Immiserizing Growth and Mobility of Capital
with and without Land in Harris-Todaro Model

Tadashi Inoue
Hiroshima Shudo University

Abstract

Under various frameworks of Harris-Todaro two sector model, the effects of the change in capital, population and land on outputs, employment of labor, employment ratio of urban sector and welfare are examined. The possibility of immiserizing growth is seen to give rise with tariff ridden imports and foreign capital inflow with or without full interest payment repatriation for both sector specific capital and mobile capital irrespective of the existence of land in production function.

Key words: Harris-Todaro model, sector-specific capital, mobile capital, immiserizing growth
JEL Classification Codes: F12, F16
I. Introduction

This paper tries to investigate the Harris-Todaro (1970, H-T) model focusing on the mobility of capital with and without land as an additional productive factor. Especially the possibility of immiserizing growth is examined in H-T model in various framework. This possibility has been first advocated by Bhagwati (1968) and Johnson (1967), who show that the real income of a small host country which imports capital intensive good lowers with foreign capital inflow when interest payment is fully repatriated. This proposition has been furthered by Brecher-Alejandro (B-A 1977) in the standard 2x2 small open country economy.

Khan (1982) confirmed B-A’s results employing generalized H-T model, i.e., with urban unemployment, three factors (labor, sector-specific capital and land) and fixed urban formal wage rate. Beladi and Naqvi (1988) showed that the case of non-immiserizing growth caused by technological progress within the standard H-T model.

Furthermore Grinols (1991) questioned the immiserizing growth by emphasizing the role of an urban informal sector within the standard H-T model. Chandra and Khan (1993) further extended Grinols’ results employing three sector (i.e., rural, urban formal and urban informal) and two factor model.

The role of urban informal sector in the H-T model is further investigated by Kar and Marjit (2001), Marjit and Beladi (2002) and Marjit (2003), among others.

Lastly the possibility or impossibility of immiserizing growth was considered on the basis of the envelope property by Marjit and Beladi (2003) and Choi and Yu (2006).

In the next section first in II. 1, as a benchmark we review the basic properties of the standard 2 x 2 H-T model in case of sector specific capital without foreign capital inflow following basically the framework of Choi and Yu (2006). Here there are two sectors, agricultural or rural sector (sector 1) and industrial or urban sector (sector 2), and two productive factors labor and capital. Here labor is mobile, while capital is immobile (i.e., sector specific). Urban wage rate is fixed, while rural wage rate is flexible. There exists unemployment of urban labor. Rural wage rate is equal to expected urban wage rate (i.e., urban employment rate times urban wage rate). Under these setting, the effects of the change in capital and population on outputs, employment and welfare are considered in Proposition 1.

Here we obtain as Proposition 1, that the increase in sector specific capital of agriculture and total population raises its employment, wage rate and production, and social welfare, but does not affect the production of industrial sector, while the increase in sector specific capital of industrial sector raises its employment, wage rate and production, and
social welfare, but decreases output of agricultural sector. Next in II. 2, the above results are examined in the economy with foreign capital inflow and with tariff ridden imports. Here the above results are shown to remain valid except the effects of change in sector specific capital of industry (i.e., foreign capital inflow). Especially as for the welfare effects of foreign capital inflow into industry, a necessary and sufficient condition for the occurrence of immiserizing growth is given in terms of the level of import tariff rate, generalizing Choi and Yu(2006)’s results (Proposition 2).

Furthermore in II. 3, the model is generalized into the case with land as an additional factor, and the results of Proposition 2 are shown to remain valid with additional results of the effects of the change in land. (Proposition 3)

Henceforth the case of mobile capital is discussed. First in III. 1, 2x2 H-T model (without land) with mobile capital is considered. This is the simplest case since every price of good and wage rates are fixed. Then the envelope property holds, and hence, the well known standard immiserizing growth rises in case of capital intensive import good (which is an industrial good).

Next in III. 2, the case of mobile capital with land is considered. As seen the introduction of land makes the analysis fairly complicated. First the effects of changes in factor endowments on outputs and agricultural wage rate are considered. (Proposition 4) Then in III. 3, the welfare effects of foreign capital inflow, especially the possibility of immiserizing growth are discussed (Proposition 5). Here we show again that a necessary and sufficient condition for the occurrence of immiserizing growth is given in terms of the level of import tariff rate, which is similar to that of sector specific capital (proposition 3). This further implies that our Propositions 2, 3 and 5 are robust irrespective of the sector specific capital or not (i.e., mobile capital). Our results are new to our best knowledge.

Lastly in IV. we conclude.

II. 1. Sector Specific Capital without Foreign Capital Inflow
Here, first we review the specific capital model analyzed by Choi and Yu (2006). Let $F_i(L_i, K_i)$ be the neoclassical production function of sector $i$, $i=1, 2$ which is homogeneous of degree one in $(L_i, K_i)$, and concave where $L_i$ and $K_i$ being respectively the amounts of labor and capital employed in this sector, $i=1$ and 2 being respectively agriculture (rural sector) and industry (urban sector). The price of products of agriculture is one and that of industry is $p$, which is fixed due to the small country assumption. Labor can move freely between two sectors, but due to the
existence of urban labor unemployment, the following equilibrium condition holds
\[ w_1 = \ell \cdot w_2 \]  
where \( w_1 \) (respectively \( w_2 \)) is the wage rate in the agricultural sector (respectively industrial sector) and \( \ell = L_2 / N_2 \leq 1 \) is the employment rate in the industrial sector, and \( N_2 \) is the urban population. Eq. (1) tells that in equilibrium the expected urban wage rate \( \ell w_2 \) equals the rural wage rate \( w_1 \). Here the full employment of labor prevails in the agricultural sector due to its flexible wage rate \( w_1 \) while unemployment of labor can exists in the industrial sector due to its sticky wage rate \( w_2 \). Let \( N_i \) be the population of sector \( i, \ i = 1, 2 \). Then \( N_i = L_i \) and \( N_2 = L_2 / \ell \) hold, and hence
\[ L_1 + L_2 / \ell = N \]  
follows where \( N \) being the total population.

Let \( X \) and \( Y \) be respectively the amounts of products of agriculture and industry so that \( X = F_i(L_i, K_i) \) and \( Y = F_2(L_2, K_2) \) hold by definition. Capital is sector-specific so that the rental prices of capital can be different, i.e.,
\[ r_i = F_{ik}(L_i, K_i) \]  
and
\[ r_2 = pF_{k2}(L_2, K_2) \]  
hold where \( r_i \) being the rental price of capital of sector \( i, \ i = 1, 2, \) and \( F_{ik} = \partial F_i(L_i, K_i) / \partial K_i \) being the marginal products of capital of sector \( i, \ i = 1, 2 \). Similarly as for the labor markets
\[ w_1 = F_{le}(L_1, K_1) \]  
and
\[ w_2 = pF_{le2}(L_2, K_2) \]  
where \( F_{le} = \alpha F_i(L_1, K_1) / \alpha L_1 \) being the marginal products of labor of sector \( i, \ i = 1, 2 \). Then in view of the duality theory
\[ 1 = G_i(w_i, r_i) \]  
and
\[ p = G_2(w_2, r_2) \]  
hold where \( G_i \) being the unit cost function of sector \( i, \ i = 1, 2 \), which is concave and homogeneous of degree one in \((w_i, r_i)\). Here we note from (9), since both \( p \) and \( w_2 \) are fixed, so is \( r_2 \). Then from \( r_2 = p f_2(k_2) \) where \( f_2(k_2) = F_2(1, k_2) \) being the labor productivity function and \( k_2 = K_2 / L_2 \) being the capital-labor ratio of sector 2. Then recalling \( r_2 \) to be fixed, so is \( k_2 \), implying \( L_2 \) is fixed given \( K_2 \), or more generally
\[ L_2 = k_2^{-1} K_2 \]  
holds where \( k_2 \) is fixed. Here by totally differentiating (2), we obtain
\[ w_1 dL_1 + w_2 dL_2 = (L_2 / \ell) d\ell w_1 + w_1 dN \]
Eq. (11) implies that $w_1 dL_1 + w_2 dL_2 \neq w_i dN$ and hence that so called envelope property does not hold in this case, as shown below.

In fact by observing
\[
dX + pdY = w_1 dL_1 + w_2 dL_2 + r_1 dK_1 + r_2 dK_2 = (L_2 / \ell) dw_1 + w_1 dN + r_1 dK_1 + r_2 dK_2
\]
and hence the envelope property
\[
dX + pdY = w_1 dN + r_1 dK_1 + r_2 dK_2
\]
does not follow.

Let $E(p, u)$ be the expenditure function which is the minimum value of the expenditure to obtain the utility level $u$, i.e.,
\[
E(p, u) = D_1 + pD_2 = \min \left\{ \ell \mid U(D_1, D_2) = u, D_1 + pD_2 \leq I \right\} = X + pY
\]
where $D_i$ is the level of consumption of good $i$, $i = 1, 2$, and $U(D_1, D_2)$ is the aggregate utility function.

Then the change in the level of social welfare $W$ is defined to be
\[
dW = E_u d_u = dD_1 + pdD_2 = dX + pdY.
\]

Next we derive the effects of the change in the sector specific capital $K_1, K_2$, total population $N$ on the amounts of products $X, Y$ and social welfare $W$.

**Proposition I**

1. $K_1 \uparrow \rightarrow w_1 \uparrow, L_1 \uparrow, \ell \uparrow, X \uparrow, W \uparrow$
   
   no change in $Y, L_2$

2. $K_2 \uparrow \rightarrow w_1 \uparrow, L_2 \uparrow, \ell \uparrow, L_1 \uparrow, X \uparrow, Y \uparrow, W \uparrow$

3. $N \uparrow \rightarrow w_1 \uparrow, L_1 \uparrow, \ell \uparrow, X \uparrow, W \uparrow$
   
   no change in $Y, L_2$

That is to say,

1. the increase in capital stock of agriculture $K_1$ raises employment of labor, the production of agriculture wage rate, employment rate of the industrial sector and social welfare, but does not change the production and employment of industrial sector. ($\ell \uparrow$ and no change in $L_2$ imply $N_2 \uparrow$.)

2. The increase in capital stock of industrial sector $K_2$ lowers employment and the production of agriculture, but raises wage rate of agriculture, employment, employment rate, production of the industrial sector and social welfare

3. The increase in the total population $N$ lowers wage rate of the agricultural sector,
but raises its employment and production of the agriculture employment rate of the industrial sector and social welfare but does not change industrial production and its employment.

Proof
1. From (11) with \( dN = dK_2 = dL_2 = 0 \)
   \[ (dK_2 = dL_2 = 0 \text{ follows from } (10)), \quad w_1 dL_4 = (L_2 / \ell)dw_1 \text{ follows.} \]
   Then observing from (6) and the above, \( dw_1 = F_{1LL} dL_4 + F_{1dK} dK_1 = F_{1dK} L_2 \ell^{-1}w_1^{-1}dw_1 + F_{1dK} dK_1 \), we obtain
   \[ (1 - F_{1LL} L_2 \ell^{-1}w_1^{-1})dw_1 = F_{1dK} dK_1, \]
   and hence \( K_1^+ \rightarrow w_1^+ \) and hence \( L_1^+, X^+ \) and \( \ell^+ = (w_1/w_2) \). Next recalling \( dK_2 = dL_2 = 0 \), and hence no change in \( L_2 \) and \( Y \), and hence \( w^+ \) from (15).

2. From (10) \( K_2^+ \rightarrow L_2^+ \) follows and hence \( Y^+ \). From (6) with \( dK_1 = 0 \), we obtain \( dw_1 = F_{1LL} dL_4 \). Combining this with (11) with \( dN = 0 \), we observe
   \[ (L_2 \ell^{-1} - w_1 F_{1dK}^{-1})dw_1 = w_2 dL_2, \]
   and hence \( w_1^+, L_1^+, X^+ \) and \( \ell^+ \). \( W^+ \) follows from (12) and (15) with \( dw_1 = F_{1LL} dL_4 \)
   \[ \ell^+ = L_2, \ell^+ = L_2, \]
   and hence \( N^+ \rightarrow L_1^+, w_1^+, \ell^+ \) and \( X^+ \). From (10) no change in \( L_2 \) and \( Y \), and hence \( W^+ \) follows.

In short the increase in the agricultural capital stock \( K_1 \) raises all variables except employment and production of the industrial sector, while the increases in the industrial capital stock raises its employment, employment rate production, and social welfare but lowers the wage rate, employment and production of the agriculture. Lastly the increase in the total population \( N \) raises agricultural employment and production, lowers its wage rate, and does not change industrial employment and production (and hence raises social welfare).

In the other words, the increase in the agricultural capital stock develops agriculture but does not affect industrial sector, while the increase in the industrial capital stock advances the industrial sector at the costs of the decline of the agricultural sector. Lastly the increase in the total population raises agricultural production with decline in its wage rate, but does not affect industrial sector.
II. 2. Sector Specific Capital with Foreign Capital Inflow

Next we generalize the above results by introducing foreign capital inflow. We assume the agricultural goods are non traded, and the payment for the foreign capital is done in term of the industrial good. Let \( t \) be the tariff rate for the imports of the industrial good, and \( p^* \) (resp. \( p \)) be the international (resp. domestic) price of the industrial good. Then by construction \( p = (1 + t)p^* \) and the expenditure function is expressed as

\[
E(p, u) = D_i + pD_2 = X + pY + tp^*(D_2 - Y) - r_i \cdot K'_i, \quad i = 1, 2
\]  

(16)

Case 1. If the paid interest rate is equal to its domestic interest rate \( r_i \) in sector \( i \), \( i = 1, 2 \) where \( K'_i \) is the foreign capital inflow into sector \( i \), \( M = D_2 - Y \) is the amounts of imports of industrial good, \( tp^*M \) is its tariff proceeds, and \( r_iK'_i \) is the interest payment for the foreign capital.

Case 2. If the paid interest is equal to the international interest rate \( r^* \), then the expenditure function is expressed as

\[
E(p, u) = D_i + pD_2 = X + pY + tp^*(D_2 - Y) - r^* \cdot K'_i.
\]  

(17)

We assume \( r^* < r_i \), \( i = 1, 2 \).

Let \( m = p \cdot h_2 \) be the marginal propensity to consume good 2 where \( h_2(p, I) = (D_2) \) being the demand function for good 2 and \( I = E \) being the amount of income. Both goods are normal good so that \( 0 < m < 1 \). Then for Case 1 from (11) with \( dN = 0 \) and (17), (15) is rewritten as

\[
\left(1 - \frac{t}{t+1}m\right)dw = L_2 \varepsilon^{-1}dw_i - tp^*dY.
\]  

(18)

noting \( dK_i = dK'_i \) and \( dK_j = 0 \) for \( j \neq i \).

For Case 2, i.e., \( r^* < r_i \), \( i = 1, 2 \), (15) is expressed as

\[
\left(1 - \frac{t}{t+1}m\right)dw = L_2 \varepsilon^{-1}dw_i + (r_i - r^*)dK'_i - tp^*dY, \quad i = 1, 2.
\]  

(19)

Then we obtain

**Proposition 2**

1. With Foreign Capital inflow and tariff on imports of industrial sector for both Cases 1 and 2 the conclusions of Proposition 1 remain valid except for 2, \( K_2 \uparrow \rightarrow w^*_2 \). The inflow of the foreign capital into agricultural sector always increase social welfare.

2. However particularly \( dW / dK'_i < 0 \) for Case 1 if and only if \( L_2 \varepsilon^{-1}w^*_2/(L_2 \varepsilon^{-1} - w^*_2F^{-1}_{11}2) < (t/(1+t))(w^*_2 + r^*_2) \).
and $dW / dK_2^t < 0$ for Case 2 if and only if
\[ L_2^e w_2 (L_2^e - w_1 F_{1tt}^{-1}) + (r_2 - r^*)k_2 < (t/(1+t)) (w_2 + r_2 k_2) \]

**Proof**

The arguments of the proof of Proposition 1 go through except for 2, $K_2^* \to w_2^*$. It is easy to derive for Case 1, from $Y = L_2 f_2^r(k_2), (6), (10)$ and (11)
\[ dW / dK_2^t < 0 \iff dW < 0 \iff L_2^e w_2 (L_2^e - w_1 F_{1tt}^{-1}) - (t/(1+t)) p f_2^r(k_2) < 0 \iff L_2^e w_2 (L_2^e - w_1 F_{1tt}^{-1}) - (t/(1+t)) (w_2 + r_2 k_2) < 0. \]

Similarly for Case 2
\[ dW < 0 \iff L_2^e w_2 (L_2^e - w_1 F_{1tt}^{-1}) - (t/(1+t)) p f_2^r(k_2) + (r_2 - r^*) k_2 < 0. \]

1 of Proposition 2 shows that the introduction of the foreign capital inflow and tariff on imports does not alter the almost all the results of the basic sector specific capital. 2 shows that the immiserizing growth $dW / dK_2^t < 0$ occurs if the tariff rate $t$ is fairly high. The second inequality is more stringent than the first one, implying it occurs more likely for Case 1 rather than for Case 2.

Furthermore as seen from 2 of Proposition 2, the immiserizing growth is negated if the tariff rate $t$ is low. Especially such an absence of immiserizing growth has been noticed by Choi and Yu (2006) in case of no-tariff (i.e., $t = 0$).

Next we introduce the third productive factor land in the agriculture and generalize the above proposition.

**II.3. Sector Specific Capital with Land in Agriculture**

Here we introduce land in agriculture and generalize Proposition 2. This case is discussed by Kahn (1982) also. Here the production function of agriculture (sector 1) is expressed as $X = F_1(L_1, K_1, T)$ where $T$ is the amount of land, which is concave and homogenous degree one in $L_1, K_1$ and $T$, and (4) and (6) are expressed as
\[ r_i = F_{1i}(L_1, K_1, T) \quad \text{(5)}' \]
\[ w_i = F_{1i}(L_1, K_1, T) \quad \text{(6)}' \]
and
\[ t = F_{1r}(L_1, K_1, T) \quad \text{(20)} \]
where $t$ is the rental price of land.

Here we assume any two productive factors of sector 1 to be substitute, i.e., $F_{1jk} > 0$
for \( j, k = L_i, K_j, T \) and \( j \neq k \). That is, the marginal products of any production factor increases by the increase of any other factor. Further its unit cost function is rewritten as

\[
1 = G(w_1, r_1, t)
\]

For Case 1, i.e., with full repatriation of interest payment, (18) is rewritten as

\[
\left(1 - \frac{t}{t+1} \right) dW = \ell L_2 dw_1 - tp^* dY + tdT
\]

and for Case 2, i.e., interest payment is based on international interest rate \( r^* \) (19) is expressed as

\[
\left(1 - \frac{t}{t+1} \right) dW = \ell L_2 dw_1 + (r_1 - r^*) dK'_j - tp^* dY + tdT, i = 1, 2.
\]

Then we obtain

**Proposition 3**

With introduction of land as the third productive factor in agriculture, in addition to the results of Proposition 2, the following further holds, i.e.,

4. \( T^\uparrow \rightarrow w_1^\uparrow, L_1^\uparrow, X^\uparrow, r_1^\uparrow, W^\uparrow, t^\downarrow \), and \( L_2 \) and \( Y \) do not change.

That is to say, 4. the increase in land \( T \) raises the wage rate of agriculture \( w_1 \), its employment \( L_1 \), its output \( X \), and its interest rate \( r_1 \), social welfare \( W \), but lowers rental price of land \( t \), and do not affect the industrial employment \( L_2 \) and its output \( Y \).

**Proof.**

If suffices to prove 4. Noting \( dK_j = 0 \), from (6)' it follows \( dw_1 = F_{112} dL_1 + F_{113} dT \).

Next from (11) with \( dL_2 = dN = 0 \) ( \( dL_2 = 0 \) follows from (10) with \( dK_2 = 0 \)) \( w_i dL_1 = L_2 \ell^\downarrow dw_1 \) is derived. Combining these two equations we obtain

\[
(1 - F_{112} L_2 (w_i \ell)^\downarrow) dw_1 = F_{113} dT.
\]

Here \( T^\uparrow \rightarrow w_1^\uparrow \rightarrow L_1^\uparrow \rightarrow X^\uparrow \). Here noting \( L_1^\uparrow \) and \( T^\uparrow, r_1^\uparrow \) follow from (5)', \( F_{112} > 0 \) and \( F_{113} > 0 \). Then \( t^\downarrow \) follows from (8)'. \( Y \) does not change since \( K_2 \) and \( L_2 \) do not change. \( W^\uparrow \) follows from (18)' and (19)'.

Next we investigate the robustness of this result in case of mobile capital.

**III.1. Mobile Capital without Land in Agriculture**

In case of mobile capital without land as a production factor, (8) and (9) are replaced by
\[ 1 = G_i(w_1, r) \] (8)’

and
\[ p = G_2(w_2, r) \] (9)’

Here we note since \( p \) and \( w_2 \) are fixed, so are \( r \) and \( w_1 \). In short the economy is reduced to the fixed price economy. Then since \( dw_1 = 0 \), the envelope property (13) holds, and as equilibrium condition in capital market, we obtain
\[ b_{ik} X + b_{iz} Y = K \] (21)

where \( b_{ik} = \partial G_i(w_i, r)/\partial Y, \ i = 1, 2 \) being the amount of capital used for production of one unit of good \( i \), which is const. here.

Next from \( b_{1i} X = L_1 \) and \( b_{2i} Y = L_2 \) where \( b_{ii} = \partial G_i(w_i, r)/\partial w_i, \ i = 1, 2 \) being the amounts of labor used for production of one unit of good \( i \), which is also const. here, and from (2), we obtain
\[ b_{1i} X + b_{2i} Y / \ell = N \] (22)

Then from (21) and (22) noting \( \ell = w_1/w_2 \) to be fixed, we obtain the generalized Rybczynski property, i.e.,
\[ \begin{pmatrix} X \\ Y \end{pmatrix} = \frac{1}{|D|} \begin{pmatrix} b_{ik} N - b_{iz} \ell^{-1} K \\ b_{ii} K - b_{ik} N \end{pmatrix} \] (23)

where \(|D| = b_{ii} b_{2k} - b_{ik} b_{2i} / \ell = b_{ii} b_{2i} \ell^{-1} (k_i \ell - k_i)\) and \( k_i = b_{ik}/b_{ii} \) being the capital-labor ratio of sector \( i, \ i = 1, 2 \). Here we define sector 2 to be capital (resp. labor) intensive if and only if \( k_i \ell > \) (resp. <) \( k_i \).

Then the Rybczynski property holds;
\( K^+ \) or \( N^+ \) \( \Leftrightarrow X^+ \leftrightarrow Y^+ \leftrightarrow k_i \ell > k_i \).

Next we note that the welfare effects of the foreign capital inflow, (18) and (19) are simplified as
\[ \left(1 - \frac{t}{t+1} m\right) dW = -tp^* dY \] (18)”

for Case 1, and
\[ \left(1 - \frac{t}{t+1} m\right) dW = (r - r^*) dK^+ - tp^* dY \] (19)”

for Case 2.

As seen from (23) and (18)” for Case 1 (i.e., full repatriation of interest payment) if the industrial is capital intensive, then \( dW < 0 \), i.e., immiserizing growth occurs. This is a straightforward generalization of the results for the standard 2x2 no distortion case shown by Bhagwati (1968), Johnson (1967) and Brecher-Alejandro (1977).
III.2. Mobile Capital with Land

Lastly we analyze the case of mobile capital with land as a productive factor for the agricultural sector. Then again (5)', (6)', (20),

\[ 1 = G_i(w_1, r_i, t) \]  

and

\[ p = G_z(w_2, r) \]

follow. Here we assume

\[ k_2 \ell > k_1, \]

i.e., the industrial sector is more capital intensive. This is a necessary and sufficient condition for the stability of the dynamic adjustment process of the 2X2 distortion case as shown by Gordon-Findlay (1975) and Khan (1980b) and (1981). Furthermore the production function of agriculture obeys C.E.S.

Let \( \sigma_{ij}^X \), \( i, j=L, K, T \) be the partial elasticity of substitution between productive factors \( i \) and \( j \) \( (i \neq j) \) where, say

\[ \sigma_{ik}^X = \frac{G_{x_{ul}}/G_{x_{u}} \cdot G_{x_k}}{G_{x_{ul}}} = \partial^2 G_x / \partial w_i \partial r, \]

\[ G_{x_{ul}} = \partial G_x / \partial w_i, \quad G_{x_k} = \partial G_x / \partial r. \]

The other elasticity of substitution is similarly defined. Similarly let \( \sigma_{ik}^Y = p G_{y_{ul}} / G_{y_{u}} \cdot G_{y_k} \) where \( G_{y_{ul}} = \partial^3 G_y / \partial w_i \partial r, \)

\[ G_{y_{ul}} = \partial G_y / \partial w_i, \quad G_{y_k} = \partial G_y / \partial r \]

be the partial elasticity of substitution between \( L \) and \( K \) in sector 2. Let \( x_i \) be the input of productive factor \( i \), \( i=L, K, T \) and \( v_i \) be its factor price. Then \( \sigma_{ij}^X \) is also expressed on \( \sigma_{ij}^X = \frac{\partial^2 \ln (x_i / x_j)}{\partial \ln (v_i / v_j)}. \)

For the C.E.S. production function, \( \sigma_{ij} = \sigma \geq 0 \) holds for any \( i, j \) \( (i \neq j) \). Then C.E.S. production function is expressed as

\[ X = (aL^n + bK^n + cT^n)^{1/\rho} \]

with \( \sigma = 1/(1-\rho), \quad \rho < 1 \) where \( a, b, c > 0 \) are const. Next let

\[ \lambda_{11} = b_{1l} X / N \]

\[ \lambda_{12} = b_{2l} Y / N \ell \]

be respectively rural population rate and urban population rate where

\[ b_{1l} = L_1 / X = \partial G_x / \partial w_1, \quad b_{2l} = L_2 / Y = \partial G_y / \partial w_2. \]

Similarly let

\[ \lambda_{x1} = b_{1x} X / K \]

\[ \lambda_{x2} = b_{2x} Y / K \]

be respectively proportion of capital employed in sector 1 and sector 2 where

\[ b_{1x} = K_1 / X = \partial G_x / \partial r \quad \text{and} \quad b_{2x} = K_2 / X = \partial G_y / \partial r. \]

By definition
\[ \lambda_{t_1} = 1 = b_{t_1} \cdot X / T \] where \( b_{t_1} = \partial G_x / \partial t = T / X \).

Next let \( \theta_{t_1} = w_j b_{t_1} / p_j, \ j = 1, 2 \) with \( p_1 = 1 \) and \( p_2 = p \) be the labor share in sector \( j \), \( \theta_{k_1} = r b_{t_1} / p_j \) be the capital share in sector \( j \) and \( \theta_{t_1} = t b_{t_1} \) be the land share in sector 1. Then by construction

\[
\begin{align*}
\theta_{t_1} + \theta_{k_1} + \theta_{t_1} &= 1, \\
\theta_{t_2} + \theta_{k_2} &= 1, \\
\hat{\theta}_1 &= 0 = \theta_{t_1} \hat{w}_1 + \theta_{k_1} \hat{r} + \theta_{t_1} \hat{t}
\end{align*}
\]

where \( \hat{x} = dx / x \) being the relative change rate of variable \( x \).

\[
\hat{p} = \theta_{t_2} \hat{w}_2 + \theta_{k_2} \hat{r}.
\]

First we recall from (9)' \( r \) is fixed since both \( p \) and \( w_2 \) are fixed. Then (25) is rewritten as

\[
\theta_{t_1} \hat{w}_1 + \theta_{t_1} \hat{r} = 0
\]

By construction, we obtain

\[
\begin{align*}
\theta_{t_1} \sigma_{x_1}^x + \theta_{k_1} \sigma_{x_k}^x + \theta_{t_1} \sigma_{x_t}^x &= 0, \ i = L, K, T \\
\theta_{t_2} \sigma_{x_1}^x + \theta_{k_2} \sigma_{x_k}^x &= 0, \ i = L, K
\end{align*}
\]

where say

\[
\sigma_{x_1}^x = G_{x_1 w_1} / G_{x_1}^2 < 0.
\]

Furthermore, from (21) and (22) by taking total differentiation, we obtain

\[
\lambda_{t_1} \cdot \hat{X} + \lambda_{t_2} \cdot \hat{Y} - \lambda_{t_2} \cdot \hat{\theta} = \hat{N} - \hat{b}_{t_1} \lambda_{t_1}
\]

and

\[
\lambda_{k_1} \cdot \hat{X} + \lambda_{k_2} \cdot \hat{Y} = \hat{K} - \lambda_{k_1} \cdot \hat{b}_{k_1}
\]

Furthermore from \( b_{t_1} = T / X \), we observe

\[
\hat{b}_{t_1} + \hat{X} = \hat{T}
\]

Noting \( \hat{r} = 0 \) and (27), by calculation it follows that

\[
\begin{align*}
\hat{b}_{t_1} &= \theta_{t_1} \sigma_{x_1}^x \hat{w}_1 + \theta_{k_1} \sigma_{x_k}^x \hat{r} + \theta_{t_1} \sigma_{x_t}^x \hat{i} = -(\theta_{t_1} \sigma_{x_1}^x + \theta_{t_1} \sigma_{x_t}^x) \hat{w}_1 + \theta_{t_1} \sigma_{x_t}^x \hat{i} \\
\hat{b}_{k_1} &= \theta_{t_1} \sigma_{x_1}^x \hat{w}_1 + \theta_{k_1} \sigma_{x_k}^x \hat{r} + \theta_{t_1} \sigma_{x_t}^x \hat{i} = \theta_{t_1} \sigma_{x_1}^x \hat{w}_1 + \theta_{t_1} \sigma_{x_t}^x \hat{i} \\
\hat{b}_{t_1} &= \theta_{t_1} \sigma_{x_1}^x \hat{w}_1 + \theta_{k_1} \sigma_{x_k}^x \hat{r} + \theta_{t_1} \sigma_{x_t}^x \hat{i} = \theta_{t_1} \sigma_{x_1}^x \hat{w}_1 - (\theta_{t_1} \sigma_{x_1}^x + \theta_{k_1} \sigma_{x_k}^x) \hat{i}
\end{align*}
\]

The above three equations are rewritten in the following, using (25)'

\[
\begin{align*}
\hat{b}_{t_1} &= -\{\theta_{k_1} \sigma_{x_k}^x + \theta_{t_1} \sigma_{x_t}^x (\theta_{t_1} / \theta_{t_1} + 1)\} \hat{w}_1 \\
\hat{b}_{k_1} &= \theta_{t_1} (\sigma_{x_1}^x - \sigma_{x_k}^x) \hat{w}_1
\end{align*}
\]
\[ \hat{b}_i = \{ \theta_i \sigma^x_i + (\theta_i, \sigma^x_i + \theta_k, \sigma^x_k) \theta_i \}/\theta_i \{ \hat{w}_i \}. \]

Substituting these three equations into (29), (30) and (31), noting \( \hat{k} = \hat{w}_1 - \hat{w}_2 = \hat{w}_3 \), we obtain

\[
\begin{bmatrix}
\hat{\lambda}_{l1} & \hat{\lambda}_{l2} & -\hat{\lambda}_{l2} - \hat{\lambda}_{l1} \{ \theta_{k1} \sigma^x_{lk} + (\theta_{l1} + \theta_{l1}) \sigma^x_{lr} \} \\
\hat{\lambda}_{k1} & \hat{\lambda}_{k2} & \hat{\lambda}_{k1} \theta_{l1} \{ \sigma^x_{kl} - \sigma^x_{kt} \} \\
1 & 0 & \theta_{l1} \{ \theta_{l1} \sigma^x_{nl} + \theta_{l1} \sigma^x_{nl} + \theta_{k1} \sigma^x_{nk} \} / \theta_{l1}
\end{bmatrix}
\begin{bmatrix}
\hat{X} \\
\hat{Y} \\
\hat{w}_1
\end{bmatrix}
= \begin{bmatrix}
\hat{N} \\
\hat{K} \\
\hat{T}
\end{bmatrix}. \tag{34}
\]

Here the coefficient matrix \( E \) is simplified as

\[
E = \begin{bmatrix}
\hat{\lambda}_{l1} & \hat{\lambda}_{l2} & -\hat{\lambda}_{l2} - \hat{\lambda}_{l1} \{ \theta_{k1} \sigma^x_{lk} - \theta_{l1} \sigma^x_{lk} \} \\
\hat{\lambda}_{k1} & \hat{\lambda}_{k2} & \hat{\lambda}_{k1} \theta_{l1} \{ \sigma^x_{kl} - \sigma^x_{kt} \} \\
1 & 0 & \theta_{l1} \{ \sigma^x_{nl} - \sigma^x_{tr} \}
\end{bmatrix} \tag{35}
\]

noting \( \theta_{k1} \sigma^x_{lk} + \theta_{l1} \sigma^x_{lk} = -\theta_{l1} \sigma^x_{ll} \) and \( \theta_{l1} \sigma^x_{nl} + \theta_{k1} \sigma^x_{nk} = -\theta_{l1} \sigma^x_{tt} \). Recalling \( \sigma^x_{jk} > 0 \),

\( i, k = L_1, K_1, T \) and \( j \neq k \) from \( F_{ijk} > 0 \), \( j \neq k \),

\[ |E| = (\lambda_{l1} \lambda_{k2} - \lambda_{k1} \lambda_{l2}) \theta_{l1} \{ \sigma^x_{nl} - \sigma^x_{tt} \} + \lambda_{l2} \lambda_{k1} \theta_{l1} \{ \sigma^x_{kl} - \sigma^x_{kt} \} + \lambda_{k2} (\lambda_{l2} + \lambda_{k1} \theta_{l1}) (\sigma^x_{nl} - \sigma^x_{tr}) \].

In case of C.E.S. function of sector 1, since \( \sigma^x_{ij} = \sigma > 0 \) for \( i, j = L_1, K_1, T, i \neq j \) and

\[ \sigma^x_{ii} = (1 - 1/\theta_i) \sigma < 0 \], \( i = L_1, K_1, T, \), \( \sigma^x_{ij} \) is expressed as

\[ |E| = (\lambda_{l1} \lambda_{k2} - \lambda_{k1} \lambda_{l2}) \theta_{l1} / \theta_{l1} + \lambda_{k2} (\lambda_{l2} + \lambda_{l1} \theta_{l1}) \sigma \] \( \tag{37} \)

and hence it follows that \( |E| > 0 \) if

(1) \( \lambda_{l1} \lambda_{k2} - \lambda_{k1} \lambda_{l2} > 0 \Leftrightarrow k_2 \ell - k_1 > 0 \) \( \tag{24} \)

or

(2) if \( \lambda_{k2} > \lambda_{k1} \theta_{l1} / \theta_{l1} \)

or

(3) if \( \lambda_{k2} \lambda_{l1} > \lambda_{l2} \lambda_{k1} \theta_{l1} / \theta_{l1} \Leftrightarrow k_2 \ell - k_1 \theta_{l1} / \theta_{l1} > 0 \).

Since (24) is assumed, \( |E| > 0 \) holds. Then we obtain effects of the change in \( K, T \) and \( N \) on \( w_1, X \) and \( Y \) from (35) in the following.

**Proposition 4**
In case of mobile capital, with C.E.S. production function of agricultural sector 1, and capital intensive industrial sector 2, from (35),

\[ \frac{dw_1}{dK} = [E]^{-1} \cdot w_1 \cdot K^{-1} \lambda_{t_2} > 0, \]

\[ \frac{dw_1}{dT} = [E]^{-1} \cdot w_1 \cdot T^{-1} b_{t_1} b_{t_2} e^{-1} (k_{t_2} - k_t) > 0, \]

\[ \frac{dw_1}{dN} = -[E]^{-1} \cdot w_1 \cdot N^{-1} \lambda_{K_2} < 0, \]

\[ \frac{dX}{dK} = -[E]^{-1} X \cdot K^{-1} \lambda_{z_2} \theta_{t_1} \sigma \theta_{r_1}^{-1} < 0, \]

\[ \frac{dX}{dT} = [E]^{-1} X \cdot T^{-1} \lambda_{K_2} (\lambda_{z_2} + \lambda_{t_1} \sigma) > 0, \]

\[ \frac{dX}{dN} = [E]^{-1} X \cdot N^{-1} \lambda_{K_2} \theta_{t_1} \sigma \theta_{r_1}^{-1} > 0, \]

\[ \frac{dY}{dK} = [E]^{-1} Y \cdot K^{-1} (\lambda_{z_2} \theta_{t_1} \theta_{r_1}^{-1} \sigma + \lambda_{t_2} + \lambda_{t_1} \sigma) > 0, \]

\[ \frac{dY}{dT} = [E]^{-1} Y \cdot T^{-1} \lambda_{K_2} (\lambda_{z_2} + \lambda_{t_1} \sigma) > 0, \]

\[ \frac{dY}{dN} = -[E]^{-1} Y \cdot N^{-1} \lambda_{K_1} \sigma \theta_{t_1} \theta_{r_1}^{-1} < 0. \]

**Proof.** (See Appendix I.)

First we note that the Rybczynski Property holds for this case, i.e., since industrial sector is capital intensive in the sense of (24), the increase in capital \( K \) increases the industrial output \( Y \) and decreases agricultural output \( X \). Exactly the opposite results holds when the increase in labor \( N \) gives rise, again conforming to the Rybczynski Property. It matters a course that the increase in land \( T \) increases agricultural output \( X \) since land is exclusively used in agriculture, but it is interesting that this also increases industrial output \( Y \). Such an increase in industrial output is made possible because some portion of agricultural labor and capital is freed from this sector into industrial sector. It is noteworthy that these Rybczynski Properties hold even if the envelope property (13) does not hold, or put it differently, \( dx + p dy = t L x dw_1 + w_1 dN + r dK \) holds instead.

The effects on the agricultural wage rate \( w_1 \) of the changes in \( K, T \) and \( N \) are similar as before (Propositions 1, 2 and 3). That is to say, increases in capital \( K \) and land \( T \) raises \( w_1 \) but increases in population \( N \) lowers \( w_1 \).
III. 3 Welfare Effects of Foreign Capital Inflow

With tariff ridden imports and foreign capital inflow, the change in social welfare $W$ are expressed by (18) (Case 1 full interest payment repatriation) and by the following (19)” (Case 2 international interest $r^*$ is paid).

$$\left(1 - \frac{t}{t+1}ight) dW = \ell L dw + (r - r^*)dK^f - tp^* dY$$

(19) ”

Then we observe

**Proposition 5**

For Case 1

$$dW / dK^f < 0 \iff \lambda_{L2} \theta_{L2} < \frac{t}{t+1} (\lambda_{L1} \sigma (1+\theta_{L1}/\theta_{L1}) + \lambda_{L2})$$

For Case 2

$$dW / dK^f < 0 \iff \lambda_{L2} \theta_{L2} + (r-r^*)|E[K/PY] < \frac{t}{t+1} (\lambda_{L1} \sigma (1+\theta_{L1}/\theta_{L1}) + \lambda_{L2})$$

**Proof (See Appendix II.)**

As seen from the above two inequalities, the second inequality is more stringent than the first one. In other words, the immiserizing growth $dW/dK^f < 0$ occurs more likely for the Case 1 (i.e., case of interest rate payment full repatriation). In particular for Case 1, as seen from the first inequality, it occurs if $\theta_{L2} < t/(t+1)$.

IV. Concluding Remarks

By comparing the results of Proposition 3 (i.e., sector specific capital with land), and Propositions 4 and (i.e., mobile capital with land) it is easily seen that the former results are more extreme e.g., changes in sector specific capital $K_1$, population $N$ and land $T$ do not affect industrial production $Y$, and hence the latter results are more realistic. However if we interprets these differences from the length of time span – i.e., the former’s time span is shorter than the latter’s –, then these can be understood without conflict.

As seen from the arguments of mobile capital with land, the assumption of C.E.S. agricultural production function simplifies the analysis considerably. Therefore the robustness of the model comes about naturally. If we calculate the effects of the change in $K$, $N$ and $T$ on $w_1$, $X$ and $Y$ based on (35) without assuming C.E.S., instead assuming $\sigma_{ij}^x > 0$, $i,j = L, K, T$, $i \neq j$ and $\sigma_{ii}^x \sigma_{jj}^x$, then except $sgn \; dY/dN$,
all the other results are obtained. Furthermore the similar results as Proposition 5 are also obtained for this more general case, although expressions are somewhat more complicated.

We could admit the scope of our model to be rather limited. The two good model where all 3 factors are used in two sectors are more general. But this would make the analysis extremely complicated as can be guessed by the 2x3 model dicussed by Batra and Casas (1976). No consideration is taken for Stiglitz’s (1974) labor turnover model, Stiglitz’s (1971) efficiency wage model, Calvo’s (1978) trade union model, absolute or proportional wage model, and Khan’s (1980a and b) endogenously determind urban wage model which covers these models.

Perhaps the extension to 3 sector model (urban formal, urban informal and rural sector) seems immediate, which would be our next step.

**Appendix I**

From (35), we obtain

$$|E| \hat{w}_i = \begin{bmatrix} \lambda_{L1} & \lambda_{L2} & \hat{N} \\ \lambda_{K1} & \lambda_{K2} & \hat{K} \\ 1 & 0 & \hat{T} \end{bmatrix},$$  \hspace{1cm} (A-1)

from which

$$Kw_i^{-1}E|dw_i / dK = \lambda_{L2} > 0,$$

$$Tw_i^{-1}E|dw_i / dT = \lambda_{L1}\lambda_{K2} - \lambda_{K1}\lambda_{L2} = b_1 b_2 (k_2 \ell - k_1) > 0,$$

and

$$NW_i^{-1}E|dw_i / dN = -\lambda_{K2} < 0$$

are derived. Similarly from (35), we observe

$$|E| \hat{X} = \begin{bmatrix} \hat{N} & \lambda_{L2} & -\lambda_{L2} - \lambda_{N1} \sigma \\ \hat{K} & \lambda_{K2} & 0 \\ \hat{T} & 0 & \theta_{L1} \theta_{T1}^{-1} \end{bmatrix},$$  \hspace{1cm} (A-2)

from which

$$NX^{-1}E|dX / dK = -\lambda_{L2} \theta_{L1} \sigma \theta_{T1}^{-1} < 0,$$
\[ TX^{-1}E dX / dT = \lambda_{K_2}(\lambda_{L_2} + \lambda_{L_1}\sigma) > 0, \]

and

\[ NX^{-1}E dX / dK = \lambda_{K,2}\theta_{L_1}\sigma\theta_{R_1}^{-1} > 0, \]

are derived. Lastly again from (35), we obtain

\[
|E| \hat{Y} = \begin{bmatrix} \lambda_{L_1} & \hat{N} & -\lambda_{L_2} - \lambda_{Y_1}\sigma \\ \lambda_{K_1} & \hat{K} & 0 \\ 1 & \hat{T} & \lambda\theta_{L_1}\theta_{R_1}^{-1} \end{bmatrix},
\]

from which

\[ NY^{-1}E dY / dK = \lambda_{L_1}\theta_{L_1}\theta_{R_1}^{-1}\sigma + \lambda_{L_2} + \lambda_{L_1}\sigma > 0, \]

\[ TY^{-1}E dY / dT = \lambda_{K_1}(\lambda_{L_2} + \lambda_{L_1}\sigma) > 0, \]

and

\[ NY^{-1}E dY / dK = -\lambda_{K_1}\sigma\theta_{L_1}\theta_{R_1}^{-1} < 0 \]

are derived.

**Appendix II**

For case 1, from (18) and Proposition 4, we observe

\[
\ell^{-1}L_2 dw_1 - tp^* dY = \left[ \ell^{-1}L_2 |E|^{-1} w_1 K^{-1} \lambda_{L_2} - tp^* |E|^{-1} YK^{-1}(\lambda_{L_1}\theta_{L_1}\theta_{R_1}^{-1}\sigma + \lambda_{L_2} + \lambda_{L_1}\sigma) \right] dK.
\]

Hence

\[ dW / dK' < 0 \iff \ell^{-1}L_2 w_1 \lambda_{L_2} < tp^* Y(\lambda_{L_1}\theta_{L_1}\theta_{R_1}^{-1}\sigma + \lambda_{L_2} + \lambda_{L_1}\sigma). \]

Here noting \( \ell^{-1}L_2 w_1 = L_2 w_2 = b_{2L} Y w_2 = p \theta_{L_2} Y \) and \( tp^* Y = \frac{t}{1+t} p Y \), we observe

\[ dW / dK' < 0 \iff \theta_{L_2} \lambda_{L_2} < \frac{t}{1+t}(\lambda_{L_1}\sigma(1 + \theta_{L_1} / \theta_{R_1}) + \lambda_{L_2}). \]

For Case 2, from (19)''' and Proposition 4, we obtain
\[ E\left( t^{-1}L_2 dw_1 - tp^* dY + (r - r^*) dK^f \right) \]

\[ = \left\{ t^{-1}L_2 w_1 K^{-1} \lambda_{l2} - tp^* Y K^{-1} (\lambda_{l1}, \theta_{l1}, \theta_{r1}, \sigma + \lambda_{l2} + \lambda_{l1}, \sigma) + |E| (r - r^*) \right\} dK^f. \]

Here noting \( L_2 w_1 = \theta_{l2} pY \)

RHS of the above equation is seen to equal

\[ \left[ p \theta_{l2} Y K^{-1} \lambda_{l2} - \frac{t}{1 + t} p Y K^{-1} (\lambda_{l1}, \theta_{l1}, \theta_{r1}, \sigma + \lambda_{l2} + \lambda_{l1}, \sigma) + |E| (r - r^*) \right] dK^f \]

we obtain

\[ dW / dK^f < 0 \Leftrightarrow \theta_{l2} \lambda_{l2} + |E| K / p Y < \frac{t}{1 + t} (\lambda_{l1}, \sigma (1 + \theta_{l1} / \theta_{r1}) + \lambda_{l2}) \).

**Notes**

1. Let \( F_i (L_i, K_i, T) = f(x_i, x_2, x_3), \ (v_1, v_2, v_i) = (w_i, r, t) \). Then for example

\[ \sigma^x_{l,k_i} = d \ln (x_i / x_2) d \ln (v_2 / v_i) = \frac{x_2}{x_1} \frac{v_2}{v_1} \frac{d (x_i / x_2)}{d (v_2 / v_i)} = \frac{x_i}{x_2} \frac{v_i}{v_2} \frac{d (x_i / x_2)}{d (v_i / v_1)} \]

from \( v_i = f_i = \partial f / \partial x_i, \ i = 1, 2 \). Here recalling \( f_i dx_i + f_2 dx_2 = 0 \) from cost minimization, 

\[ \sigma^x_{l,k_i} = f_i f_2 (f_i x_i + f_2 x_2) / x_i x_2 \]

Hence \( \sigma^x_{l,k_i} > 0 \) if \( F_i x_k > 0 \).

2. Since \( \theta_{l1} \sigma^x_{u} = -(\theta_{l1} + \theta_{k1}) \sigma = -(1 - \theta_{l1}) \sigma \)

\[ \sigma^x_{u} = (1 - 1 / \theta_{l1}) \sigma \] follows.

3. When the production function \( F_i \) is not under C.E.S., then \( E > 0 \) still holds if (24) and 

\( \sigma^x_{ki} > \sigma^x_{kr}, \ i.e., \) the elasticity of substitution between capital and labor is not less than the one between capital and land in sector 1.

**References**

Heckscher-Ohlin and the Neoclassical Models of International Trade”, *Journal of International Economics* 6, 21-38.


