Equilibrium Leadership in Tax Competition When Capital Supply is Endogenous

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Abstract

In this paper, we model a timing game in asymmetric tax competition. As a feature of this model, capital competed among the two countries are supplied not only by the residents of the countries, but also from the outside, depending on rate of return to capital of the integrated market. It is found that, due to capital inflow from the outside of the two countries, sequential-move equilibria can be realized, in which one country leads and the other country follows.

Keywords: Tax competition; Endogenous timing; Capital supply; Regional disparities.

JEL classification H73, H77

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1 Introduction

The issue of the endogenized leadership has been one of important strands in the literature of tax competition, since the seminal work of Kempf and Rota-Graziosi (2010, hereafter K-RG). The study directs questions at one of basic assumptions that traditional models of tax competition have set from Zodrow and Mieszkowski (1986) and Wilson (1986), asking whether regional governments are really competing in Nash manner? In other words, is simultaneous setting of tax rates truly commitment robust for governments, if they can choose the relative timing to determine tax policies? To give an answer to the question, the study of K-RG applies the timing game, which is introduced by Hamilton and Slutsky (1990), to the model of tax competition; a pre-play stage to choose timing is set before the stage governments determine their tax rates. As the result, it is found that the simultaneous-move outcome does not emerge as the subgame perfect Nash equilibrium (hereafter, SPE) of the timing game, and only the sequential-move, or Stackelberg outcome, emerges, contrary to the assumption of canonical models of tax competition.

In regard to the result of K-RG, Ogawa (2013) indicates that it heavily depends on the assumption of absentee of capital ownership, and shows that only the simultaneous-move outcome is emerged, if it is assumed that capital competed among the countries are owned by residents of the countries, as standard models assume. Then, the argument of endogenized leadership in tax competition begins among researchers, varying settings of model in many ways to clarify a decisive factor of the leadership.1 In particular, Kemp and Rota-Graziosi (2015) and Hindriks and Nishimura (2017) succeed to describe the intermediate situation of the two polar cases of K-RG and Ogawa (2013), and give a detailed analysis on the role of capital ownership.

However, in the models of Kemp and Rota-Graziosi (2015) and Hindriks and Nishimura (2017), ratio of capital ownership (i.e., how much capital competed among countries is initially owned by residents within the economy and by non-residents) is exogenously given. It implies that amount of capital investment from the outside is assumed not to respond the rate of return determined in the market. Under the assumption, the governments engaged in the capital tax competition do not have incentive to attract capital investment from the outside to the market, but they do have incentive to attract capital initially owned by residents within their economy. It seems a lack of consistency.

In this paper, we reconsider the endogenized leadership of tax competition, focusing on the role of capital supply from the outside of the economy. Simply, it is assumed that there exist two asymmetric countries where the residents are initially endowed with a certain amount of capital, and capital is freely mobile between the countries through the integrated market. However, capital owner reside not only within the two countries, but also outside of the economy potentially. As a feature of this model analyzed in this paper, total amount of capital competed by the two countries is undetermined initially and depend on the return to capital investment, or price of capital in the market: a higher return attracts more capital investment from the outside, while a lower return attracts less capital investment.

As a result, it is found that the SPEs of this timing game are classified in three ways: i) two sequential-move equilibria emerge in which one country leads and the other country follows, ii) one sequential-move equilibrium emerges, in which the country with higher production technology leads and the country with lower production technology follows, and iii) one simultaneous-move equilibrium emerges in which both of the two countries determine their tax rates simultaneously. In addition, it is shown that, when

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1For instance, Eichner (2014) focuses on preference for public goods, which is financed by capital tax revenue, while Kemp and Rota-Graziosi (2010, 2015), Ogawa (2013), and Hindriks and Nishimura (2015, 2017) assume capital tax revenue is redistributed in lump-sum manner. Kawachi et al. (2015) adds one more stage of public investment competition to increase productive efficiency of capital, before the stage of tax competition. Ogawa and Susa (2017) consider heterogeneity of not only the countries, but also residents in a country, using the framework of majority voting. Pi and Chen (2017) present further analysis on the risk-dominant equilibrium in the tax leadership game.
capital invest from the outside two countries is more responsive to the rate of return in the market, the sequential-move outcome is more likely to emerge, and the simultaneous-move outcome is less likely to emerge.

The rest of this paper is organized as follows: Section 2 presents the model of timing game between two asymmetric countries. In Section 3, the main result of our analysis is proposed. Finally, we conclude in Section 4.

2 The Model

The economy we focus on is composed of two countries, country $H$ and country $L$. Firms in a country produce private goods using labor and capital with constant-returns-to-scale technology. Here, we assume that the market of private goods is perfectly competitive and production per capita is expressed by $f_i(k_i) = (A_i - k_i)k_i$, where $k_i$ represents capital per labor and $A_i$ stands for productive efficiency, or technology, of the firms in country $i$. In this model, productive efficiency $A_i$ is country specific parameter and the only factor asymmetry between the two countries. Without loss of generality, $A_H > A_L$ is assumed. The firm’s profit in country $i$ is given by $\pi_i = f_i(k_i) - (r + T_i)k_i - w_i$, where $r$ denotes the price of capital in the integrated market between the two countries, $T_i$ the unit tax rate on capital imposed by the government of country $i$, and $w_i$ the wage rate paid to labor.

The residents in each country are homogenous with respect to the amount of labor and initial endowment of capital. Particularly, a resident in each country is initially endowed with a unit of labor and 0.5 unit of capital. A resident in country $i$ has a simple preference, $u_i(c_i) = c_i$, where $c_i$ denotes the consumption of a numeraire private good. The total income of a resident in country $i$ consists of wage $w_i$, return to capital $0.5\kappa$, and a lump-sum transfer from the government $g_i$. Therefore, the budget constraint of the resident in country $i$ is given by

$$c_i = w_i + \frac{1}{2}r\kappa + g_i. \quad (1)$$

where $w_i = f_i(k_i) - f'_i(k_i)k_i = k_i^2$.

The government of country $i$ imposes unit tax $T_i$ on mobile capital employed by firms in the country, in order to finance the lump-sum transfer to the residents. Hence, the budget constraint of the government of country $i$ is given by

$$g_i = T_i k_i. \quad (2)$$

Using this budget constraint of the government, the utility function $u_i(c_i)$ can be rewritten as

$$u_i(c_i) = (A_i - k_i)k_i + r \left( \frac{1}{2}r\kappa - k_i \right), \quad (3)$$

which indicates that the utility of a resident in country $i$ consists of its GDP per capita and the net return from capital investment. With this formulation of utility, it enables to describe the situation where the manipulation of the terms-of-trade, or capital price in the integrated market, is the sole incentive to control capital tax $T_i$ (Peralta and Van Ypersele, 2005; Itaya et al., 2008; Ogawa, 2013).

As the feature of this model, the residents of other (outside) countries invest their capital into the two countries ($H$ or $L$). We assume here that the amount of capital flowing into the integrated market is positively related with the capital price and given by $br$. Thus, the total amount of capital in this economy is $\kappa + br$.

With the assumption that capital is freely mobile between the two countries, the market-clearing conditions are
Table 1. Payoff Matrix

<table>
<thead>
<tr>
<th>Country $H$ / Country $L$</th>
<th>First</th>
<th>Second</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>$u_H^N, u_L^N$</td>
<td>$u_H^{FS}, u_L^{FS}$</td>
</tr>
<tr>
<td>Second</td>
<td>$u_H^{SF}, u_L^{SF}$</td>
<td>$u_H^N, u_L^N$</td>
</tr>
</tbody>
</table>

Note. The first (second) coordinate in each pair is the utility in country $H$ ($L$).

$$r = A_i - 2k_i - T_i,$$

$$k_H + k_L = \kappa + br.$$

Under these conditions, the amount of capital in country $i$ and capital price in the market are derived as follows:

$$k_i = \frac{(1 + 2b)(A_i - T_i) - (A_j - T_j) + 2\kappa}{4(1 + b)},$$

$$r = \frac{A_H + A_L + T_H + T_L - 2\kappa}{2(1 + b)}.$$

3 Equilibrium

3.1 Timing Game

Following K-RG, Ogawa (2013), Hindriks and Nishimura (2015, 2017), and Ogawa and Susa (2017), we consider the timing game, which is introduced by Hamilton and Slutsky (1990). The timing game is composed of two stages: the period announcement stage and the tax determination stage. In the period announcement stage, the governments in countries $H$ and $L$ have two possible time periods—First and Second—for the choice of a tax rate. The both governments simultaneously decide and announce the period in which it will choose its tax rate. After that, in the tax determination stage, they have to choose their tax rates in only one of these two periods, recognizing the period in which the other government decides its tax rate.

If both governments determine to set their tax rates in the same period, a simultaneous-move game emerges in tax competition. This game is denoted as $G^N$. If the governments determine to set their tax rates in different periods from each other, a sequential-move game, or Stackelberg game, emerges. If country $H$ is the first mover and country $L$ is the second mover, we denote this game as $G^{FS}$. Similarly, country $H$ is the second mover and country $L$ is the first mover, we denote this game as $G^{SF}$. Based on comparison of residents’ utility, the SPEs of this timing game can be derived.

3.2 Equilibria of the Second Stage

3.2.1 Game $G^N$: Simultaneous-move Game

There are two possibilities of the simultaneous-move game: both of the two governments choose either the first or second period. As the preliminary results for the main argument of this study, we derive a lemma that shows the tax rates and utilities of residents in both countries.
Lemma 1. The tax rates, when chosen simultaneously for both countries, are yielded as follows:

\[
T^N_H = \frac{(1 + 4b^2 + 6b)A_H - (1 + 2b)A_L - 4b(1 + b)\kappa}{4(1 + b)(1 + 4b + 2b^2)}, \tag{8}
\]

\[
T^N_L = \frac{(1 + 4b^2 + 6b)A_L - (1 + 2b)A_H - 4b(1 + b)\kappa}{4(1 + b)(1 + 4b + 2b^2)}. \tag{9}
\]

Then, the utilities of the residents in each country are obtained as

\[
u^N_H = \frac{(1 + 2b)(3 + 2b)[(1 + 6b + 4b^2)A_H - (1 + 2b)A_L]^2}{64(1 + b)^2(1 + 4b + 2b^2)^2}
+ \frac{(4 + 33b + 86b^2 + 80b^3 + 24b^4)A_H + b(3 + 2b)(1 + 2b)^2A_L}{8(1 + b)(1 + 4b + 2b^2)^2} \kappa
- \frac{1 + 8b + 17b^2 + 8b^3}{4(1 + 4b + 2b^2)^2} \kappa^2, \tag{10}
\]

\[
u^N_L = \frac{(1 + 2b)(3 + 2b)[(1 + 6b + 4b^2)A_L - (1 + 2b)A_H]^2}{64(1 + b)^2(1 + 4b + 2b^2)^2}
+ \frac{(4 + 33b + 86b^2 + 80b^3 + 24b^4)A_L + b(3 + 2b)(1 + 2b)^2A_H}{8(1 + b)(1 + 4b + 2b^2)^2} \kappa
- \frac{1 + 8b + 17b^2 + 8b^3}{4(1 + 4b + 2b^2)^2} \kappa^2. \tag{11}
\]

Proof. See Appendix A.

The superscripts on variables are relevant to the game in which they are derived (e.g., \(u^N_H\) is the utility of resident in country \(H\) in game \(G^N\)). These expressions are also applied to the subsequent analysis.

3.2.2 Game \(G^{FS}\): H Leads, L Follows

There are two cases for sequential-move game in our model: one of the two countries chooses first, and the other chooses second. To begin with, we derive the tax rates and utilities of the residents in game \(G^{FS}\), where country \(H\) moves first to choose its tax rate and country \(L\) follows.

Lemma 2. When country \(H\) leads and country \(L\) follows, the tax rates are yielded as follows:

\[
T^{FS}_H = \frac{2(1 + b)(1 + 6b + 4b^2)A_H - 2(1 + b)(1 + 2b)A_L - 8b(1 + b)^2\kappa}{(1 + 6b + 4b^2)(5 + 10b + 4b^2)}, \tag{12}
\]

\[
T^{FS}_L = \frac{-(1 + 6b + 4b^2)A_H + (1 + 12b + 20b^2 + 8b^3)A_L - 2b(3 + 2b)(1 + 2b)\kappa}{(1 + 6b + 4b^2)(5 + 10b + 4b^2)}. \tag{13}
\]

Then, the utilities of the residents in each country are obtained as
Lemma 3 summarizes the results of the Stackelberg games. We derive the tax rates and utilities of
residents in game $G$. See Appendix B.

3.2.3 Game $G^H$: H leads, L follows

Lemma 3. When country L leads and country H follows, the tax rates and utilities of residents are
yielded as follows:

\[
\begin{align*}
T_H^S &= \frac{-(1 + 6b + 4b^2)A_L + (1 + 12b + 20b^2 + 8b^3)A_H - 2b(3 + 2b)(1 + 2b)}{(1 + 6b + 4b^2)(5 + 10b + 4b^2)} \kappa, \\
T_L^S &= \frac{2(1 + b)(1 + 6b + 4b^2)A_L - 2(1 + b)(1 + 2b)A_H - 8b(1 + b)^2}{(1 + 6b + 4b^2)(5 + 10b + 4b^2)} \kappa.
\end{align*}
\]

Then, the utilities of the residents in each country are obtained as

\[
\begin{align*}
u_H^S &= \frac{(1 + 2b)(3 + 2b) \left[-A_L(1 + 6b + 4b^2) + (1 + 12b + 20b^2 + 8b^3)A_H\right]^2}{4(1 + 6b + 4b^2)^2(5 + 10b + 4b^2)^2} \\
&+ \frac{\kappa}{4(6b + 4b^2 + 1)^2(10b + 4b^2 + 5)^2} \left[4b(1 + 6b + 4b^2)(3 + 2b)^2(1 + 2b)^2A_L\right] \\
&+ 2(25 + 382b + 2128b^2 + 5672b^3 + 8032b^4 + 6176b^5 + 2432b^6 + 384b^7)A_L \\
&- \kappa(25 + 400b + 2332b^2 + 6496b^3 + 9600b^4 + 7680b^5 + 3136b^6 + 512b^7), \\
\end{align*}
\]

\[
\begin{align*}
u_L^S &= \frac{\left[(1 + 6b + 4b^2)A_L - (1 + 2b)A_H\right]^2}{4(1 + 6b + 4b^2)(5 + 10b + 4b^2)^2} \\
&+ \frac{\kappa}{4(1 + 6b + 4b^2)(5 + 10b + 4b^2)^2} \left[2(5 + 6b)(1 + 6b + 4b^2)A_L\right] \\
&+ 8b(1 + b)(1 + 2b)A_H - (5 + 40b + 68b^2 + 32b^3)\kappa.
\end{align*}
\]

Proof. See Appendix C.
3.3 SPEs of the Timing Game

We obtain the SPEs of this timing game from the lemmas and a comparison of the residents’ utilities. The main results can be summarized as follows:

**Proposition 1.** In the timing game, we derive three types of equilibria based on technological asymmetry between the two countries and magnitude of capital supply from the outside. The equilibria are classified as follows:

(i) When $P_{TH}^T < 0$ and $Q_{TL}^T > 0$, there are two sequential-move equilibria: one country chooses its tax rate in the first period and the other in the second period,

(ii) When $P_{TH}^T > 0$ and $Q_{TL}^T > 0$, there is one sequential-move equilibrium: country $H$ chooses its tax rate in the first period and country $L$ in the second period,

(iii) When $P_{TH}^T > 0$ and $Q_{TL}^T < 0$, there is one simultaneous-move equilibrium: both countries choose their tax rates in the same (i.e., the first) period,

where

\[
\begin{align*}
P_{TH}^T & = (1 + 4b + 2b^2)(A_H - A_L) - 2b(1 + b)(A_H + A_L - 2\kappa), \\
Q_{TL}^T & = (1 + 4b + 2b^2)(9 + 80b + 168b^2 + 128b^3 + 32b^4)(A_L - A_H) \\
& + 2b(1 + b)(11 + 80b + 168b^2 + 128b^3 + 32b^4)(A_L + A_H - 2\kappa).
\end{align*}
\]

**Proof.** See Appendix D.

Figure 1: Equilibrium classification when $b = 0.3$

Notice that (iii) corresponds to the case of Ogawa (2013), in which there is no capital inflow from the outside of the two countries ($b = 0$). For reference, Figure 1 graphically depicts our results when $b = 0.3$ as an example, in which the shaded area presents the domain of definition of this model. The areas (i) to (iii) in the figure correspond to propositions (i) to (iii).
Overall, it is observed that the simultaneous-move outcome is derived when the asymmetry between the two countries is large. However, the Stackelberg outcome is derived with a little asymmetry between the countries.

In order to give an interpretation to the result shown in Figure 1, it is required to distinguish two types of incentive to control tax rate $T_i$ for the governments: a) Both of the two governments has the incentive to attract capital from the outside and obtain more domestic production, lowering capital tax rate. b) Each government has the incentive to control the price of capital in the market, depending on the position of the country (i.e., whether the country is a capital exporter or capital importer in each sub-game), which is called the terms-of-trade effect. These are reflected in the first and the second term of the utility function, $u_i = (A_i - k_i)k_i + r(\kappa_i/2 - k_i)$, respectively. In addition, we can point out that the former is the factor for a community of interest, while the latter is the factor for a conflict of interest between the two countries. The type of equilibrium emerging as the SPEs depends on whether the community or the conflict of interest works more significantly, compared to the other.

As well known in the literature of endogenous timing game, the emergence of the sequential-move game depends on whether the second mover can derive a benefit from and accepts the choice of the first mover. If there is not much benefit (or some damage) for the second mover, it refuses to become the second mover and the simultaneous-move game emerges as the SPE as a result of the timing game. As mentioned above, the first-mover advantage in this model is to manipulate the price of capital by changing the tax rate as the government wants: On one hand, if a government wants to attract capital invested from the outside countries, it can raise the price of capital in the market, by lowering tax rate. On the other hand, if a government as capital importer/exporter wants to lower/raise the price of capital, it has an incentive to set a higher/lower tax rate.

Under the situation where $b = 0$ shown in Figure 1, there is no capital investment from the outside. It implies that both of the two governments do not have incentive to attract capital from the outside and just compete for capital initially owned by themselves. As a result, their positions are clearly separated and their incentive for manipulating the price of capital differs from each other; country $H$ becomes a capital importer, due to its higher level of technology, and wants to set a higher tax rate, while country $L$ becomes capital exporter, due to its lower level of technology, and wants to set a lower tax rate. Only a conflict of interest exists between the two countries, so one country can not accept the action of the other country as the first mover. Therefore, there emerges only the simultaneous-move equilibrium as the SPE of this timing game, when $b = 0$.

However, under the situation where $b > 0$ shown in Figure 2, capital can be supplied also from the outside, so both of the two governments do have the common incentive to attract the capital, in order to increase their domestic production. In this case, there is not only the conflict of interest as capital importer and exporter, but also the community of interest between the two governments. Particularly, in the area close to the line of 45 degree, where regional asymmetry is not large, the community of interest dominates the conflict of interest; one country can accept the action of the other country as the first mover, hence, the sequential-move equilibria emerge as the SPE. Nevertheless, when the regional asymmetry is sufficiently large, the conflict of interest dominates the community of interest, and the simultaneous-move outcome emerges.

In addition, we examine the situation where $b$ is increased, or where capital outside of the two countries is more likely to respond to the rate of return in the market. The result can be summarized as follows:

**Proposition 2.** As $b$ increases, the classification of the equilibrium described is changed as

\[\text{From (7), it is observed that the capital price in the market is lowered (raised) if the tax rate in a country is raised (lowered): } \frac{\partial r}{\partial T_i} < 0. \] Hence, if a country is a capital importer (exporter), meaning that the amount of capital initially endowed is less (more) than the capital employed per capita in the country in an equilibrium, the government has an incentive to lower (raise) the capital price by raising (lowering) the tax rate in the country.
(i') The area of (i) in Figure 2 is widened, which implies that the two-sequential-move-equilibria is more likely to emerge as the SPEs of this timing game.

(ii') The area of (ii) in Figure 2 is shrunk, which implies that the one-sequential-move-equilibrium is less likely to emerge as the SPEs of this timing game, and eventually vanished.

(iii') The area of (iii) in Figure 2 is shrunk, which implies that the simultaneous-move-equilibrium is less likely to emerge as the SPEs of this timing game.

Proof. See Appendix E.

This result can be interpreted with the same manner mentioned above. When the capital market between the two countries is more opened for investment from the outside, one government, which is a potential second mover, is more likely to accept the action of the other government as the first mover, because both of the two governments does have the same incentive to attract capital through manipulating the price of capital in the market.

4 Concluding Remarks

In this study, we reexamined the endogenous leadership in asymmetric tax competition model, focusing on the role of capital supply. Particularly, we model the situation where capital owner reside not only in the two countries, but also outside of the economy potentially, so the amount of capital competed between the two countries depend on the return to capital, or the price capital in the market. This setting enables us to consider the incentive of the governments to attract capital from the outside.

As the main results of the analysis, we found that the SPEs of this timing game are classified as follows: i) two sequential-move outcome emerge, or one country leads and sets a tax policy and the other follows, ii) one sequential outcome emerges, or the country with higher level of production technology leads and the country with lower level of production technology follows, iii) one simultaneous-move outcome emerges, or both of the two governments set their tax policies at the same time. Additionally, it is found that, when capital of outside of the economy is more responsive to the return to capital in the market, the SPE of type i) is more likely to emerge.

In order to simply consider the role of capital supply from outside of the economy, we assume that its amount is determined linearly with respect to the return to capital. However, it can be non-linear, or more minutely, we have to take account of existence of “other market” that capital owners of outside of the economy can invest. These are remained for our future research.

Appendices

Appendix A. First, we begin with game $G^N$, where the two governments determine their tax rates simultaneously. The maximization problem of government of country $i$ is

$$\max_{T_i} u_i(c_i) = (A_i - k_i)k_i + r \left( \frac{1}{2}k_i - k_i \right),$$

s.t. (6) and (7).

The tax rates in the equilibrium of this game can be derived by solving the following simultaneous equations, consisting of the reaction functions of countries $H$ and $L$, which are derived from the maximization problems above, respectively:
\[ T_H(T_L) = \frac{T_L + (1 + 2b)A_H - A_L - 2bk}{(1 + 2b)(3 + 2b)}, \]  
\[ T_L(T_H) = \frac{T_H + (1 + 2b)A_L - A_H - 2bk}{(1 + 2b)(3 + 2b)}. \]  

Thus, we obtain (8) and (9). By substituting (8) and (9) into (6) and (7), we obtain the equilibrium values as follows:

\[ k^N_H = \frac{(1 + 2b)(1 + 6b + 4b^2)A_H - (1 + 2b)^2A_L + 4(1 + b)(1 + 3b)\kappa}{8(1 + b)(1 + 4b + 2b^2)}, \]  
\[ k^N_L = \frac{(1 + 2b)(1 + 6b + 4b^2)A_L - (1 + 2b)^2A_H + 4(1 + b)(1 + 3b)\kappa}{8(1 + b)(1 + 4b + 2b^2)}, \]  
\[ r^N = \frac{(1 + 2b)(A_i + A_j - 2\kappa)}{2(1 + 4b + 2b^2)}. \]

Using (22)-(24), we derive the utilities of residents in countries H and L in the simultaneous-move game as in (10) and (11).

**Appendix B.** Here, we derive the sequential equilibrium in game \( G^{FS} \), where country H leads and country L follows. The maximization problem of the government of country H is

\[ \max_{T_H} u_H(c_H) = (A_H - k_H)k_H + r \left( \frac{1}{2}\kappa - k_H \right), \]

s.t. \( (6), (7), \) and \( (20) \).

The first-order condition gives the equilibrium tax rate of country H as (12). Substituting (12) into (21) gives the equilibrium tax rate of country L as (13). Similarly, using the equilibrium tax rate, we obtain the equilibrium values as follows:

\[ k^FS_H = \frac{(1 + 6b + 4b^2)A_H - (1 + 2b)A_L + (5 + 6b)\kappa}{2(5 + 10b + 4b^2)}, \]  
\[ k^FS_L = \frac{1}{2(1 + 6b + 4b^2)(5 + 10b + 4b^2)} \left[ (1 + 2b)(1 + 6b + 4b^2)A_H 
+ (1 + 2b)(1 + 12b + 20b^2 + 8b^3)A_L + (5 + 34b + 56b^2 + 24b^3)\kappa \right], \]  
\[ r^{FS} = \frac{2(1 + b)(1 + 6b + 4b^2)A_H + (3 + 2b)(1 + 2b)^2A_L - (5 + 28b + 40b^2 + 16b^3)\kappa}{(1 + 6b + 4b^2)(5 + 10b + 4b^2)}. \]

From (25)-(27), we derive the utilities of residents of country H and L in the sequential-move game \( G^{FS} \) as (14) and (15).

**Appendix C.** Finally, we derive the equilibrium of the sequential-move game \( G^{SF} \), where country L leads and country H follows. The maximization problem of the government of country L is

\[ \max_{T_L} u_L(c_L) = (A_L - k_L)k_L + r \left( \frac{1}{2}\kappa - k_L \right), \]

s.t. \( (6), (7), \) and \( (20) \).
The first-order condition yields the equilibrium tax rate of country \( L \) as (17). Substituting (17) into (20), we obtain the equilibrium tax rate of country \( H \) as (16). The equilibrium values are derived from the equilibrium tax rates as follows:

\[
k^T_L = \frac{1}{2(5 + 10b + 4b^2)} \left[ -(1 + 2b)(1 + 6b + 4b^2)A_L \
+ (1 + 2b)(1 + 12b + 20b^2 + 8b^3)A_H + (5 + 34b + 56b^2 + 24b^3)\kappa \right],
\]

(28)

\[
k^T_L = \frac{(1 + 6b + 4b^2)A_L - (1 + 2b)A_H + (5 + 6b)\kappa}{2(5 + 10b + 4b^2)},
\]

(29)

\[
r^SF = \frac{2(1 + b)(1 + 6b + 4b^2)A_L + (3 + 2b)(1 + 2b)^2A_H - (5 + 28b + 40b^2 + 16b^3)\kappa}{(1 + 6b + 4b^2)(5 + 10b + 4b^2)}.
\]

(30)

From (28)-(30), the utilities of residents in countries \( H \) and \( L \) in the sequential-move game \( G^SF \) are (18) and (19).

**Appendix D.** In order to derive the SPEs of this timing game, we compare the utilities of the residents of the two countries. First, the utilities of residents of country \( H \) can be compared as follows:

\[
u^H_W - u^N_H = \frac{[-(1 + 6b + 4b^2)A_H + (1 + 2b)A_L + 4b(1 + b)\kappa]^2}{64(1 + 6b + 4b^2)(5 + 10b + 4b^2)(1 + b)^2(1 + 4b + 2b^2)^2},
\]

\[
u^H_W < u^N_H, \quad \therefore \quad u^H_W > u^N_H, \quad (31)
\]

\[
u^N_L - u^H_L = \frac{(3 + 2b)(1 + 2b)P^T_HQ^T_H}{64(1 + b)^2(1 + 4b + 2b^2)^2(5 + 10b + 4b^2)^2(1 + 6b + 4b^2)^2},
\]

\[
u^N_L > u^H_L, \quad \therefore \quad u^N_L < u^H_L \Leftrightarrow P^T_HQ^T_L \geq 0, \quad (32)
\]

where

\[
P^T_H \equiv (1 + 4b + 2b^2)(A_H - A_L) - 2b(1 + b)(A_H + A_L - 2\kappa),
\]

\[
Q^T_H \equiv (1 + 4b + 2b^2)(9 + 80b + 168b^2 + 128b^3 + 32b^4)(A_H - A_L)
+ 2b(1 + b)(11 + 80b + 168b^2 + 128b^3 + 32b^4)(A_H + A_L - 2\kappa).
\]

With assumption \( A_H > A_L \) and \( r^N = \frac{(1 + 2b)(A_H + A_L - 2\kappa)}{2(1 + 4b + 2b^2)} > 0 \), \( Q^T_H > 0 \) holds. Hence, (32) can be reduced as

\[
u^N_L \geq u^H_L \Leftrightarrow P^T_H \geq 0.
\]

Then, the utilities of residents of country \( L \) can be compared as follows:

\[
u^L_W - u^N_L = \frac{[-(1 + 6b + 4b^2)A_L + (1 + 2b)A_H + 4b(1 + b)\kappa]^2}{64(1 + 6b + 4b^2)(5 + 10b + 4b^2)(1 + b)^2(1 + 4b + 2b^2)^2},
\]

\[
u^L_W > u^N_L, \quad \therefore \quad u^L_W < u^N_L, \quad (34)
\]

\[
u^N_L - u^L_L = \frac{(3 + 2b)(1 + 2b)P^T_LP^T_S}{64(1 + b)^2(1 + 4b + 2b^2)^2(5 + 10b + 4b^2)^2(1 + 6b + 4b^2)^2},
\]

\[
u^N_L > u^L_L, \quad \therefore \quad u^N_L < u^L_L \Leftrightarrow P^T_LP^T_S \geq 0, \quad (35)
\]

where

\[
P^T_L \equiv (1 + 4b + 2b^2)(A_L - A_H) - 2b(1 + b)(A_L + A_H - 2\kappa),
\]

\[
Q^T_L \equiv (1 + 4b + 2b^2)(9 + 80b + 168b^2 + 128b^3 + 32b^4)(A_L - A_H)
+ 2b(1 + b)(11 + 80b + 168b^2 + 128b^3 + 32b^4)(A_L + A_H - 2\kappa).
\]
Similarly, with assumption $A_H > A_L$ and $r^N = \frac{(1+2b)(A_H + A_L - 2 \kappa)}{4(1+4b+2b^2)} > 0$, $P^T_L < 0$ holds. Hence, (35) can be reduced as

$$u^N_L \geq u^F_L \iff Q^T_L \leq 0.$$  

(36)

Therefore, in the domain of definition, we have three areas of the SPEs of this timing game: i) When $P^T_H < 0 \iff u^N_H < u^F_H$ and $Q^T_L > 0 \iff u^N_L < u^F_L$, there exist two sequential-move equilibria, where one leads and the other follows, ii) When $P^T_H > 0 \iff u^N_H > u^F_H$ and $Q^T_L > 0 \iff u^N_L < u^F_L$, there exists one sequential-move equilibrium, where country $H$ leads and country $L$ follows, and iii) When $P^T_H > 0 \iff u^N_H > u^F_H$ and $Q^T_L < 0 \iff u^N_L > u^F_L$, there exist one simultaneous-move equilibrium, where the two countries determine their tax rate in the first period.

In particular, when $b = 0$, $u^N_H - u^F_H = u^N_L - u^F_L = 27(A_H - A_L)^2 / 1600 > 0$. Taking account of the fact (31) and (34) always hold, it can be pointed out that only the simultaneous-move equilibrium emerges in the domain of definition.

Appendix E. As shown in Footnote 2, the line of $P^T_H = 0$ on $A_H - A_L$ plane is expressed as

$$A_L = \frac{1 + 2b}{1 + 6b + 4b^2} A_H + \frac{4b(1 + b) \kappa}{1 + 6b + 4b^2}.$$  

Then, we examine changes of its slope and its vertical intercept as $b$ increases:

$$\frac{\partial}{\partial b} \left( \frac{1 + 2b}{1 + 6b + 4b^2} \right) < 0 \quad \text{and} \quad \frac{\partial}{\partial b} \left( \frac{4b(1 + b) \kappa}{1 + 6b + 4b^2} \right) > 0,$$

which indicate that the slope of the line $P^T_H = 0$ becomes less steeper and the vertical intercept is increased. To see the values when $b$ is sufficiently large, we take the limits of both of them as

$$\lim_{b \to \infty} \left( \frac{1 + 2b}{1 + 6b + 4b^2} \right) = 0 \quad \text{and} \quad \lim_{b \to \infty} \left( \frac{4b(1 + b) \kappa}{1 + 6b + 4b^2} \right) = \kappa.$$  

Hence, the line of $P^T_H = 0$ eventually becomes a horizontal line with vertical intercept $\kappa$.

Similarly, the line of $Q^T_L = 0$ on $A_H - A_L$ plane is expressed as

$$A_L = \frac{(1 + 6b + 4b^2)(9 + 40b + 48b^2 + 16b^3)}{(3 + 2b)(3 + 44b + 200b^2 + 352b^3 + 256b^4 + 64b^5)} A_H + \frac{4b(1 + b)(11 + 80b + 168b^2 + 128b^3 + 32b^4) \kappa}{(3 + 2b)(3 + 44b + 200b^2 + 352b^3 + 256b^4 + 64b^5)} + \frac{(3 + 2b)(3 + 44b + 200b^2 + 352b^3 + 256b^4 + 64b^5)}{(3 + 2b)(3 + 44b + 200b^2 + 352b^3 + 256b^4 + 64b^5)}.$$  

Then, we examine changes of its slope and its vertical intercept as $b$ increases:

$$\frac{\partial}{\partial b} \left( \frac{(1 + 6b + 4b^2)(9 + 40b + 48b^2 + 16b^3)}{(3 + 2b)(3 + 44b + 200b^2 + 352b^3 + 256b^4 + 64b^5)} \right) < 0,$$

$$\frac{\partial}{\partial b} \left( \frac{4b(1 + b)(11 + 80b + 168b^2 + 128b^3 + 32b^4) \kappa}{(3 + 2b)(3 + 44b + 200b^2 + 352b^3 + 256b^4 + 64b^5)} \right) > 0,$$

which indicate that the slope of the line $Q^T_L = 0$ becomes less steeper and the vertical intercept is increased. To see the values when $b$ is sufficiently large, we take the limits of both of them as

$$\lim_{b \to \infty} \left( \frac{(1 + 6b + 4b^2)(9 + 40b + 48b^2 + 16b^3)}{(3 + 2b)(3 + 44b + 200b^2 + 352b^3 + 256b^4 + 64b^5)} \right) = 0,$$

$$\lim_{b \to \infty} \left( \frac{4b(1 + b)(11 + 80b + 168b^2 + 128b^3 + 32b^4) \kappa}{(3 + 2b)(3 + 44b + 200b^2 + 352b^3 + 256b^4 + 64b^5)} \right) = \kappa.$$  

Hence, the line of $Q^T_L = 0$ eventually becomes a horizontal line with vertical intercept $\kappa$. 

12
References


