^{priv} \neq b_1^{sp}$*. Then, assuming* τ_s *is small enough, the optimal tax on the profits of international investors is characterized by the equation below.*

$$\tau_s = \Gamma \frac{-u_0^T \left(\Gamma b_1^2 - \frac{db}{d\tau}\right) - \beta \Xi \left[(1 + r_* - 2\Gamma b_1) \frac{db}{d\tau} - \Gamma b_1^2\right]}{2u_0^T \left[\Gamma b_1^2 + b_1 (1 - \beta \Gamma \Xi) \frac{db}{d\tau}\right]} \tag{21}$$

where $\Xi = \int_{\underline{\phi}}^{\overline{\phi}} u_1^T - \beta \Gamma b_2^2 \frac{db_2}{db_1(1+r_1)} u_2^T - \int_{\underline{\phi}}^{\phi^c} \frac{1}{db_1(1+r_1)} \left[db_2 \left(u_1^T - \beta (1 + r_2) u_2^T\right) + d\mu \left(u_1^T - \beta (1 + \overline{r}) u_2^T\right)\right]$ *and* $\Xi > 0$.

2. If $b_1^{priv} = b_1^{sp}$*, then the optimal tax on the profits of international investors turns out to be*

$$\tau_s = \frac{-\frac{1}{2}\beta r_* + \beta \Gamma b_1 \Xi}{u_0^T - \beta \Gamma b_1 \Xi} \tag{22}$$

Thus, if the optimal decision is to borrow, then the solution of the social planning is to subside international investors. On the contrary, if the optimal decision is to lend, then the solution is to tax international investors for most sets of plausible parameter values.

3. Suppose $\frac{d^2(-b_2)}{dIR^2} < 0$*. With the optimal tax* τ_s *on international investors,* $\widehat{IR}$ *rises if* c_o^T *increases with the optimal tax. On the contrary* $\widehat{IR}$ *falls if* c_o^T *decreases with the optimal tax.*

Even without the assumption $\frac{d^2(-b_2)}{dIR^2} < 0$*, the statement above mostly holds.*

Proof) See the Appendix.

The equation (20) characterizes the optimal taxation on international investors when the borrowing is away from the optimum for the EME. Please notice that the right hand side also includes the taxes so that the optimal tax is the solution of the fixed point problem. Although the equation is the most material expression we have found, it is hard to see an intuition behind the equation. To see it, let's express the equation in a slightly abstract form. The first order condition can be presented as below.

$$\frac{\partial V}{\partial \pi} \frac{d\pi}{d\tau} + \frac{\partial V}{\partial b_1} \frac{db_1}{d\tau_s} = \frac{\partial V}{\partial \Gamma} \Gamma \tag{23}$$

where V is the summation of utilities over the three periods as it was in proposition 3, and π is the collected taxes from the international investors. As it is revealed in the equation (22), we can decompose the effects of the taxation into three components. First, obviously the taxes on the international investors provide additional incomes to the households ($\frac{\partial V}{\partial \pi} \frac{d\pi}{d\tau} > 0$). Second, the taxation affects the amounts of the borrowing.

[24] Since the optimization for the government is not well defined when (gamma) is zero, the equation holds only if Γ is strictly positive.

We know in the absence of optimal taxation in lemma 3 or with insufficiently high tax rates, the overborrowing arises so that we have $\frac{\partial V}{\partial b_1} > 0$. Also, taxes on international investors depress the borrowing through the additional income and higher elasticity of the interest rate to the borrowing – higher Γ. Thus $\frac{db_1}{d\tau_s} > 0$. In overall, the tax on the capital inflow discourage the socially undesirable overborrowing ($\frac{\partial V}{\partial b_1}\frac{db_1}{d\tau_s} > 0$). Third, as we noted earlier, beside the effect from the additional income from the tax, the imposing taxes on the profits of international investors is equivalent to the control on Γ; taxes raise Γ whereas subsidies lower it. Subsequently, the higher Γ raises the cost of borrowing, thereby hindering households from smoothing their consumption streams. If the government subsidizes, vice versa.

The second statement in Proposition 5, equation (21) is more interesting. The most clear interpretation of the equation (22) is as follows. First, we ignore the negligible number $-\frac{1}{2}\beta r_*$. Then, given the borrowing is the "optimal" borrowing, the government should subsidize,$\tau_s < 0$, if the optimal decision is to borrow against future, while it should tax if the optimum for households is to save for the future. Apparently, it may sound counter-intuitive. If the borrowing is the optimum although the benevolent social planner can raise income from outside by imposing taxes, that implies a "smoother" consumptions stream is more desirable . The small number number $-\frac{1}{2}\beta r_*$ reflects the fixed cost of one unit of the borrowing.

The last statement gives a very intuitive answer to an important question in this paper. If the optimal taxation is so, then how is it related with the reserves accumulation? The answer is very clear: the optimal amount of reserves is likeky to increase in the subsidy if the subsidy raises the current consumption, c_o^T, while the reserves is likely to decrease if the subsidy depresses the consumption. This can be understood in the context of the function of the taxation; the role of the taxation is to make it easier to smooth consumption streams. If the current consumption falls according to the introduction of the optimal taxation, it means the government want to leave more resources for the future, which reduces the demands for reserves. On the other hand, If the current consumption rises according to the introduction of the optimal taxation, this is the case that the government want to transfer resources from the future to now, leaving less resources for the future, which calls for more demands for the reserves. Hence, in contrast to the taxes on the borrowers, the optimal tax on international investors, which turns out to be subsidies under the optimal tax on the borrowers, can either raise or suppress the demand for international reserves, although it surely enhance the effectiveness of holding reserves. We highlight these relations in the following remark

Remark 2. The optimal tax on the borrowing may substitute the demand for international reserves, while the optimal tax on international investors under the optimal tax on the borrowing can either raise or reduce the demand for international reserves.

Discussion of Non-optimal taxations As an academic interest or even a purpose of suggesting a policy alternative, figuring out an analytic description of the policies is worth spending time and getting attentions. However, despite of its worthiness, one can easily question its validity in the reality: how do policy makers 'in the real world, not in a model' can find the optimal taxes? Such criticism is fair as it is usual in any optimal taxation problem. Furthermore, being able to find a magic formula of an optimal policy doesn't necessarily mean it can be implemented right away, in particular for taxations. Any taxation, which has to go through a legislation process, is an interest for many different stake holders, thereby hardly being an optimal. To adjust the tax rates flexibly according to changes in economic conditions should be even much harder than setting an optimal tax.

Of course, these limitations don't necessarily imply that the formulas above are meaningless; it corresponds to an academic interest and can be a useful guide for policy makers. However, it would be more realistic to assume that taxations in the reality are, to some extent, away from the optimum taxations. This is particularly important for the investigation of how those taxations are related with other policy tools, – here, international reserves management.

Finding a relation between reserves management and non-optimal taxations is mainly a job in the next section; numerical illustration of our results. However, it should be worth noting how the relation of optimal reserves management with the taxations will change according to the deviations of the optimal taxations. Of course, this is because whether each of the two taxation will increase or decrease the optimal amount of reserve accumulation isn't conclusive. We highlight this inconclusiveness in the following remark. In addition, we provide little intuition behind the inconclusiveness.

Remark 3. 1. The optimal reserves accumulation in Proposition 2, denoted as IR^*, may increase or decrease in non-optimal taxations. That is, the signs of $\frac{dIR^*}{d\tau_d}$ and $\frac{dIR^*}{d\tau_s}$ are inconclusive.

To discuss intuitions behind the remark, reacll the equation characterizing the optimal reserves accumulation without the optimal capital controls.

$$\beta \int_{\underline{\phi}}^{\phi^c} \frac{\partial (-b_2)}{\partial IR} \left(u_1^T - \beta (1+r_2) u_2^T\right) d\mathcal{F}_\phi + \beta IR \int_{\underline{\phi}}^{\phi^c} \frac{\partial \mu}{\partial IR} \left(u_1^T - \beta (1+\overline{r}) u_2^T\right) d\mathcal{F}_\phi - \beta^2 \Gamma \int_{\underline{\phi}}^{\overline{\phi}} b_2 \frac{\partial b_2}{\partial IR} u_2^T d\mathcal{F}_\phi$$
$$= u_0^T \left(\frac{1}{1+r_*}\right) - \beta \left(\int_{\underline{\phi}}^{\phi^c} u_1^T \mu d\mathcal{F}_\phi + \int_{\phi^c}^{\overline{\phi}} u_1^T d\mathcal{F}_\phi\right) - \beta^2 \int_{\underline{\phi}}^{\phi^c} u_2^T (1-\mu)(1+\overline{r}) d\mathcal{F}_\phi - \frac{d(-b_1)}{dIR} \frac{dV}{d(-b_1)}$$

For the tax on the borrowing, τ_d, any tax on the borrowing must discourage the borrowing so as to lower the possibility of a crisis and the pain of the crisis as well. This supresses the right hand side. Moreover, less borrowing curtails the spread between the borrowing rate of the EME and the yield on the reserves, which reduces $u_0^T \left(\frac{1}{1+r_*}\right) - \beta \left(\int_{\underline{\phi}}^{\phi^c} u_1^T \mu d\mathcal{F}_\phi + \int_{\phi^c}^{\overline{\phi}} u_1^T d\mathcal{F}_\phi\right) - \beta^2 \left(\int_{\underline{\phi}}^{\phi^c} u_2^T (1-\mu)(1+\overline{r}) d\mathcal{F}_\phi\right)$ in the right hand side. On the other hand, the less borrowing by the tax on the borrowing will lessen the overborrowing. In the equation above, it let $\frac{dV}{d(-b_1)}$ down so as the cost of reserves as well. In overall, the same intuition with the case that the taxation is optimal applies here.

Same as the tax on the borrowers, the tax on the "suppliers", τ_s, must depress the borrowing of the households, $-b_1$. However, the taxation makes the interest rate higher by raising Γ on the other hand; the tax on the international investors is ex-post identical to the case of having more endowements of tradable goods in period 0 and higher Γ as a cost of more y_0^T. Hence, the relation between the "optimal" reserves accumulation and the "non-optimal" taxation at first depends on whether it increases or decreases the NFA of the EME, hence the sign of How taxes on international investors, τ_s, will impact the reserves accumulation depends on whether it will lead either of lower borrowing rate or higher borrowing rate[25]. On the right hand side of the equation, if the tax raises the borrowing rate, then it is very likely the first cost of reserves, the spread between the borrowing rate and the yield on the reserves, rises. How the moral hazard term,

[25]Remember $NFA = IR - b_1 (1+r_1)$. Hence, what matters here is the sign of $\frac{d(b_1(1+r_1))}{d\tau_s} = (1+r_* - 2\Gamma b_1) \frac{db}{d\tau} - \Gamma b_1^2$.

$\frac{dV}{d(-b_1)}$, will change to the introduction of the non-optimal tax is also indeterministic. Because of the these offsetting forces with each other, we can't assert that the "optimal" holding of reserves increases or decrease after imposing tax on international investors at an arbitrary rate[26].

4 Numerical Illustration

[TBW]

5 Concluding Remarks

In this paper, we first build a new model of international reserves. The purpose of the reserve accumulation in our model is to prevent a sudden stop stop crisis and lessen the pain of a crisis. We constructed our model in an environment where the fisherian deflation mechanism exacerbates the severity of a sudden stop crisis. Unlike some preceding works, we present a setup where holding reserves is actually effective in coping with unexpected capital outflows. First, following Shousha (2017), we assumed that reserves can work as a collateral so as to give EMEs a little higher leverage. This assumption not only provides an incentive of holding costly reserves, but also explains why EMEs were hesitant to deplete their reserves during the GFC and the subsequent market turmoil in 2013 after the announcement of QE tapering; during a crisis, the planner of an EME in our model faces a trade off between holding reserves and depleting reserves; holding reserves allow the EME for more "costly" borrowings, while using the reserves to pay back debts gives less liquidity, but leads to a lower cost of the borrowing. Second, following recent paper of foreign exchange market interventions, we assumed that no arbitrage condition doesn't perfectly hold in foreign exchange markets in EMEs. Because of the assumption, reserve accumulation tends to raise the NFA of the EME, as it is believed among practitioners.

Based on the newly constructed model, we explored the relation between reserve accumulation and capital controls. As usual in the literature, we derived the optimal taxation on foreign borrowings. Similarly with Acharya and Krishnamurthy (2018), we found that one of roles on the optimal control on the foreign borrowing is to eliminate the concern over the moral hazard; in the presence of a large amount of reserves, decentralized private agents tend to borrow more because the agents expect their government will subside them during a crisis, and in turn the concern depresses the demand for reserves. However, the magnitude of the moral hazard is lower in our model since the reserve accumulation should result in higher borrowing rates so as to "punish" the moral hazard. As a result, the optimal holding of reserves might decrease in our model if the optimal taxation on the foreign borrowings is introduced. The market incompleteness gives a little different quantitative prediction of the impact of the taxation on the optimal reserve accumulation, but it also gives the possibility of introducing another taxation: governments in EMEs can tax the profits of international investors. This "supply side"

[26] In another aspect, a larger Γ driven by the taxation incentivize the government to acquire more reserves. As we showed already, taxes on the profits of international investors make the supply of foreign capitals inflows less sensitive to the yield difference $r_t - r_*$. In other words, the supply curve will be steeper. (See Figure 1). Because of the steeper curve, with the same amount of intervention, the government can derive a larger improvement in the current account, so as to induce higher net foreign assets. Then the government can achieve a same goal with lower costs – less reserves accumulation.

taxation is ex-post to identical to the policy of adjusting the elasticities of foreign capital inflows with some costs. While one of roles of the taxation on international investors is, like the taxation on the borrowers, to control the overborrowing, another role of the taxation is to adjust frictions in the capital flows so that households can smooth their consumption streams in a more efficient way. This is highlighted by the case that the both optimal controls are deployed: if the borrowing is already optimal, then it is optimal to subsidize international investors to let capital inflow to the EME smoother.

It is important to note that we abstracted from few important features in the reality. Although the abstraction was necessary for the simplicity and to focus on the key mechanisms on which we wanted to shed light, we list some of the features here since we believe that the missed features are avenues for future research. First, like the most papers in the literature, we implicitly assumed that the cost of crisis is to disrupt the consumption smoothing of households in EMEs. However, as it was revealed in the recent GFC, the cost of a financial crisis would be much larger than the temporary drop of consumptions, and it could be more persistent. This is particularly important for the answer to the question that hasn't been answered clearly: what would be the optimal amount of reserve holding? Second, we didn't consider the "composition" of the external liabilities in EMEs, especially the presence of the liabilities denominated in their own "local" currencies in EMEs. After 2000's, many EMEs begin diversifying their liabilities. It might be a result of the efforts of the EMEs to avoid the heavy reliance on the short-term debts denominated in US dollars, or has been driven by global banks who has searched for higher yields. The literature is relatively new in exploring the external liabilities composition in EMEs. (See Avdjiev et al. (2017) for the empirical evidence. In the theory side, see Korinek (2018) and Wei and Zhou (2018)). Regarding the local currency external liabilities of EMEs, there have been a few works such Alfaro and Kanczuk (2013), Hale et al. (2016), Du and Schreger (2015), and Ottonello and Perez (2018), but as it is still relatively new in the literature. We believe that understanding the reasons and effects of local currency liabilities in EMES is much important in answering questions on sudden stop crisis, reserves management and macroprudential captial controls. We considered adding local currency debts to our model, but we found many difficult issues arise once we add local currency debts; we need to consider, for example, the market segmentation between local currency debt market and key currency debt market or speculations of foreign investors in EME Foreign exchange markets. Besides these issues in modeling, the absence of credible data is also a hindrance to make a progress[27]. Because of the issues, we discarded the local currency debts in our paper. Third, we abstracted from the beliefs of international investors on EMEs, and the beliefs of EMEs on international financial markets on the other hand. On the side of beliefs of international investors on EMEs, it seems that international investors overreact to negative news, and it causes unexpected capital outflows from EMEs. On the other side of beliefs of EMEs on the international financial market, one related interesting fact is EMEs began accumulating reserves after experiencing crises. This sluggish behavior of EMEs may imply that policy makers update their beliefs about the international financial market through a learning process. To the best of our knowledge, the only paper in a similar idea is Hur and Kondo (2016). However, in their model, the beliefs of agenst are stick to the fundamentals. On the contrary, if we, following few recent papers in the literature of informational friction in macroeconomics, allow the beliefs to be slightly off

[27]The database of BIS provides the amounts of local currency debts of some EMEs. We chceked the data of Korea in the database. Unfortunately, it seems that the dtat is missing much of Korean Won external debts, reflecting on our knowledge. The issue of data reliability in researches of cross-border capital flows is well known. To see the difficulty, see Shin (2012), and Bruno and Shin (2015)

from the fundamentals, then we can explain many facts: policy makers in East Asian countries realized they are also prone to sudden stop crises in the late 1990', and hence began building a massive amount of reserves. Also, since policy makers in EMEs can't figure out the exact optimal amount of reserves to hold, they easily get a peer pressure to have more reserve, watching rising stocks of reserves of other EMEs. Besides these three issues, building a more general model for a more quantitative research, incorporating nominal rigidities into the model, and international policy coordination using reserves in EMEs will be fruitful as well.

We believe all the issues above give us hard, but interesting questions unanswered in this paper. We leave these issues to future research

References

[1] Acharya, V. V. and Krishnamurthy, A . 2018. "Capital Flow Management with Multiple Instruments." *NBER Working Paper* No 24443.,

[2] Verdelhan, A., Du, W. and Tepper A. 2016. "Deviations from Covered Interest Rate Parity." *Journal of Finance* 73 (3): 915-957.

[3] Aizenman, J. 2011. "Hoarding International Reserves versus a Pigovian Tax-cum-Subsidy Scheme." *Journal of Economic Dynamics and Control* 35(9):1502–1513.

[4] Aizenman, J., Chinn, M. D. and Iti, H. 2017. "Financial Spillovers and Macroprudential Policies." *NBER Working Paper* No. 24105.

[5] Aizenman, J. and Hutchison, M. 2012. "Exchange Market Pressure and Absorption by International Reserves: Emerging Markets and Fear of Reserve Loss During the 2008-09 Crisis." *Journal of International Money and Finance* 31 (5): 1076-1091.

[6] Aizenman, J. and Lee, J. 2007. "International Reserves: Precautionary Versus Mercantilist Views, Theory and Evidence." *Open Economies Review* 18 (2): 191–214

[7] Aizenman, J. and Sun, Yi. 2012. "The Financial Crisis and Sizable International Reserves Depletion: From "Fear of Floating" to the "Fear of Losing International Reserves?" *International Review of Economics & Finance* 24:250–269.

[8] Alfaro, L., Cuñat, A., Fadinger, H., and Liu, Y. 2018. "The Real Exchange Rate, Innovation and Productivity: Regional Heterogeneity, Asymmetries and Hysteresis." *NBER Working Paper* No 24633.

[9] Alfaro, L., and Kanczuk, F. 2009b. "Optimal Reserve Management and Sovereign Debt." *Journal of International Economics* 77 (1): 23–36.

[10] ——. 2013. "Debt Redemption and Reserve Accumulation." *NBER Working Paper* No 19098.

[11] Alvarez, F., Atkeson A., and Kehoe, P. J. 2009. "Time-Varying Risk, Interest Rates, and Exchange Rates in General Equilibrium," *Review of Economic Studies* 76 (3): 851–878.

[12] Avdjiev, S., Binder S., and Sousa, R. 2017. "External Debt Composition and Domestic Credit Cycles," *BIS Working Papers* No 627.

[13] Benigno, G., Chen, H., Otrok, C., Rebucci, A., and Young, E. R. 2016. "Optimal Capital Controls and Real Exchange Rate Policies: A Pecuniary Externality Perspective," *Journal of Monetary Economics* 84:147–165.

[14] Bacchetta, P. and Wincoop, E.V. 2010. "Infrequent Portfolio Decisions: A Solution to the Forward Discount Puzzle." *American Economic Review* 100 (3): 870–904.

[15] Benigno, G. and Fornaro, L. 2014. "Reserve accumulation, growth and financial crises." *Manuscript, London School of Economics and Political Science.*

[16] Bianchi, J. 2011. "Overborrowing and Systemic Externalities in the Business Cycle." *American Economic Review* 101 (7): 3400–3426.

[17] Bianchi, J., Hatchondo, J. C. and Martinez, L. 2012. "International Reserves and Rollover Risk." *Forthcoming, American Economic Review.*

[18] Bigio, S. 2015. "Endogenous Liquidity and the Business Cycle." *American Economic Review* 105 (6): 1883-1927.

[19] Bruno, V. and Shin, H.S. 2014b. "Cross-Border Banking and Global Liquidity." *Review of Economic Studies* 82 (2): 535–564.

[20] Bocola, L. and Lorenzoni, G. 2018. "Financial Crisis, Dollarization, and Lending of Last Resort in Open Economie." *Manuscript, Northwestern University.*

[21] Brunnermeier, M. and Pedersen, L. H. 2009. "Market Liquidity and Funding Liquidity." *Review of Financial Studies*, 22 (6):2201–2238.

[22] Bussiere, M., Cheng, G., Chinn, M., and Lisack, N. 2015. "For a Few Dollars More: Reserves and Growth in Time of Crises." *Journal of International Money and Finance* 52:127–145.

[23] Caballero, R. J. and Krishnamurthy, A. 2003. "Excessive Dollar Debt: Financial Development and Underinsurance." *Journal of Finance,* 58 (2):867–894.

[24] Cavallino, P. 2017. "Capital Flows and Foreign Exchange Intervention." *Forthcoming, American Economic Journal: Macroeconomics.*

[25] Cerutti, E., Claessens, S., Rose, A.K., 2017a. "How Important is the Global Financial Cycle? Evidence from Capital Flows." *NBER Working Paper* No. 23699.

[26] Chang, R. 2018. "Foreign Exchange Intervention Redux." *NBER Working Paper* No 24463.

[27] Choi, W. J. and Taylor, A. M. 2017. "Precaution versus Mercantilism: Reserve Accumulation, Capital Controls, and real exchange rate." *NBER Working Paper,* No. 23341

[28] K. Dominguez, Y. Hashimoto, and T. Ito. 2012 "International Reserves and the Global Financial Crisis." *Journal of International Economics* 88 (2): 388-406

[29] Dominguez, Kathryn M.E. 2010. "International Reserves and Underdeveloped Capital Markets." *In NBER International Seminar on Macroeconomics 2009*, edited by Lucrezia Reichlin and Kenneth West, 193–221. University of Chicago Press.

[30] Du, W.and Schreger, J. 2016. "Sovereign Risk, Currency Risk, and Corporate Balance Sheets." *Manuscript, Columbia Business School.*

[31] Durdu, C. B., Mendoza, E. G., and Terrones, M. E. 2009. "Precautionary Demand for Foreign Assets in Sudden Stop Economies: An Assessment of the New Mercantilism." *Journal of development Economics* 89 (2):194–209.

[32] Fanelli, S. and Straub, L. 2017. "A Theory of Foreign Exchange Interventions." *Manuscript, Massachusetts Institute of Technology.*

[33] Farhi, E. and Werning, I. 2014. "Dilemma not Trilemma? Capital Controls and Exchange Rates with Volatile Capital Dlows." *IMF Economic Review*, 62:569–605.

[34] Frankel, J. and Saravelos, G. 2012. "Are Leading Indicators of Financial Crises Useful for Assessing Country Vulnerability? Evidence from the 2008-09 Global Crisis." *Journal of International Economics*, 87 (2): 216-231.

[35] Gabaix, X. and Maggiori, M. 2015. "International Liquidity and Exchange Rate Dynamics." *Quarterly Journal of Economics* 130 (3): 1369–1420.

[36] Geanakoplos, J. D. 2009. "The Leverage Cycle." *NBER Macroeconomics Annual 2009*, 24:1–65.

[37] Gottardi, P., Maurin, V. and Monnet, C. 2016. "A Theory of Repurchase Agreements, Collateral Re-use, and Repo Intermediation." *Manuscript, European University Institute.*

[38] Hernandez, J. 2017. "How International Reserves reduce the Probability of Debt Crises." *Manuscript, University of Pennsylvania.*

[39] Hur, S. and Kondo I. O. 2016. "A Theory of Rollover Risk, Sudden Stops, and Foreign Reserves." *Journal of International Economics* 103:44–63.

[40] Jeanne, O. 2015. "The Macroprudential Role of International Reserves." *American Economic Review* 106 (5):570–573.

[41] Jeanne, O. and Rancière, R. 2011. "The Optimal Level of International Reserves for Emerging Market Xountries: a New Formula and Some Applications." *Economic Journal* 121 (555):905–930.

[42] Jeanne, O. and Korinek, A. 2010b. "Managing Credit Booms and Busts: A Pigouvian Taxation Approach." *NBER Working Paper*, No. 16377.

[43] Jeanne O and Korinek, A. 2013. "Macroprudential Regulation Versus Mopping Up After the Crash". *NBER Working Paper*, No. 18675.

[44] Korinek, A. 2018. "Regulating Capital Flows to Emerging Markets: An Externality View." *Journal of International Economics* 111: 61-80

[45] Korinek, A. and Sandri, D. 2016 "Capital Controls or Macroprudential Regulation?" Journal of International Economics 99 (S1): 27-42.

[46] Korinek, A. and Servén, L. 2016. "Undervaluation through Foreign Res, erve Accumulation: Static Losses, Dynamic Gains." *Journal of International Money and Finance* 64:104–136.

[47] Kiyotaki, N. and Moore, J. 1997. "Credit Cycles." *Journal of Political Economy* 105 (2):211–248.

[48] Maggiori, M. 2017. "Financial Intermediation, International Risk Sharing, and Reserve Currencies." *American Economic Review* 107 (10): 3038-3071.

[49] Mendoza, E. G. 2010. "Sudden Stops, Financial Crises, and Leverage." *American Economic Review* 100 (5):1941–1966.

[50] Mendoza, E. G. and Bianchi, J. 2018. "Optimal Time-Consistent Macroprudential Policy." *Forthcoming, Journal of Political Economy.*

[51] Parlatore, C. 2018. "Collateralizing Liquidity." *Forthcoming, Journal of Financial Economics.*

[52] Rey, H., 2015. Dilemma Not Trilemma: the Global Financial Cycle and Monetary Policy Independence. National Bureau of Economic Research

[53] Rodrik, D. 2006. "The Social Cost of Foreign Exchange Reserves." International Economic Journal 20 (3): 253–266.

[54] Shousha, S. F. 2017. "International Reserves, Credit Constraints, and Systemic Sudden Stops." *Board of Governors of the Federal Reserve System International Finance Discussion Papers* No. 1205.

[55] Obstfeld, M., Shambaugh J. C., and Taylor, A. M. 2010. "Financial stability, the trilemma, and international reserves." *American Economic Journal: Macroeconomics* 2 (2):57–94.

[56] Ottonello, P. and Perez, D. J. 2017. "The currency composition of Soverign Debt." *Manuscript, University of Michigan.*

[57] Wei, S. J. and Zhou, J. 2018. "Quality of Public Governance and the Capital Structure of Nations and." *NBER Working Paper* No. 24184.

[58] Winant P., Ostry, J., Ghosh, A. and Basu, S. 2016. "Managing Capital Outflows: The Role of Foreign Exchange Intervention." *2016 Meeting Papers 756, Society for Economic Dynamics.*

A Omitted Algebras and Proofs

A.1 Derivation of (6), (7) and (8)

First, we derive (6) and solve further to derive the closed from of the borrowing in the states of unbinding credit constraint.

If the credit constraint (the equation (3) doesn't bind, then the b_1 is determined by the following Euler equation.

$$\frac{1}{u_1^T} = \beta(1+r_2)\frac{1}{u_2^T} \tag{24}$$

Given other states, solving for b_2 yields the equation (6). If we solve for $b_2$fully, then we can derive the closed form; r_2 in the equation (6) is a function of b_2.

$$-b_2 = \frac{-W_1 + \sqrt{W_1^2 + 4\left(1+\beta\right)\Gamma y_2^T}}{2\left(1+\beta\right)\Gamma} \tag{25}$$

where $W_1 = \left(1+\overline{r}\right)\left(1+\beta\right) + \beta\Gamma\left(y_1^T + b_1\left(1+r_1\right) + IR\right)$.

Next, we derive the equation (7). We know $p_1 = \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{c_1^T}{c_1^N}\right)$ and $r_2 = -\Gamma b_2 + \overline{r}$. Plugging these into the equation (3), and then solving the quadratic equation yields the equation (7). Obviously, (8) is derived from $r_2 = -\Gamma b_2 + \overline{r}$.

Some readers might worry if the equation isn't well defined for small values of Γ. To see it is not, applying L'Hospital's rule, on can show as $\Gamma \to 0$, $-b_2$ converges to

$$\left(\frac{1}{1+r_* - \phi\left(\frac{1-\alpha}{\alpha}\right)}\right)\left(\phi\left(y_1^T + \frac{1-\alpha}{\alpha}\left(y_1^T + b_1\left(1+r_1\right) + \mu IR\right)\right) + \theta\left(1-\mu\right)IR\right) \tag{26}$$

A.2 Proof of Lemma 1

To show $b_1^{priv} < b_1^{sp}$, we first derive the first order condition of b_1 for the social planner. b_1^{sp} is characterized as below

$$\begin{aligned} -u_0^T + \beta\int_{\underline{\phi}}^{\overline{\phi}}\left[\left(1+r_* - 2\Gamma b_1\right)\left(1 - \frac{db_2}{db_1\left(1+b_1\right)}\right) + IR\frac{d\mu}{db_1}\right]u_1^T d\mathcal{F}_\phi + \\ \beta^2\int_{\underline{\phi}}^{\overline{\phi}}\left[\left(1+r_* - 2\Gamma b_2\right)\frac{db_2}{db_1\left(1+b_1\right)}\left(1+r_* - 2\Gamma b_1\right) - IR\left(1+\overline{r}\right)\frac{d\mu}{db_1}\right]u_2^T d\mathcal{F}_\phi = 0 \end{aligned} \tag{27}$$

This can be represented by

$$\begin{aligned} -u_0^T + \beta\int_{\underline{\phi}}^{\overline{\phi}}\left(1+r_1\right)u_1^T d - \Gamma b_1 u_1^T d\mathcal{F}_\phi + \beta\int_{\underline{\phi}}^{\phi^d} IR\frac{d\mu}{db_1}\left(u_1^T - \beta\left(1+\overline{r}\right)u_2^T\right)d\mathcal{F}_\phi \\ \left(1+r_* - 2\Gamma b_1\right)\beta^2\int_{\underline{\phi}}^{\overline{\phi}}\frac{db_2}{db_1\left(1+b_1\right)}\left(u_1^T - \left(1+r_2 - \Gamma b_2\right)u_2^T\right)d\mathcal{F}_\phi = 0 \end{aligned} \tag{28}$$

Suppose $b_1 = b_1^{priv}$ and $b_2 = b_2^{priv}$. Then we have $u_0^T = \beta \int_{\underline{\phi}}^{\overline{\phi}} (1+r_1)\, u_1^T d\mathcal{F}_\phi$ and also $u_1^T = \beta (1+r_2)\, u_2^T \; if \; \phi > \phi^c$. This gives us

$$\beta \int_{\underline{\phi}}^{\overline{\phi}} -\Gamma b_1 u_1^T d\mathcal{F}_\phi + (1+r_* - 2\Gamma b_1)\, \beta \int_{\underline{\phi}}^{\overline{\phi}} \frac{-db_2}{db_1 (1+b_1)} \Gamma b_2 u_2^T d\mathcal{F}_\phi + \beta \int_{\underline{\phi}}^{\phi^d} IR \frac{d\mu}{db_1} \left(u_1^T - \beta (1+\overline{r})\, u_2^T\right) d\mathcal{F}_\phi$$

$$\beta (1+r_* - 2\Gamma b_1) \int_{\underline{\phi}}^{\phi^c} \frac{db_2}{db_1 (1+r_1)} \left(u_1^T - \beta (1+r_2)\, u_2^T\right) d\mathcal{F}_\phi > 0 \quad (29)$$

$\frac{d\mu}{db_1} > 0$ as it will be shown in the proof of lemma 2. If $\phi \in [\underline{\phi}, \phi^c)$, then $u_1^T - \beta (1+r_2)\, u_2^T > 0$, and then obviously $u_1^T - \beta (1+\overline{r})\, u_2^T$ since $\overline{r} < r_2$. Absolutely the remaining terms are all positive. It implies that $b_1^{priv} < b_1^{sp}$.

To help readers familiar with the setup of Lagrange multiplier, it can be easily shown $\beta \int_{\underline{\phi}}^{\overline{\phi}} -\Gamma b_1 u_1^T d\mathcal{F}_\phi + (1+r_* - 2\Gamma b_1)\, \beta \int_{\underline{\phi}}^{\overline{\phi}} \frac{db_2}{db_1(1+b_1)} -\Gamma b_2 u_2^T d\mathcal{F}_\phi = -\eta\Gamma$ and $(1+r_* - 2\Gamma b_1)$ $\int_{\underline{\phi}}^{\phi^c} \frac{db_2}{db_1(1+r_1)} \left(u_1^T - \beta (1+r_2)\, u_2^T\right) d\mathcal{F}_\phi = \xi \phi \psi \frac{dp_1}{db_1}$ where η and ξ are the multipliers (co-state variables) for the constraints $r_1 = -\Gamma b_1 + r_*$ and $-b_2(1+r_2) \leq \phi(\omega)(y_1^T + \psi p_1 y_1^N) + \theta(1-\mu) IR$ respectively.

To prove the seoond statement in Lemma 2, we can ignore the first term $\int_{\underline{\phi}}^{\overline{\phi}} -\Gamma b_1 u_1^T d\mathcal{F}_\phi +$ $(1+r_* - 2\Gamma b_1)\, \beta \int_{\underline{\phi}}^{\overline{\phi}} \frac{db_2}{db_1(1+b_1)} -\Gamma b_2 u_2^T d\mathcal{F}_\phi$. Then it is trivial $|\; b_1^{sp} - \; b_1^{priv}|$ is increasing in the term $\int_{\underline{\phi}}^{\phi^c} \frac{db_2}{db_1(1+r_1)} \left(u_1^T - \beta (1+r_2)\, u_2^T\right) d\mathcal{F}_\phi$. It can be shown more formally after we introduce the tax on the borrowing. For the convenience, we denote

$$\frac{db_2}{db_1 (1+r_1)} = \frac{1}{\sqrt{\left(r_* - \phi\psi \left(\frac{1-\alpha}{\alpha}\right)\right)^2 + 4\Gamma\kappa}} \phi\psi \left(\frac{1-\alpha}{\alpha}\right)$$

where $\kappa = \phi \left(y_1^T + \frac{1-\alpha}{\alpha} \psi \left(y_1^T + b_1 (1+r_1) + \mu IR\right)\right) + \theta (1-\mu)\, IR$.

Further, let's define

$$h(\phi) \equiv \frac{1}{\sqrt{\left(r_* - \phi\psi \left(\frac{1-\alpha}{\alpha}\right)\right)^2 + 4\Gamma\kappa}} \phi\psi \left(\frac{1-\alpha}{\alpha}\right) \; and \; \gamma(\phi) \equiv u_1^T - \beta (1+r_2)\, u_2^T$$

Obviously, $h' > 0$ for sufficiently low ϕ, whcih is a very likely case. However, $\gamma' < 0$ because the borrowing in period 1, $-b_2$ increases in ϕ. Combining h and γ, let $\mathcal{H}(\mathcal{F}) \equiv \int_{\underline{\phi}}^{\phi^c} h(\phi)\, \gamma(\phi)\, d\mathcal{F}_\phi$. See $(h(\phi)\, \gamma(\phi))'$ is indeterminate.

To prove the three cases in the second statement in Lemma 2, we only need to see how $\int_{\underline{\phi}}^{\phi^c} h(\phi)\, \gamma(\phi)\, d\mathcal{F}_\phi$ changes in another distrobution of ϕ. We denote the cdf and the pdf of the other distributon by $\mathcal{G}$ and g respectively. Now we show the magnititude of the externality decreases in the first moment of ϕ. Now assume $\int x d\mathcal{F}_x > \int x d\mathcal{G}_x$. Except for the first moment, all the moments of $\mathcal{F}$ and $\mathcal{G}$ are equivalent to each other. Define $\mathcal{H}(\mathcal{F}) \equiv \int_{\underline{\phi}}^{\phi^c} h(\phi)\, \gamma(\phi)\, d\mathcal{F}_\phi$ and $\mathcal{H}(\mathcal{G}) \equiv \int_{\underline{\phi}}^{\phi^c} h(\phi)\, \gamma(\phi)\, d\mathcal{G}_\phi$. We previously assumed $f' > 0$ for all $\phi < \phi^c$ (same for g as well) and g is a left shift of f. Thus, we have $f - g < 0$ for all $\phi < \phi^c$. Then it immediately follows that

$$\mathcal{H}(\mathcal{F}) - \mathcal{H}(\mathcal{G}) = \int_{\underline{\phi}}^{\phi^c} h(\phi)\, \gamma(\phi)\, (f-g)\, d\phi < 0 \quad (30)$$

Next, consider $\mathcal{G}$ as a mean-preserving spread of $\mathcal{F}$. The case in which $\mathcal{H}(\mathcal{F}) - \mathcal{H}(\mathcal{G}) < 0$

is trivial. Thus, we provide an opposite case to establish the indeterminancy. Suppose there exists an interval $[\phi^s, \phi^c]$ such that $g < f$ for all $\phi \in (\phi^s, \phi^c)$. The existence of such internval doesn't violate our assumption that $\mathcal{G}$ is a mean-preserving spread of $\mathcal{F}$. If $(h(\phi)\gamma(\phi))' > 0$ and $h(\phi)\gamma(\phi)$ is large enogh for $\phi \in (\phi^s, \phi^c)$, then we may have

$$\mathcal{H}(\mathcal{F}) - \mathcal{H}(\mathcal{G}) = \int_{\underline{\phi}}^{\phi^s} h(\phi)\gamma(\phi)(f-g)\,d\phi + \int_{\phi^s}^{\phi^c} h(\phi)\gamma(\phi)(f-g)\,d\phi > 0 \tag{31}$$

Lastly, to show the indeterminancy for the third moment, assume $\mathcal{G}$ has a less negative skewness than $\mathcal{F}$. However, again there may exist an interval $[\phi^s, \phi^c]$ such that $g < f$ for all $\phi \in (\phi^s, \phi^c)$. Following the same logic above, we can show the sign of $\mathcal{H}(\mathcal{F}) - \mathcal{H}(\mathcal{G})$ is either of negative or positive. □

A.3 Proof of Proposition 1

We begin by deriving the derivative of the object function with all the constraints. This yields

$$\frac{dV}{d\mu} = \frac{d(-b_2)}{d\mu}\left[u_1^T - \beta(1+r_2)u_2^T\right] + \frac{d(-b_2)}{d\mu}\Gamma b_2 u_2^T + IR\left[u_1^T - \beta(1+\bar{r})u_2^T\right] \tag{32}$$

where $\frac{d(-b_2)}{d\mu} = \frac{IR\left(\phi\psi\left(\frac{1-\alpha}{\alpha}\right)-\theta\right)}{\sqrt{\left(r_* - \phi\psi\left(\frac{1-\alpha}{\alpha}\right)\right)^2 + 4\Gamma\kappa}}$ and $\kappa = \phi\left(y_1^T + \frac{1-\alpha}{\alpha}\psi\left(y_1^T + b_1(1+r_1) + \mu IR\right)\right) + \theta(1-\mu)IR$.

In equation (31), the term $\left[u_1^T - \beta(1+r_2)u_2^T\right]$ will disappear if the credit constraint doesn't bind since the households satisfy their Euler equations as long as the constraint doesn't bind. By assumption 1, $\frac{d(-b_1)}{d\mu} < 0$ and $u_1^T - \beta(1+\bar{r})u_2^T > 0$ because $\bar{r} < r_2$. Hence, $dV/d\mu > 0$ as long as the credint constraint doesn't bind. Then it is trivial to see that the government will deplete all the reserves if the credit constraint doesn't bind. This estabilshes the first statement in the proposition.

Next, we prove the second statement in the proposition, which characterizes the partial depletion of reserves during a sudden stop crisis. Before we proceed, notice that the equation (31) is continuous with respect to ϕ on the interval $\left[\underline{\phi}, \phi^c\right]$. This is because ϕ^c is defined as ϕ such that the constrained borrowing is same as the unconstrained borrwing under the ϕ and $-b_2\,|_{\phi\leq\phi^c}$ is continuous with respect to ϕ.

To finalize the proof, we need to have $\frac{d^2(-b_2)}{d\mu d\phi} > 0$. That is, the marginal decrease of borrowing according to reserves depletion decreases in ϕ. It is easily satisfied by the second condition in the Assumption 1; there exists $\phi \in \left(\underline{\phi}, \phi^c\right)$, say ϕ^d, such that $\frac{\partial(-b_2)}{\partial\mu}\,|_{\phi\leq\hat{\phi^d}} < -IR$. This condition implies $\frac{\left(\phi\psi\left(\frac{1-\alpha}{\alpha}\right)-\theta\right)^2}{\left(r_* - \phi\psi\left(\frac{1-\alpha}{\alpha}\right)\right)^2} > \frac{\left(\phi\psi\left(\frac{1-\alpha}{\alpha}\right)-\theta\right)}{\sqrt{\left(r_* - \phi\psi\left(\frac{1-\alpha}{\alpha}\right)\right)^2 + 4\Gamma\kappa}} > 1$. Let $\Lambda(\phi) \equiv \frac{\left(\phi\psi\left(\frac{1-\alpha}{\alpha}\right)-\theta\right)^2}{\left(r_* - \phi\psi\left(\frac{1-\alpha}{\alpha}\right)\right)^2}$. Since κ is increasing in ϕ, $\Lambda' < 0$ implies $\frac{d^2(-b_2)}{d\mu d\phi} > 0$. One can easily see $\frac{\left(\phi\psi\left(\frac{1-\alpha}{\alpha}\right)-\theta\right)^2}{\left(r_* - \phi\psi\left(\frac{1-\alpha}{\alpha}\right)\right)^2}$ implies $\theta > r_*$. With $\theta > r_*$, we have $\Lambda' < 0$.

The above results establishes $\frac{d(-b_2)}{d\mu}\left[u_1^T - \beta(1+\bar{r})u_2^T\right]$ is continuously decreasing in ϕ. Since $\frac{d^2(-b_2)}{d\mu^2} > 0$ and it is also continuous, there exists $\delta > 0$ such that $\frac{d(-b_2)}{d\mu}\left[u_1^T - \beta(1+r_2)u_2^T\right] + \frac{d(-b_2)}{d\mu}\Gamma b_2 u_2^T + IR\left[u_1^T - \beta(1+\bar{r})u_2^T\right] < 0$ for $\mu = 1$ and $\phi \in (\phi^c - \delta, \phi^c]$.

$\frac{d^2(-b_2)}{d\mu d\phi} > 0$ also implies $u_1^T - \beta(1+r_2)u_2^T$ is decreasing in ϕ, and for $u_1^T - \beta(1+\bar{r})u_2^T$ as well. Hence, $\frac{d^2(-b_2)}{d\mu d\phi} > 0$ gives the other desired results. □

A.4 Proof of Lemma 2

Let μ^* be the solution of reserves depletion in Proposition 1. Hence, it is the solution of the equation (31). Consider an alternative policy to μ^*, say μ^{**}. We construct that alternative policy μ^{**} that dominates μ^*. We define

$$\mu^{**}(\phi) = \mu^*(\phi) + \varepsilon(\phi) \tag{33}$$

where $\varepsilon(\phi) > 0$ for all ϕ on the support. Now, we define a "metric" for μ^{**}.

$$\epsilon = E_0[\varepsilon(\phi)] \tag{34}$$

Hence, ϵ measures how much μ^{**} departs from μ^*on average. We only look at ϵ,while implicitly assuming $\varepsilon(\phi)$ is "optmially allocated" along with the states in period 1.

To proceed, we first show the borrowing in period 0 $-b_1$ decreasing in ϵ. Because of the assumption that $\frac{\partial(-b_2)}{\partial\mu}\mid_{\phi\leq\hat{\phi}^d}< -IR$, c_2^T decreases in μ the states where $\phi \leq \hat{\phi}^d$. Recall b_1 is determined by the Euler equation.

$$u_0^T = \beta(1+r_1)E_0\left[u_1^T\right]$$

Higher ϵ means higher μ. It decreases the consumptions of tradable goods in the states where $\mu \in (0,1)$. Thus higher ϵ yields a higher $E_0\left[u_2^T\right]$, which drives up u_1^T through the Euler equation above. Of course, this establishes the desired property that $-b_1$ decreasing in ϵ.

Next, without formulating the problem of determination of ϵ explicitly, we present the following first order condition.

$$\frac{dV}{d(-b_1)}\frac{d(-b_1)}{d\epsilon}+E_0\left[\frac{d(-b_2)}{d\mu^{**}}\left[u_1^T-\beta(1+r_2)u_2^T\right]+\frac{d(-b_2)}{d\mu^{**}}\Gamma b_2 u_2^T+IR\left[u_1^T-\beta(1+\overline{r})u_2^T\right]\right]=0 \tag{35}$$

ITwhere $V = u\left(c_0^T, y_0^N\right) + E_0\left[\beta u\left(c_1^T, y_1^N\right) + \beta^2 u\left(c_2^T, y_2^N\right)\right]$.

By lemma 1, $\frac{dV}{d(-b_1)} < 0$, and $\frac{d(-b_1)}{d\epsilon} < 0$ by the discussion above. Hence, $\frac{dV}{d(-b_1)}\frac{d(-b_1)}{d\epsilon} > 0$. See if $\epsilon = 0$, then $E_0\left[\frac{d(-b_2)}{d\mu^{**}}\left[u_1^T-\beta(1+r_2)u_2^T\right]+\frac{d(-b_2)}{d\mu^{**}}\Gamma b_2 u_2^T+IR\left[u_1^T-\beta(1+\overline{r})u_2^T\right]\right] = 0$. It implies the solution of (34) is a strictly positive number. That is, $\epsilon > 0$. □

A.5 Proof of Proposition 2

We first take the first order condition.

$$\frac{d(-b_1)}{dIR}[u_0^T-\beta\int_{\underline{\phi}}^{\overline{\phi}}\left((1+r_*-2\Gamma b_1)\left(1-\frac{\partial b_2}{\partial b_1(1+b_1)}\right)+IR\frac{\partial\mu}{\partial(-b_1)}\right)u_1^T d\mathcal{F}_\phi$$

$$-\beta^2\int_{\underline{\phi}}^{\overline{\phi}}\left((1+r_*-2\Gamma b_2)\frac{\partial b_2}{\partial b_1(1+b_1)}(1+r_*-2\Gamma b_1)-IR(1+\overline{r})\frac{\partial\mu}{\partial(-b_1)}\right)u_2^T d\mathcal{F}_\phi] =$$

$$\frac{-1}{1+r_*}u_0^T+\beta\int_{\underline{\phi}}^{\overline{\phi}}\left(\frac{\partial(-b_2)}{\partial IR}+\mu+IR\frac{\partial\mu}{\partial IR}\right)u_1^T d\mathcal{F}_\phi$$

$$+\beta^2\int_{\underline{\phi}}^{\overline{\phi}}\left((1+r_*-2\Gamma b_2)\frac{\partial(-b_2)}{\partial IR}+(1-\mu)(1+\overline{r})-(1+\overline{r})IR\frac{\partial\mu}{\partial IR}\right)u_2^T d\mathcal{F}_\phi \tag{36}$$

We use the facts that $u_0^T = \beta(1+r_1)E_0\left[u_1^T\right]$ and $u_1^T = \beta(1+r_1)u_2^T$ if the credit constraint doesn't bind. Take these terms out of the equation (35) and then rearranging the equation yields the equation (13).

The terms in the bracket is $\frac{dV}{d(-b_1)}$. Of course $\frac{dV}{d(-b_1)} < 0$. This establishes the second statement in Proposition 2. □

A.6 Proof of Corollary 1

If $\theta = 0$ so that reserves can't be used as acollateral, then we always have $\mu = 1$. Then the equation (13) is reduced to

$$\beta \int_{\underline{\phi}}^{\phi^c} \frac{\partial(-b_2)}{\partial IR}\left(u_1^T - \beta(1+r_2)u_2^T\right)d\mathcal{F}_\phi - u_0^T\left(\frac{1}{1+r_*}\right) + \beta\int_{\underline{\phi}}^{\overline{\phi}} u_1^T d\mathcal{F}_\phi = \frac{dV}{d(-b_1)}\frac{d(-b_1)}{dIR} \tag{37}$$

where $\frac{dV}{d(-b_1)} = u_0^T - \beta(1+r_*-2\Gamma b_1)\int_{\underline{\phi}}^{\overline{\phi}}\left(1-\frac{\partial b_2}{\partial b_1(1+b_1)}\right)u_1^T + \beta\left((1+r_*-2\Gamma b_2)\frac{\partial b_2}{\partial b_1(1+b_1)}\right)u_2^T$. This can be rearranged to

$$\left(\frac{-1}{1+\overline{r}}\right)u_0^T + \beta\int_{\underline{\phi}}^{\overline{\phi}} u_1^T d\mathcal{F}_\phi + \frac{d(-b_1)}{dIR}\left[\beta\Gamma\int_{\underline{\phi}}^{\overline{\phi}} b_1 u_1^T + \beta(1+r_*-2\Gamma b_2)\frac{\partial b_2}{\partial b_1(1+b_1)} b_2 u_2^T d\mathcal{F}_\phi\right] =$$
$$-\beta^2\Gamma\int_{\underline{\phi}}^{\overline{\phi}} b_2\frac{\partial b_2}{\partial IR}u_2^T d\mathcal{F}_\phi + \beta\int_{\underline{\phi}}^{\phi^c}\left(\frac{\partial(-b_2)}{\partial IR} + \frac{\partial(-b_2)}{\partial(-b_1)}\frac{d(-b_1)}{dIR}\right)\left(u_1^T - \beta(1+r_2)u_2^T\right)d\mathcal{F}_\phi \tag{38}$$

See $\frac{\partial(-b_2)}{\partial IR} + \frac{\partial(-b_2)}{\partial(-b_1)}\frac{d(-b_1)}{dIR} = \frac{\partial(-b_2)}{\partial(NFA)}\frac{dNFA}{dIR}$. Except for $\beta\int_{\underline{\phi}}^{\phi^c}\frac{\partial(-b_2)}{\partial(NFA)}\frac{dNFA}{dIR}\left(u_1^T - \beta(1+r_2)u_2^T\right)d\mathcal{F}_\phi$, all the terms are identical to the equation (13). From the equation of the borrowing in the states of binding credit constraints, we can immediately see $\frac{\partial(-b_2)}{\partial(NFA)} > 0$. Hence, the right hand side of the equation (38) has a positive sign. With non-negligible probabilities of sudden stop crisis, the solution of the equation (38) $IR*$can be strictly positive. □

A.7 Proof of Lemma 3

First, we show the borrowing of decentralized households under the optimal taxation is same as the direct choice of the social planner. The Euler equation of the households under the taxation will be

$$u_0^T(b_1,;) = \beta(1+r_1)(1+\tau_d)\left(\int_{\underline{\phi}}^{\phi^c} u_1^T(b_1, b_{2,c},;)\,d\mathcal{F}_\phi + \int_{\phi^c}^{\overline{\phi}} u_1^T(b_1, b_{2,u},;)\,d\mathcal{F}_\phi\right) \tag{39}$$

The solution of the planning is τ_d such that the equation (39) is ex-post identical to the equation (27), which characterizes the borrowing determined by the government. Of course, it implies $b_1^{priv}(\tau_d) = b_1^{sp}$.

Ignoring the term $r_1\tau_d$, solving for τ_d such that the equation (39) is same as the equation (27) yields the characterization of the optimal taxation in the equation (18). □

A.8 Proof of Proposition 4

To derive the result, we modify the equation (20) as follows.

$$b_t = \frac{1}{\Gamma\left(1+\tau_s\right)}\left(r_t - r_*\right) \tag{40}$$

Then, the tax revenue for the government, say π_t is

$$\pi_t = \tau_s\left(r_{t+1} - r_*\right)\left(-b_{t+1}\right) \tag{41}$$

The government problem of optimal taxation can be formulated as below.

$$\max_{t_s} u\left(c_0^T,\, y_0^N\right) + E_0\left[\beta u\left(c_1^T,\, y_1^N\right) + \beta^2 u\left(c_2^T,\, y_2^N\right)\right] \qquad \textit{subject to}$$

$$\begin{aligned}
c_0^T &= y_0^T + b_0(1+r_0) - b_1 - T + \pi_1 \\
c_1^T &= y_1^T + b_1(1+r_1) - b_2 + \mu IR \\
c_2^T &= y_2^T + b_2(1+r_2) + (1-\mu)\, IR\,(1+\overline{r}) \\
b_1 &= \textit{the solution of } (10) \\
-b_2 &= \begin{cases} \frac{-W_1+\sqrt{W_1^2+4(1+\beta)\Gamma y_2^T}}{2(1+\beta)\Gamma} & if \quad \phi \in \left[\underline{\phi}, \phi^c\right] \\ \frac{-r_*+\phi\psi\left(\frac{1-\alpha}{\alpha}\right)+\sqrt{\left(r_*-\phi\psi\left(\frac{1-\alpha}{\alpha}\right)\right)^2+4\Gamma\kappa}}{2\Gamma} & if \quad \phi \in \left[\phi^c, \overline{\phi}\right] \end{cases} \\
\mu &= \begin{cases} \textit{the solution of } (11) & if \quad \phi \in \left[\underline{\phi}, \phi^c\right] \\ 1 & if \quad \phi \in \left[\phi^c, \overline{\phi}\right] \end{cases} \\
r_t &= \begin{cases} -\Gamma\left(1+\tau_s\right) b_1 + r_* & if \quad t=1 \\ -\Gamma b_2 + r_* & if \quad t=2 \end{cases}
\end{aligned}$$

The first order condition is

$$u_0^T\left[\Gamma b_1^2\left(2\tau_s+1\right) + \frac{db_1}{d\tau}\left(\Gamma\left(\tau_s+\tau_s^2\right)2b_1 - 1\right)\right] = \beta H\left(\tau_s\right)\left[\left(1+r_*-2b_1\Gamma\left(1+\tau\right)\right)\frac{db_1}{d\tau_s} - \Gamma b_1^2\right] \tag{42}$$

where $H\left(\tau_s\right) = \int_{\underline{\phi}}^{\overline{\phi}}\left[\left(1-\frac{db_2}{db_1(1+b_1)}\right) + IR\frac{d\mu}{db_1}\right]u_1^T + \beta\left[\left(1+r_*-2\Gamma b_2\right)\frac{db_2}{db_1(1+b_1)} - IR\left(1+\overline{r}\right)\frac{d\mu}{db_1}\right]u_2^T d\mathcal{F}_\phi$

To reduce $H\left(\tau_s\right)$ to Ξ defined in Proposition 4, we use the fact that $u_1^T = \beta\left(1+r_1\right)u_2^T$ in the states that the credit constrain)t doesn't bind.After cancelling out some terms in $H\left(\tau_s\right)$ and rearranging the function, we can see $H\left(\tau_s\right) = \Xi$. To show $\Xi > 0$, we use $\frac{dV}{db_1} > 0$ because of the overborrowing. See $\frac{dV}{db_1} = -u_0^T + \left(1+r_*-2b_1\Gamma\left(1+\tau\right)\right)\Xi > 0$. We can immediately see $\Xi > 0$.

We ignore τ_s^2, assuming the tax rate is small enough. Then solving for τ_2 yields the equation (21).

Next, we show the optimal tax rate with the optimal borrowing is characterized by the equation (22) in Proposition 4. Once the borrowing is optimal, we can treat b_1 as another choice variable. Then we can elliminate $\frac{db_1}{d\tau_s}$ in the equation 42. From the first order condition of b_1, we can have

$$u_0^T = \beta\left(1+r_*-2b_1\Gamma\left(1+\tau\right)\right)\Xi \tag{43}$$

Plugging in the equation (43) into the equation (21) yields the equation (22).

We show $\widehat{IR}$ increases under the optimal taxation τ_s if c_o^T increases under the taxation. Once we showed that $\widehat{IR}$ increases under the optimal taxation τ_s if c_o^T increases, to show the other case that c_o^T decreases under the taxaton is trivial. If the optimal decision is to lend, there is no need to accumulate reserves. Thus, we only consider cases of $b_1^{sp} < 0$. By the equation, the optimum for the government is to subsidize international investors. One can notice it is same as lowering Γas a return to some payments of tradable goods. That is, imposing the optimal subsidizes is identical to different parameter values such that $y_0^T > y_0^T(\tau_s)$ and $\Gamma > \Gamma(\tau_s)$, where $x(\tau_s)$ is the hypothetical parameter values that derive the same result with the one under the optimal tax on international investors. It is trivial that theses changes in parameter values must increase the borrowing, $-b_1$. By the assumption, c_0^T rises, which implies the external liablilites in period 1 rise since the tradable goods consumption can't increase in both periods. That is, $-b_1(1+r_1)$ becomes large. Hence, c_1^T muct fall. After all, the optimal taxation on international investors drives down u_0^T, while driving up u_1^T and u_2^T accordingly. Because of the control on the borrowing τ_d, the government now doesn't have a concern over the moral hazard. To finalize the proof, recall the first order condition of reserves accumulation under the optimal taxation on the borrowing.

$$\beta \int_{\underline{\phi}}^{\phi^c} \frac{d(-b_2)}{dIR} \left(u_1^T - \beta(1+r_2)u_2^T\right) d\mathcal{F}_\phi + \beta IR \int_{\underline{\phi}}^{\phi^c} \frac{d\mu}{dIR} \left(u_1^T - \beta(1+\overline{r})u_2^T\right) d\mathcal{F}_\phi =$$

$$u_0^T\left(\frac{1}{1+\overline{r}}\right) - \beta\left(\int_{\underline{\phi}}^{\phi^c} u_1^T \mu d\mathcal{F}_\phi + \int_{\phi^c}^{\overline{\phi}} u_1^T d\mathcal{F}_\phi\right) - \beta^2\left(\int_{\underline{\phi}}^{\phi^c} u_2^T(1-\mu)(1+\overline{r})\, d\mathcal{F}_\phi\right)$$

See lower u_0^T, and higher u_1^T and u_2^T lead to a lower value of the right hand side in the equation above. Obviously, more external liabilities and the change of the marginal utilities raise the all the terms in the left hand side, except for $\frac{\partial\mu}{\partial IR}$. To determine how $\frac{\partial\mu}{\partial IR}$ changes according to the changes isn't obvious. However, see $\frac{d(-b_2)}{dIR} = \frac{\partial(-b_2)}{\partial IR} + \frac{\partial(-b_2)}{\partial\mu}\frac{d\mu}{dIR}$. Since $\frac{\partial(-b_2)}{\partial\mu} > 1$, if $\frac{d\mu}{dIR}$ decreases in $-b_1(1+r_1)$, the value of the right hand side rises. On the contrary, if $\frac{d\mu}{dIR}$ increases in $-b_1(1+r_1)$, we can use the assumption that $\frac{d^2(-b_2)}{dIR^2} > 0$. Obviously $\frac{d^2(-b_2)}{dIR^2} < 0$ if and only if $\frac{d^2(-b_2)}{dIRd(-b_1(1+r_1))} < 0$. Then all the terms in the left hand side increases if $\frac{d\mu}{dIR}$ increases in $-b_1(1+r_1)$. Hence, under the optimal taxationin, the right hand side fakks while the right hand side falls in either case. Since the equation above is the first order condition of a concave programming, this completes the proof. □