

# On Separation between Payment and Saving Instruments

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## Abstract

This paper presents a parsimonious model to analyze why the central bank, which issues payment instruments, and commercial banks, which supply saving instruments, must be separated from each other as observed in the modern banking system. The supply of private banknotes by commercial banks improves risk sharing in the economy. If there is sunk cost of production, however, commercial banks issuing private banknotes are subject to the possibility of a self-fulfilling bank run. There exists a case in which the possibility of a self-fulfilling bank run can be eliminated if the central bank re-discounts commercial bills held by commercial banks.

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# 1 Introduction

Historically, privately-held banks used to issue private banknotes. Subsequently, however, the central bank monopolized note issue in most countries. As a result, payment instruments, i.e., central-bank notes, and saving instruments, i.e., bank deposits, are separated in the modern banking system.

This paper presents a parsimonious model to endogenize this structure of the banking system. The model features credit creation by commercial banks, in which commercial banks swap IOUs with payers. Payees become the ultimate creditors of payers via commercial banks when they receive commercial-bank IOUs paid by payers. This arrangement improves welfare, as a commercial-bank IOU backed by a pool of payers' IOUs diversifies idiosyncratic default risk in payers' IOUs for a payee. Commercial-bank IOUs in this arrangement can be regarded as private banknotes, as they serve as payment instruments.

Commercial banks, however, are subject to a self-fulfilling run through payees' mass refusal of their IOUs, if payers' production involves sunk cost. In this case, if payers have already borrowed commercial-bank IOUs in exchange for their IOUs, then they must default on their IOUs issued to commercial banks in the past. Resulting default losses to commercial banks justify payees' mass refusal of commercial-bank IOUs. This result can be interpreted as a run on private banknotes.

Commercial banks can shrink the parameter space in which a self-fulfilling run is possible by creating a separate entity that issues only payment instruments. The separate entity in the model can be regarded as the central bank in practice. It will be shown that the central bank policy to prevent a self-fulfilling bank run resembles short-term re-discounting of commercial bills in practice. It also implies that the optimal supply of central-bank notes is elastic.

In the parameter space in which the possibility of a self-fulfilling run remains, it can be shown that introducing the central bank still reduces the aggregate amount of bank capital in the banking system that is necessary to eliminate the possibility of a self-fulfilling run.

Thus, the separation of payment and saving instruments in the modern banking system can be regarded as a capital-saving mechanism.

This paper derives the need for payment instruments given the availability of trade credit in the economy. This approach contrasts with the standard theory of money, such as Samuelson (1958), Townsend (1980), and Kiyotaki and Wright (1989) for example, in which people need money for the transfer of value because they cannot arrange credit. In this regard, this paper is similar to Freeman's (1996) model, which highlights the complementarity between money and credit in a spatial economy with limited communication. Freeman's model also implies the optimality of elastic money supply through the central bank's discount window. This paper is different from Freeman's work in highlighting credit creation by commercial banks and the endogenous use of central-bank notes, whereas Freeman's model features fiat money in an overlapping-generations economy.

A self-fulfilling run on banknotes featured in this paper is related to Diamond and Dybvig's (1983) model, which characterizes a run on demand deposits as a coordination failure. It is a new contribution to the literature to show that separating the central bank from commercial banks can prevent a self-fulfilling bank run.

## 2 Baseline model

Time is discrete and indexed by 0, 1, and 2. There exist two types of agents, manufacturers and loggers, each of which is a unit continuum. Each agent is indexed by  $(i, j) \in \{M, L\} \times [0, 1]$ , where  $i$  is  $M$  and  $L$  if the agent is a manufacturer and a logger, respectively, and  $j$  is the index among agents of the same type in a unit continuum. Each agent is risk-averse, maximizing the expected utility function,  $Eu(c_{i,j})$ , where  $E$  is the expectation operator in period 0;  $u' > 0$  and  $u'' < 0$ ; and  $c_{i,j}$  denotes each agent's consumption of goods in period 2.

Each logger is endowed with a unit of wood in period 0, and can produce an amount  $\underline{\alpha}$  of goods in period 2 by using a unit of wood. In contrast, manufacturers receive no endowment

of wood in period 0. If a manufacturer uses a unit of wood in period 0, however, it can produce an amount  $\bar{\alpha}$  of goods with probability  $\mu_j$  and no goods with probability  $1 - \mu_j$  in period 2, where  $j$  ( $\in [0, 1]$ ) is the index for each manufacturer. For all  $j \in [0, 1]$ ,  $\mu_j$  is an i.i.d. random variable following a uniform distribution over  $[0, 1]$ . Assume that

$$\bar{\alpha} > 4\underline{\alpha} > 0 \tag{1}$$

The value of  $\mu_j$  is revealed publicly in period 1 for  $j \in [0, 1]$ .

In period 0, each manufacturer is randomly matched with a logger, and vice versa. Thus, every agent has one match. In each match, a manufacturer can make a take-it-or-leave-it offer to buy wood from a logger in exchange for the manufacturer's IOU that promises to deliver goods in period 2. A manufacturer can be committed to delivering any share of goods produced in period 2 upon successful production. Call a manufacturer's IOU a "commercial bill". Assume each commercial bill is divisible.

There exists a competitive mutual fund market in period 1, in which loggers can swap commercial bills received from manufacturers for mutual fund shares. There is a free entry of mutual funds into this market. Because commercial bills are divisible, a mutual fund can hold a share of every manufacturer's commercial bill for diversification.<sup>1</sup> Loggers can arrive at the mutual fund market only after the revelation of  $\mu_j$  for  $j \in [0, 1]$  in period 1.

An equilibrium is characterized by a zero profit condition for each mutual fund; a take-it-or-leave-it-offer of a commercial bill by a manufacturer to a logger in each pairwise meeting; and each agent's rational expectation. See Table 1 for a summary of the baseline model.

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<sup>1</sup>Alternatively, redistribute a unit continuum of manufacturers over a  $[0, 1] \times [0, 1]$  plane, which is possible because the density of  $[0, 1]^n$  is the same for  $n = 1, 2$ . Also, suppose that there is a unit continuum of mutual funds. For each manufacturer, the horizontal coordinate represents the index for a mutual fund buying the manufacturer's commercial bill. By the law of large numbers, the value of  $\mu_j$  for manufacturers sharing the same horizontal coordinate is distributed over  $U[0, 1]$ . Thus, it is possible to consider an alternative set-up in which each mutual fund holds a perfectly diversified portfolio of commercial bills, while each commercial bill issued by a manufacturer is not split among multiple mutual funds.

Table 1: Chronological order of events in the baseline model

Period 0	Manufacturers can make take-it-or-leave-it offers of their commercial bills for loggers' wood in pairwise meetings.
Period 1	Idiosyncratic default probability for each commercial bill is revealed publicly.  Loggers can swap commercial bills for mutual fund shares in a competitive market.
Period 2	Manufacturers repay their commercial bills by the goods they produce upon successful production.

## 2.1 Equilibrium

The zero profit condition for mutual funds implies that each commercial bill is priced fairly in the mutual fund market in period 1:

$$p_j = \mu_j b \quad (2)$$

for  $j \in [0, 1]$ , where  $p_j$  denotes the face value of a mutual fund share that is exchanged for manufacturer  $j$ 's commercial bill, given that the face value of the commercial bill equals  $b$ . Both face values,  $p_j$  and  $b$ , are in terms of the amounts of goods repayable in period 2. The value of  $b$  is the same across manufacturers, given the symmetry of manufacturers in period 0. Note that  $p_j$  involves no risk premium, as each mutual fund can diversify idiosyncratic default risk in commercial bills by holding a share of every manufacturer's commercial bill.

Given (2), each manufacturer chooses the minimum value of  $b$  that satisfies a logger's participation constraint for selling wood in a pairwise meeting in period 0:

$$\int_0^1 u(\mu b_{BL}) d\mu = u(\underline{\alpha}) \quad (3)$$

where  $b_{BL}$  denotes such a value of  $b$ . The left-hand side and the right-hand side of (3) are a logger's expected utilities when selling wood for a commercial bill whose face value equals  $b_{BL}$  and when retaining wood, respectively. It is feasible for a manufacturer to repay the

commercial bill upon successful production if and only if  $\bar{\alpha} \geq b_{BL}$ . Otherwise, loggers retain wood for their own production.

### 3 Credit creation by commercial banks

The baseline model features a time lag between a goods market transaction and a securities market transaction. The underlying assumption is that a market participant cannot synchronize different market transactions perfectly. Given this time lag, it is natural to assume that there can be a revision of the idiosyncratic default probability of each commercial bill anytime while the holders of commercial bills transit from one market to another. Such an update on idiosyncratic default risk is represented by the revelation of  $\mu_j$  for  $j \in [0, 1]$  in period 1.

#### 3.1 Introducing commercial banks

Incomplete risk sharing due to a time lag between different market transactions can be remedied if mutual funds pool commercial bills in advance, so that manufacturers can use mutual funds' IOUs as payment instruments. Given this feature of the IOUs, call such mutual funds “commercial banks”, and call the funds' IOUs “private banknotes”. See Table 2 for the classification of IOUs in the presence of commercial banks.

Table 2: Classification of IOUs in the model with commercial banks

Issuers	Name of IOUs
Manufacturers	Commercial bills
Commercial banks	Private banknotes

Suppose that there exists a unit continuum of commercial banks. At the beginning of period 0, manufacturers and commercial banks can swap IOUs with each other in a competitive market. Then, manufacturers and loggers are matched pairwise in the same period. The subsequent events are as same as in the baseline model. The definition of an equilibrium is as same as in the baseline model, except that it includes a zero profit condition

for each commercial bank, given no limit on the amount of commercial bills that can be held by each commercial bank. See Table 3 for a summary of events in the model with commercial banks. The mutual fund market in period 1 is omitted from Table 3, as it does not play any role in the presence of commercial banks.

Table 3: Chronological order of events in the model with commercial banks

Period 0	Manufacturers can swap their commercial bills for private banknotes issued by commercial banks in a competitive market.
	Manufacturers can make take-it-or-leave-it offers of their commercial bills and private banknotes for loggers' wood in pairwise meetings.
Period 1	Idiosyncratic default probability for each commercial bill is revealed publicly.
Period 2	Manufacturers repay commercial bills by the goods they produce upon successful production. Commercial banks repay private banknotes by repayments on commercial bills they hold.

### 3.2 Equilibrium with commercial banks

In the equilibrium, the zero profit condition for each commercial bank implies that commercial bills are exchanged for private banknotes at a fair price in period 0:

$$q = E[\mu_i b] = \frac{b}{2} \quad (4)$$

where  $b$  denotes the face value of each commercial bill issued by a manufacturer, and  $q$  denotes the face value of private banknotes that are exchanged for each commercial bill. Both face values,  $b$  and  $q$ , are in terms of the amount of goods repayable in period 2. Like mutual fund shares in the baseline model, private banknotes are risk-free because each commercial bank can hold a share of every manufacturer's commercial bill to diversify idiosyncratic default risk.

Given (4), a manufacturer chooses the minimum value of  $b$  that satisfies a logger's par-

ticipation constraint for selling wood in a pairwise meeting in period 0:

$$u\left(\frac{b_{BK}}{2}\right) = u(\underline{\alpha}) \quad (5)$$

where  $b_{BK}$  denotes such a value of  $b$ . The left-hand side and the right-hand side of (5) are a logger's expected utilities when selling wood for risk-less private banknotes whose face value equals  $b_{BK}/2$ , and when retaining wood, respectively. Thus,  $b_{BK} = 2\underline{\alpha}$ , given the monotonicity of  $u$ . Because  $\bar{\alpha} > b_{BK}$  as implied by (1), it is feasible for each manufacturer to issue a commercial bill whose face value equals  $b_{BK}$ .

Given  $u'' < 0$ , Jensen's inequality implies that

$$b_{BK} < b_{BL} \quad (6)$$

This difference is due to a risk premium on a commercial bill that a logger charges when receiving an individual commercial bill subject to an update on the idiosyncratic default probability. Thus, manufacturers have incentive to swap their commercial bills for private banknotes in period 0 to save a risk premium on payment instruments.

The exchange between commercial bills and private banknotes in period 0 corresponds to lending of private banknotes by commercial banks. It can be also seen as credit creation, as the extension of bank credit to manufacturers takes place before loggers become the ultimate creditors of manufacturers via commercial banks after receiving private banknotes.

## 4 Separating the central bank from commercial banks

To be written.

## 5 Conclusions

To be written.



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