Tax Policy Competition for FDI and Trade Strategy of Multinational Corporation in Bilateral FTA Environment

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Abstract

This paper shows a development of a partial equilibrium model where two FDI host countries, called home and foreign country, maximize economic welfare through tax policy competition in bilateral FTA environment. A multinational corporation (MNC) directly invests in both host countries and monopolizes each market. Concerning trade flow, with no tariffs, the role of transportation cost is crucial. If it is negligible, there exists intra-industry trade. Otherwise, only one-way trade remains.

Depending on tax policies of the two host countries, with positive transportation cost, profit maximizing trade strategy of the MNC consists of three kinds - one-way trade from home to foreign country, one-way trade from foreign to home country, and no-trade. This way, although there is no tariff, using tax policies on FDI, the two host governments control trade flows to maximize their economic welfares. In two-stage game framework, in each of the three trade strategies, profit maximizing outputs and market prices and welfare maximizing tax policies are separately analyzed.

Then, combining all the separate analyses, all three alternatives are jointly analyzed. The joint welfare analysis shows the following results - i) there is no multiple Nash equilibrium with one-way trade from home to foreign country and one-way trade from foreign to home country at the same time; ii) there are two kinds of multiple Nash equilibrium - 1) one-way trade from home to foreign country and no-trade; 2) one-way trade from foreign to home country and no-trade; iii) depending on values of market demand sizes and marginal costs of the two countries, there are situations where no Nash equilibrium exists; iv) for a special case where the two host countries are identical, both host governments choose no-trade and impose same tax to the foreign subsidiaries; v) if the two host governments choose no-trade at welfare maximizing level, both governments impose positive tax instead of offering subsidies; vi) If the two host governments choose one-way trade either from home to foreign country or foreign to home country at welfare maximizing level, there exist various kinds of tax policies. In addition, for iv), v), and vi), the associated tax policies, at Nash equilibrium, are explicitly presented.

1 Introduction

One of the keywords of the world economy in the twenty first century is globalization. There is no doubt that key players in this globalized economies are multinational corporations (MNCs). According to Chandler and Mazlish (2005), since 1970s, MNC's relative importance in the world economy has increased dramatically. They noted that, by the year 2000, 63,000 transnational corporations with more than 690,000 foreign affiliates accounted for about 25 percent of global outputs.[1]

The global activities of MNCs can be approximated by amounts and flows of foreign direct investment (FDI). Chandler and Mazlish (2005) also mention that the ratio of FDI to gross doemstic capital formation increased from 2% around 1980 to 14% in 1999; the raio of the world's stock of FDI to world GDP increased from 5% to 16% over the same period. Barba navaretti and Venables (2004) reported that, in the post 1985 data, inflows of FDI grew dramatically, far outpacing the growth of trade or income; whereas worldwide real GDP increased at a rate of 2.5% per year between 1985 and 1999 and world exports by 5.6%, world real FDI inflows increased by 17.7%.[2]

They also mention that FDI originates predominantly from advanced countries and goes to advanced countries too, but the share of developing countries has been rising, mostly in the form of horizontal integration. This is because there are some developing countries, such as Brazil, China, and India, which have (potentially) large market size and relatively low marginal cost of production which are important rationale of FDI. One of many examples is Hyundai Automobile which is headquartered in South Korea. Starting from its first FDI in Turky in 1997, the Hyundai Automobile has increased its FDI by establishing plant in Chennai, India in 1998, Beijing, China in 2002 (joint with Beijing Train, Co.) and Alabama State in U.S. in 2005. The rationale of this series of FDI is to have better market access by localization and more cost effective production by lower margial cost.

Since the mid-1980, the efforts of governments to attract FDI inflows have increased considerably, expecting positive externalities from FDI inflows including job creation, technological spillover, higher productivity and competition, higher consumer surplus, and eventually, higher economic growth and welfare. This has led tax/subsidy competitions for FDI inflows between countries.[3] This is supported by many empirical studies and surveys, such as Charlton (2003), Devereux and Griffith (2002), Fuest, Huber and Mintz (2005), Oman (2000), and UNCTAD (1996). They indicate that tax attenuation or production subsidies have had a clear impact on FDI inflows.

[1] See page 220, Chandler, Jr., Alfred D. and Bruce Mazlish (2005). *Leviathans: Multinational Corporations and The New Global History*, Cambridge University Press

[2] See page 3-4, Barba navaretti, Georgio and Anthony J. Venables (2004). *Multinational Firms in the World Economy*, Princeton University Press.

[3] There are many real world examples in Charlton, Andrew (2003). "Incentive Bidding for Mobile Investment: Economic Consequences and Potential Responses," OECD Development Centre, Working Paper No. 203.

Concerning tax attenuation and FDI inflows, there are numerous papers which can be treated as good references. Most of these papers deal with a connection between tax/subsidy competition and MNC's location choice or entry mode selection or both. Barros and Cabral (2000) examines subsidy competition in two-country model focusing on differences in unemployment of the countries. In their model, the MNC, a monopoly supplier, can invest only one of the two countries and serve the other one by exporting from its foreign subsidiary. The location choice is dependent upon subsidy offered by host countries. Janeba (1998) also examines subsidy competition in two-country model but in case of duopoly. Fumagalli (2003) develops a similar model focusing on technological difference of the host countries which are equally sized. In this paper, export is allowed as an alternative entry option. Subsidy type in this paper is assumed to be lump-sum subsidy. No profits of foreign investment are assumed to be repatriated to the MNC's headquarter.

There is a group of papers which incorpotate a regional economic integration issue into the model. Norman and Motta (1993, 1996) and Neary (2002) argue that MNC's location choice can be exclusively influenced by the interplay between both FDI incentives and regional economic integration. Haufler and Wooton (1999, 2001), similarly allowing economic integration, focus on a role of transportation cost in subsidy competition. As in Fumagalli, they use lump-sum subsidy as well. No profit repatriation to the MNC's headquarter from foreign investment is allowed. In their model, as in Barros and Cabral (2000), the MNC, a foreign owned monopolist, is allowed to invest only one of the two countries and serve the other market by exporting from its foreign subsidiary. Raff (2004) also examines investment location and entry mode choices in two-country model allowing regional economic integration. In this paper, Free Trade Agreement (FTA) and Customs Unions (CU) are clearly distinguished. Albornoz and Corcos (2005), using similar model, find a possibility of excess subsidization led by the combination of economic integration and subsidy competition. They suggest that the gains from subsidy coordination between host governments increases with regional economic integration.

In real world economy, there is a rapid increase in bilateral FTA. It is frequently observed that many MNCs directly invest in both parties of the bilateral FTA. Reflecting these considerations, the model in this paper would be different from Barros and Cabral (2000) and Haufler and Wooton (1999) by allowing the MNC to invest both host countries in bilateral FTA environment. Thus, no tariff policies exist. Moreover, unlikely to the models in the previous literatures, in real world economy, there exists so-called "footlooseness" problem meaning that it is possible for MNCs to move quickly from one to another country which gives more incentives to FDI. In that sense, lump-sum tax/subsidy policy might not be appropriate. Reflecting this consideration, the model of this paper would also be different from Fumagalli (2003) and Haufler and Wooton (1999) by adopting output tax/subsidy instead of lumpsum tax/subsidy.

The goal of this paper is to develope a general two-country model where the MNC descides to invest in both host countries, trying to maximize its profits from global investment, given welfare maximizing output tax rates imposed by the two host governments. Investing in both host countries, if foreign sales by exporting between the two host countries is allowed as in Barros and Cabral (2000) and Haufler and Wooton (1999), with no tariffs, the MNC chooses a profit maximizing trade strategy and associated output levels given tax policies. This way, although there is no tariff, using tax policies on FDI, the two host governments control trade flows to maximize their economic welfares. Moreover, without tariffs, as in Haufler and Wooton (1999), the role of transportation cost can be crucial and, thus, needs to be carefully analyzed. Approximating real world where economic variables are always varied, unlikely to Fumagalli (2003) where market demands are equally sized, market demand sizes and marginal production costs are allowed to be varied in the two host countries. Thus, impacts of these changes to determination of output levels and optimal tax policies are examined as well. In sum, this paper is to provide one single general framework where important aspects of FDI, MNCs' behaviour, and corresponding host governments' tax policies are explicitly described.

The rest of this paper is organized as follows. Section 2 describes a basic feature of the model where an MNC invests in two host countries and determine output levels given tax policies of the two host governments. The two foreign subsidiaries are allowed to serve each other's markets by exporting. In Section 3, profit maximizing trade strategies of the two foreign subsidiaries are examined. For each strategy, profit maximizing output levels and prices, given tax policies, are derived. In addition, depending on tax policies, how each subsidiary switches its profit maximizing trade strategy from one to the others is analyzed. In Section 4, for each strategy described in the Section 3, welfare maximizing level of tax policies are derived. Moreover, how differences in market demand size and marginal cost of production affect optimal tax policy determinations of the two host governments is analyzed as well. Up to the Section 4, analyses are separately conducted for each strategy. In Section 5, all feasible trade strategies are jointly analyzed. Joint analysis of welfare of all feasible trade strategies for two identical host countries is conducted first. Then, allowing differences in market demand size and marginal cost of production between the two host countries, the welfares of all feasible trade strategies for two heterogeneous host countries are jointly analyzed. Section 6 concludes.

2 Model

2.1 General Description

This paper shows a development of a partial equilibrium model of imperfectly competitive industry where two countries are accomodating foreign direct investments (FDI) by using welfare maximizing tax policies. These two countries are named home and foreign country, denoted by h and f, respectively. The two host countries might have different market demand sizes and marginal costs of production. However, consumers in the two host countries are assumed to have identical preferences.

There is a foreign multinational corporation (MNC) which is headquartered outside of the two host countries. To simplify the situation, as in Barros and Cabral (2000) and Haufler and Wooton (1999), it is assumed that both home and foreign countries have no local firms and the foreign multinational has monopoly power once it opens its subsidiary in each of the two countries. The two markets are assumed to be segmented, meaning that the foreign subsidiaries can decide their prices and quantities independently in the two different markets.

The FDI is assumed to be horizontally integrated. It incurs a fixed set-up cost regardless of the volume of ouputs. Products are assumed to be homogeneous. The foreign subsidiaries in the two host countries are allowed to serve each other's market by exporting. There exists transportation cost but no tariffs between the two countries, representing bilateral FTA environment. All profits made by foreign investment are assumed to be flown out of the host countries and delivered back to the MNC's headquarter. In other words, the repatriation rate is equal to 1 as opposed to no repatriation in Fumagalli (2003). Taxes imposed by each of the two host countries are output taxes.

In this model, it is assumed that the MNC has already decided to invest in both host countries. The production levels and directions of trade flow are dependent upon tax policies of the two host governments as well as transportation cost. The tax rates charged by the host governments are dependent upon differences in market demand sizes, marginal production costs, and transportation cost between the two host countries. The model can be represented by a sequential game with following steps of moves.

- Stage1 : The two host governments choose, simultaneously and non-cooperatively, tax rates to the foreign investment which locates in its territory.
- Stage2 : The foreign multinational decides how to supply each of the two host countries given the knowledge of tax policies of the two host governments. In other words, the foreign multinational chooses outputs for domestic sales, outputs for export, and market prices in the two host countries.

The subgame perfect Nash equilibria of this game can be characterized and solved by backward induction as usual.

2.2 Basic Model

Consumers in the two countries consume three goods - Good X, Y and K. The good X and Y are outputs produced for domestic sales and foreign sales, respectively. Good K is assumed to be a numeraire good. The utility function of the consumers in each host country is given by

$$u^i = \alpha^i q^i - \frac{1}{2}\beta(q^i)^2 + k^i \quad (i = h, f) \tag{1}$$

where q^i represents total outputs consumed by consumers in the host country i. The α^i is the measure of market demand size of the two host countries. The β is assumed to be positive. Assuming income from all sources and market price in country i is M^i and p^i, respectively, the budget constraint is $p^i q^i + k^i = M^i$. Maximizing utility function subject to the budget constraint, the linear inverse demand function in country i is obtained and given by

$$p^i = \alpha^i - \beta q^i \tag{2}$$

Unlikely most of the literatures reviewed in the Section 1, the marginal production cost in this model is assumed to be increasing, not constant. In this model, in addition to general input cost, the marginal production cost is assumed to reflect each host country's technology level and labor productivity. Moreover, it is assumed to be positive and expressed in the form of linear function given by

$$c^i = \gamma^i + \phi(x^i + y^i) \tag{3}$$

where γ^i represents the measure of zero-output marginal cost or, simply, marginal production costs of the two host countries. The ϕ is assumed to be positive.

The notations used in this paper are summarized in the below.

- Outputs for domestic sales produced by foreign subsidiary in the host country i : $x^i \quad (i = h, f)$
- Outputs for foreign sales produced by foreign subsidiary in the host country i : $y^i \quad (i = h, f)$
- Consumption in the home country's market : $q^h = x^h + y^f$
- Consumption in the foreign country's market : $q^f = x^f + y^h$
- Market price in the home country : $p^h = \alpha^h - \beta(x^h + y^f)$
- Market price in the foriegn country : $p^f = \alpha^f - \beta(x^f + y^h)$
- Foreign direct investment (FDI), or production amount, made by foreign subsidiary in the home country : $z^h = x^h + y^h$
- Foreign direct investment (FDI), or production amount, made by foreign subsidiary in the foreign country : $z^f = x^f + y^f$
- Marginal cost of the production in the home country : $c^h = \gamma^h + \phi(x^h + y^h)$
- Marginal cost of the production in the foreign country : $c^f = \gamma^f + \phi(x^f + y^f)$
- Transportation cost between the two host countries : τ
- Output tax imposed by the host country i's government : $t^i \quad (i = h, f)$

- Fixed cost incurred by foreign investment in the host country i : $f^i \quad (i = h, f)$

3 Stage 2

In this section, the profit maximizing level of outputs for domestic sales, foreign sales, and market prices given the knowledge of tax policies and transportation cost will be computed. In addition, trade flows between the two host countries are to be analyzed. The profit function of the MNC is given by

$$\Pi = \Pi^h + \Pi^f \tag{4}$$

where Π^h and Π^f represents MNC's profits from investing in the home country and the foreign country, respectively.

$$\begin{aligned}
\Pi^h &= p^h x^h + p^f y^h - [(c^h + t^h)(x^h + y^h) + \tau y^h + f^h] \\
&= p^h x^h + p^f y^h - [(\gamma^h + \phi(x^h + y^h) + t^h)(x^h + y^h) + \tau y^h + f^h] \\
\Pi^f &= p^f x^f + p^h y^f - [(c^f + t^f)(x^f + y^f) + \tau y^f + f^f] \\
&= p^f x^f + p^h y^f - [(\gamma^f + \phi(x^f + y^f) + t^f)(x^f + y^f) + \tau y^f + f^f]
\end{aligned}$$

To compute profit maximizing level of outputs, the following first order partial derivatives are obtained.

$$\frac{\partial \Pi}{\partial x^h} = p^{h\prime} x^h + p^h - [(\gamma^h + t^h) + \phi(2x^h + 2y^h)] + p^{h\prime} y^f \tag{5}$$

$$\frac{\partial \Pi}{\partial y^h} = p^{f\prime} y^h + p^f - [(\gamma^h + t^h) + \phi(2x^h + 2y^h) + \tau] + p^{f\prime} x^f \tag{6}$$

$$\frac{\partial \Pi}{\partial x^f} = p^{f\prime} y^h + p^{f\prime} x^f + p^f - [(\gamma^f + t^f) + \phi(2x^f + 2y^f)] \tag{7}$$

$$\frac{\partial \Pi}{\partial y^f} = p^{h\prime} x^h + p^{h\prime} y^f + p^h - [(\gamma^f + t^f) + \phi(2x^f + 2y^f) + \tau] \tag{8}$$

The above first order partial derivatives reveal some useful information on trade flow made by the two foreign subsidiaries between the two host countries conditional upon transporation cost. They are summarized in the Lemma 1 and 2 below.

Lemma 1 *If there is no transportation cost ($\tau = 0$), there exists intra-industry trade between the two host countries. In other words, given $\tau = 0$, the above four first order partial derivatives are all equal to zero, meaning that outputs for domestic and foreign sales produced by foreign multinationals in both host countries are all positive, that is, x^h, y^h, x^f, and $y^f > 0$.*

Proof. *For $\tau = 0$, given the first order partical derivatives (5)=(6)=(7)=0, by (5)-(8), we get*

(5)-(8)=$[(\gamma^f + t^f) + \phi(2x^f + 2y^f)] - [(\gamma^h + t^h) + \phi(2x^h + 2y^h)]$.
Since (6)=(7)=0, $(\gamma^h + t^h) + \phi(2x^h + 2y^h) = (\gamma^f + t^f) + \phi(2x^f + 2y^f)$,
meaning that (5)-(8)=0. Since (5)=0, (8) must be equal to 0.
Therefore, (5)=(6)=(7)=(8)=0, that is, $x^h > 0, y^h > 0, x^f > 0, y^f > 0$. ■

Given no transportation cost, it is a profit maximizing trade strategy that, if one of the two foreign subsidiaries in the host countries exports to the other's market, the other foreign subsidiary also needs to export. This way, intra-industry trade exists.

Lemma 2 *If there is transportation cost, intra-industry trade disappears and only one-way trade remains. Given* $\tau > 0$*, among the above four first order partial derivatives, if the equations (5),(6), and (7) are equal to zero, the remaining equation (8) is less than zero, meaning that no export from the foreign country to the home country, that is,* $x^h > 0, y^h > 0, x^h > 0$*, and* $y^f = 0$*. If, instead, the equations (5),(7), and (8) are equal to zero, the remaining equation (6) is less than zero, meaning that no export from the home country to the foreign country, that is,* $x^h > 0, x^f > 0, y^f > 0$*, and* $y^h = 0$.
Proof. *For* $\tau > 0$*, given the first order partial derivatives (5)=(6)=(7)=0, it is true that* $x^h > 0, y^h > 0, x^f > 0$*. Since (6)=(7), we get*
$(\gamma^f + t^f) + \phi(2x^f + 2y^f) = (\gamma^h + t^h) + \phi(2x^h + 2y^h) + \tau$.
Then, (8) can be re-written as
$(8) = p^{h\prime}x^h + p^{h\prime}y^f + p^h - [(\gamma^h + t^h) + \phi(2x^h + 2y^h) + 2\tau]$
which is equivalent to $(5) - 2\tau$. $(8) = (5) - 2\tau$.
Since (5)=0, $(8) = (5) - 2\tau = -2\tau < 0$*, meaning that* $y^f = 0$.
In sum, given $x^h > 0, y^h > 0$*, and* $x^h > 0$*,* $y^f = 0$.
Similarly, for (5)=(7)=(8)=0, given $x^h > 0, x^f > 0$*, and* $y^f > 0$*,* $y^h = 0$. ■

The above Lemma 2 indicates that, given positive transportation cost, a profit maximizing trade strategy is one-way trade from the home country to the foreign country or vice versa. The two lemmas are summarized in the following proposition.

Proposition 3 *If there is no transportation cost, there exists intra-industry trade between the two host countries. If there is transportation cost, intra-industry trade disappears and only one-way trade remains, either from home to foreign or foreign to home country.*

From the above proposition, concerning trade flow between the two host countries, it is known that there are three cases - (i) given $\tau = 0$, there exists intra-industry trade, that is $y^h > 0$ and $y^f > 0$; (ii) given $\tau > 0$, there is only one-way trade from the home country to the foreign country, that is, $y^h > 0$ and $y^f = 0$; (iii) given $\tau > 0$, there is only one-way trade from the foreign country to the home country, that is, $y^h = 0$ and $y^f > 0$. Assuming the tax policies of the two host governments are known, profit maximizing level of outputs and prices of the MNC for each of the three cases are shown in the next subsections.

3.1 Case 1. Intra-industry trade ($\tau = 0$, $y^h > 0$ and $y^f > 0$)

Given $\tau = 0$, equation (4), the first order conditions of the profit function would be

$$\begin{aligned}
\frac{\partial \Pi}{\partial x^h} &= p^{h\prime} x^h + p^h - [(\gamma^h + t^h) + \phi(2x^h + 2y^h)] + p^{h\prime} y^f = 0 \quad (9)\\
\frac{\partial \Pi}{\partial y^h} &= p^{f\prime} y^h + p^f - [(\gamma^h + t^h) + \phi(2x^h + 2y^h)] + p^{f\prime} x^f = 0 \quad (10)\\
\frac{\partial \Pi}{\partial x^f} &= p^{f\prime} y^h + p^{f\prime} x^f + p^f - [(\gamma^f + t^f) + \phi(2x^f + 2y^f)] = 0 \quad (11)\\
\frac{\partial \Pi}{\partial y^f} &= p^{h\prime} x^h + p^{h\prime} y^f + p^h - [(\gamma^f + t^f) + \phi(2x^f + 2y^f)] = 0 \quad (12)
\end{aligned}$$

We know that each of the two foreign subsidiaries in both countries can serve its domestic and foreign market together. This means that, as long as market demands in each country are fully satisfied by overall productions made by foreign subsidiaries in both countries, outputs for domestic sales and foreign sales by each of the two foreign subsidiaries can be any amount. In other words, as long as $q^h + q^f = z^h + z^f$ is satisfied, x^h, x^f, y^h, and y^f are indeterminants. Thus, it may not be important to derive profit maximizing output levels for domestic and foreign sales saparately. Instead, profit maximizing level of market demands and foreign investments in each country need to be derived for optimal market price computations and welfare measurements in later senctions.

It is commonly known that, if any three of the four equations above are true, the remaining one is also true. Thus, we can reduce the number of equations to use. Let's use (9), (10), and (11) only. Then, (12) is automatically true. By equating (9)=(10)=(11), the profit maximizing level of domestic consumptions of the two host countries are obtained and given by

$$\begin{aligned}
q^h &= \frac{(2\beta + \phi)\alpha^h - \phi\alpha^f - \beta\gamma^h - \beta\gamma^f - \beta t^h - \beta t^f}{4\beta(\phi + \beta)} \quad (13)\\
q^f &= \frac{-\phi\alpha^h + (2\beta + \phi)\alpha^f - \beta\gamma^h - \beta\gamma^f - \beta t^h - \beta t^f}{4\beta(\phi + \beta)} \quad (14)
\end{aligned}$$

The profit maximizing level of market prices of the two host countries are obtained and given by

$$\begin{aligned}
p^h &= \frac{(2\beta + 3\phi)\alpha^h + \phi\alpha^f + \beta\gamma^h + \beta\gamma^f + \beta t^h + \beta t^f}{4(\phi + \beta)} \quad (15)\\
p^f &= \frac{\phi\alpha^h + (2\beta + 3\phi)\alpha^f + \beta\gamma^h + \beta\gamma^f + \beta t^h + \beta t^f}{4(\phi + \beta)} \quad (16)
\end{aligned}$$

Likewise, the proift maximizing level of FDI amounts made by foreign subsidiaries in the two host countries are obtained and given by

$$z^h = \frac{\phi\alpha^h + \phi\alpha^f - (\beta+2\phi)\gamma^h + \beta\gamma^f - (\beta+2\phi)t^h + \beta t^f}{4\phi(\phi+\beta)} \quad (17)$$

$$z^f = \frac{\phi\alpha^h + \phi\alpha^f + \beta\gamma^h - (\beta+2\phi)\gamma^f + \beta t^h - (\beta+2\phi)t^f}{4\phi(\phi+\beta)} \quad (18)$$

So far, we find that the profit maximizing level of foreign investments and market prices are all endogenized by country specific economic variables such as market sizes, marginal production costs, as well as policy variables such as corporate tax levels.

3.2 Case 2. One-way trade from Home to Foreign market ($\tau > 0$, $y^h > 0$ and $y^f = 0$)

Given $\tau > 0$ and $y^f = 0$, from the profit function of the MNC (equation (4)), the first order conditions are given by

$$\frac{\partial \Pi}{\partial x^h} = p^{h\prime}x^h + p^h - [(\gamma^h + t^h) + \phi(2x^h + 2y^h)] = 0 \quad (19)$$

$$\frac{\partial \Pi}{\partial y^h} = p^{f\prime}y^h + p^f - [(\gamma^h + t^h) + \phi(2x^h + 2y^h) + \tau] + p^{f\prime}x^f = 0 \quad (20)$$

$$\frac{\partial \Pi}{\partial x^f} = p^{f\prime}y^h + p^{f\prime}x^f + p^f - [(\gamma^f + t^f) + 2\phi x^f] = 0 \quad (21)$$

By using (19), (20) and (21), the profit maximizing level of export volume from the home to the foreign country, domestic sales volumes made by foreign subsidiary in the two host countries are obtained and given by

$$y^h = \frac{-\phi\alpha^h + \phi\alpha^f - \beta\gamma^h + \beta\gamma^f - \beta t^h + \beta t^f - (\beta+\phi)\tau}{4\beta\phi} \quad (22)$$

$$x^h = \frac{(2\beta+\phi)\alpha^h - \phi\alpha^f - \beta\gamma^h - \beta\gamma^f - \beta t^h - \beta t^f + (\beta+\phi)\tau}{4\beta(\beta+\phi)} \quad (23)$$

$$x^f = \frac{\phi\alpha^h + \phi\alpha^f + \beta\gamma^h - (\beta+2\phi)\gamma^f + \beta t^h - (\beta+2\phi)t^f + (\beta+\phi)\tau}{4\phi(\beta+\phi)} \quad (24)$$

Since $y^f = 0$, the amount of domestic consumption of the home country is simply domestic sales volume of the foreign subsidiary in the home country. The profit maximizing level of domestic consumptions of the two host countries are given by

$$q^h = \frac{(2\beta+\phi)\alpha^h - \phi\alpha^f - \beta\gamma^h - \beta\gamma^f - \beta t^h - \beta t^f + (\beta+\phi)\tau}{4\beta(\beta+\phi)} \quad (25)$$

$$q^f = \frac{-\phi\alpha^h + (\phi+2\beta)\alpha^f - \beta\gamma^h - \beta\gamma^f - \beta t^h - \beta t^f - (\beta+\phi)\tau}{4\beta(\beta+\phi)} \quad (26)$$

The profit maximizing level of domestic prices in each markets are obtained and given by.

$$p^h = \frac{(2\beta + 3\phi)\alpha^h + \phi\alpha^f + \beta\gamma^h + \beta\gamma^f + \beta t^h + \beta t^f - (\beta + \phi)\tau}{4(\beta + \phi)} \quad (27)$$

$$p^f = \frac{\phi\alpha^h + (2\beta + 3\phi)\alpha^f + \beta\gamma^h + \beta\gamma^f + \beta t^h + \beta t^f + (\beta + \phi)\tau}{4(\beta + \phi)} \quad (28)$$

Since $y^f = 0$, foreign multinational's investment amount in the foreign country, z^f, is simply domestic sales volume of the foreign subsidiary in the foreign country. The proift maximizing level of FDI amounts made by foreign subsidiaries in the two host countries are obtained and given by

$$z^h = \frac{\phi\alpha^h + \phi\alpha^f - (\beta + 2\phi)\gamma^h + \beta\gamma^f - (\beta + 2\phi)t^h + \beta t^f - (\beta + \phi)\tau}{4\phi(\beta + \phi)} \quad (29)$$

$$z^f = \frac{\phi\alpha^h + \phi\alpha^f + \beta\gamma^h - (\beta + 2\phi)\gamma^f + \beta t^h - (\beta + 2\phi)t^f + (\beta + \phi)\tau}{4\phi(\beta + \phi)} \quad (30)$$

As in the Case1, all of the above profit maximizing amounts are endogenized by economic variables and policy variables (α^h, α^f, γ^h, γ^f, t^h, t^f, and τ).

3.3 Case 3. One-way trade from Foreign to Home market ($\tau > 0$, $y^h = 0$ and $y^f > 0$)

Given $\tau > 0$ and $y^h = 0$, from the profit function of the MNC (equation (4)), the first order conditions are given by

$$\frac{\partial \Pi}{\partial x^h} = p^{h\prime} x^h + p^h - [(\gamma^h + t^h) + 2\phi x^h] + p^{h\prime} y^f = 0 \quad (31)$$

$$\frac{\partial \Pi}{\partial x^f} = p^{f\prime} x^f + p^f - [(\gamma^f + t^f) + \phi(2x^f + 2y^f)] = 0 \quad (32)$$

$$\frac{\partial \Pi}{\partial y^f} = p^{h\prime} x^h + p^{h\prime} y^f + p^h - [(\gamma^f + t^f) + \phi(2x^f + 2y^f) + \tau] = 0 \quad (33)$$

By using (31), (32) and (33), the profit maximizing amounts of trade flow from the foreign to the home country, domestic sales volumes made by foreign subsidiary in the two host countries are obtained and given by

$$y^f = \frac{\phi\alpha^h - \phi\alpha^f + \beta\gamma^h - \beta\gamma^f + \beta t^h - \beta t^f - (\beta + \phi)\tau}{4\beta\phi} \quad (34)$$

$$x^h = \frac{\phi\alpha^h + \phi\alpha^f - (\beta + 2\phi)\gamma^h + \beta\gamma^f - (\beta + 2\phi)t^h + \beta t^f + (\beta + \phi)\tau}{4\phi(\beta + \phi)} \quad (35)$$

$$x^f = \frac{-\phi\alpha^h + (2\beta + \phi)\alpha^f - \beta\gamma^h - \beta\gamma^f - \beta t^h - \beta t^f + (\beta + \phi)\tau}{4\beta(\beta + \phi)} \quad (36)$$

Since $y^h = 0$, the amount of consumption in the foreign country's market is simply domestic sales volume of the foreign subsidiary in the foreign country. Then, the profit maximizing level of domestic consumptions in the two host countries are given by

$$q^h = \frac{(2\beta+\phi)\alpha^h - \phi\alpha^f - \beta\gamma^h - \beta\gamma^f - \beta t^h - \beta t^f - (\beta+\phi)\tau}{4\beta(\beta+\phi)} \quad (37)$$

$$q^f = \frac{-\phi\alpha^h + (2\beta+\phi)\alpha^f - \beta\gamma^h - \beta\gamma^f - \beta t^h - \beta t^f + (\beta+\phi)\tau}{4\beta(\beta+\phi)} \quad (38)$$

The profit maximizing level of prices in each markets are obtained and given by.

$$p^h = \frac{(2\beta+3\phi)\alpha^h + \phi\alpha^f + \beta\gamma^h + \beta\gamma^f + \beta t^h + \beta t^f + (\beta+\phi)\tau}{4(\beta+\phi)} \quad (39)$$

$$p^f = \frac{\phi\alpha^h + (2\beta+3\phi)\alpha^f + \beta\gamma^h + \beta\gamma^f + \beta t^h + \beta t^f - (\beta+\phi)\tau}{4(\beta+\phi)} \quad (40)$$

Since $y^h = 0$, foreign multinational's investment amount in the home country, z^h, is simply domestic sales volume of the foreign subsidiary in the home country. Then, the proift maximizing level of FDI amounts made by foreign subsidiaries in the two host countries are given by

$$z^h = \frac{\phi\alpha^h + \phi\alpha^f - (\beta+2\phi)\gamma^h + \beta\gamma^f - (\beta+2\phi)t^h + \beta t^f + (\beta+\phi)\tau}{4\phi(\beta+\phi)} \quad (41)$$

$$z^f = \frac{\phi\alpha^h + \phi\alpha^f + \beta\gamma^h - (2\phi+\beta)\gamma^f + \beta t^h - (\beta+2\phi)t^f - (\beta+\phi)\tau}{4\phi(\beta+\phi)} \quad (42)$$

Since the trade flow in the Case3 is symmetric to the one in the Case2, the effects of changes in economic variables, such as α^h, α^f, γ^h, γ^f, t^h, t^f, and τ, on profit maximizing level of outputs for domestic and foreign sales, market prices, and foreign investments in the Case3 are symmetric to those in the Case2.

4 Stage 1

In this section, separately for each of the three cases, welfare maximizing level of tax policies of the two host governments are derived. Moreover, effects of heterogeneity of the two host countries, in terms of market demand size and marginal cost, on the optimal tax policies are evaluated. The effects of transportation cost on the welfare maximizing tax policies are evaluated as well.

4.1 Case 1. Intra-industry trade ($\tau = 0$, $y^h > 0$ and $y^f > 0$)

In the section 3, it is assumed that profits of the foreign subsidiaries in both host countries are fully repatriated to the MNC's headquarter which locates outside of the two host countries. Thus, there is no producer's surplus in both markets. The welfare is composed of consumer surplus and government revenue. With no tariff revenue, the government revenue is simply tax revenue.

4.1.1 Welfare analysis of the Home country

The welfare function is given by

$$W_1^h = CS_1^h + GR_1^h \tag{43}$$

where CS_1^h and GR_1^h represent home country's consumer surplus and tax levied by home country's government respectively.

The CS_1^h and GR_1^h are given by

$$CS_1^h = \alpha^h q^h - \frac{1}{2}\beta(q^h)^2 - p^h q^h \tag{44}$$

$$GR_1^h = t^h(x^h + y^h) = t^h z^h \tag{45}$$

By plugging the equations (13)~(18) into (44) and (45), W_1^h is easily obtained. Since the two host countries compete each other for FDI, the welfare maximizing tax policy of the home government, denoted by t_1^{h*}, is determined as a reaction to the foreign government's tax policy. In other words, t_1^{h*} is a function of t^f. Then, from $\frac{\partial W_1^h}{\partial t^h} = 0$, t_1^{h*} is derived and given by

$$t_1^{h*} = A + Bt^f \tag{46}$$

where $A = \frac{(3\phi^2+2\beta\phi)\alpha^h+(5\phi^2+4\beta\phi)\alpha^f-(4\beta^2+11\beta\phi+8\phi^2)\gamma^h+(5\beta\phi+4\beta^2)\gamma^f}{23\phi\beta+16\phi^2+8\beta^2}$ and $B = \frac{\beta(5\phi+4\beta)}{23\phi\beta+16\phi^2+8\beta^2}$.

The equation (46) represents a reaction curve of t^h to the t^f and denoted by $R(t^f)$. Let's name this curve HH curve. The slope of the HH curve is greater than 1 in $(t^h,\ t^f)$ plane.

4.1.2 Welfare analysis of the Foreign country

As in the welfare analysis in the home country, the welfare function of the foreign country is given by

$$W_1^f = CS_1^f + GR_1^f \tag{47}$$

where CS_1^f and GR_1^f are given by

$$CS_1^f = \alpha^f q^f - \frac{1}{2}\beta(q^f)^2 - p^f q^f \quad (48)$$

$$GR_1^f = t^f(x^f + y^f) = t^f z^f \quad (49)$$

By plugging the equations (13)~(18) into (48) and (49), W_1^f is computed easily. The welfare maximizing tax policy of the foreign government, denoted by t_1^{f*}, is determined as a reaction to the home government's tax policy. Thus, t_1^{f*} is a function of t^h. From $\frac{\partial W_1^f}{\partial t^f} = 0$, t_1^{f*} is derived and given by

$$t_1^{f*} = C + Dt^h \quad (50)$$

where $C = \frac{(3\phi^2+2\beta\phi)\alpha^f+(5\phi^2+4\beta\phi)\alpha^h-(4\beta^2+11\beta\phi+8\phi^2)\gamma^f+(5\beta\phi+4\beta^2)\gamma^h}{23\phi\beta+16\phi^2+8\beta^2}$ and $D = \frac{\beta(5\phi+4\beta)}{23\phi\beta+16\phi^2+8\beta^2}$.

The equation (50) represents a reaction curve of t^f to the t^h and denoted by $R(t^h)$. Let's name this reaction curve FF curve. The slope of the FF curve is less than 1 in $(t^h,\ t^f)$ plane. We find that both $R(t^f)$ and $R(t^h)$ curves are linear and positively sloped. The slope of $R(t^f)$ is greater than the slope of $R(t^h)$. Thus, the stability condition is satisfied. There exists a Nash Equilibium.

To evaluate effects of heterogeneity of the two host countries on the optimal tax policies, initially, we need to find t^h and t^f at the Nash Equilibrium assuming the two countries are identical, thus, $\alpha^h = \alpha^f$ and $\gamma^h = \gamma^f$ and see what happen to these two tax policies as α^h, α^f, γ^h and γ^f change. Given $\alpha^h = \alpha^f = \alpha$ and $\gamma^h = \gamma^f = \gamma$, the welfare maximizing t^h and t^f are given by

$$t^h = t^f = \frac{(28\phi\beta + 16\phi^2 + 12\beta^2)(8\phi^2 + 6\beta\phi)(\alpha - \gamma)}{(23\phi\beta + 16\phi^2 + 8\beta^2)^2 - \beta^2(5\phi + 4\beta)^2} \quad (51)$$

Thus, the Nash equilibrium locates on the 45 degree line in the t^h and t^f plane. Let's name this initial Nash Equilibrium NE^I. At this initial Nash equilibrium, $A = C$ and $B = D$. From the equation (51), the following lemma is obtained and illustrated by Figure1.1.

Lemma 4 *For the two identical countries in terms of market demand size and marginal cost, with no transportation cost, at Nash equilibrium, the two host governments impose same level of tax rates.*

4.1.3 Effects of heterogeneity of the two host countries

To see what happen to initial t^h and t^f described by the equation (51), the effects of heterogeneity of the two host countries are summarized by the following facts of the first order partial derivatives.

$$\frac{\partial C}{\partial \alpha^h} > \frac{\partial A}{\partial \alpha^h} > 0,\ \frac{\partial A}{\partial \alpha^f} > \frac{\partial C}{\partial \alpha^f} > 0 \quad (52)$$

$$\frac{\partial A}{\partial \gamma^h} < 0 < \frac{\partial C}{\partial \gamma^h},\ \frac{\partial C}{\partial \gamma^f} < 0 < \frac{\partial A}{\partial \gamma^f} \quad (53)$$

Slopes of the two reaction curves are fixed. Since $\frac{\partial A}{\partial \alpha^h} = \frac{\partial C}{\partial \alpha^f}$ and $\frac{\partial C}{\partial \alpha^h} = \frac{\partial A}{\partial \alpha^f}$, the impact of changes in α^h and α^f are symmetric. Thus, as home (foreign) country's market demand increases, welfare maximizing tax policy of foreign (home) government is greater than that of home (foreign) government. Again, since $\frac{\partial A}{\partial \gamma^h} = \frac{\partial C}{\partial \gamma^f}$ and $\frac{\partial C}{\partial \gamma^h} = \frac{\partial A}{\partial \gamma^f}$, the impact of changes in γ^h and γ^f are symmetric. Thus, as home (foreign) country's marginal cost increases, welfare maximizing tax policy of foreign (home) government is greater than that of home (foreign) government.

4.2 Case 2. One-way trade from Home to Foreign market ($\tau > 0$, $y^h > 0$ and $y^f = 0$)

As in the Case1, the welfare consists of consumer surplus and government revenues. The government revenue is simply a tax revenue.

4.2.1 Welfare analysis of the Home country

The welfare function is given by

$$W_2^h = CS_2^h + GR_2^h \tag{54}$$

From the equations (25)~(30), CS_2^h and GR_2^h are given by

$$CS_2^h = \frac{[-\phi\alpha^f + (2\beta+\phi)\alpha^h - \beta\gamma^f - \beta\gamma^h - \beta t^f - \beta t^h + (\beta+\phi)\tau]^2}{32\beta(\phi+\beta)^2} \tag{55}$$

$$GR_2^h = \frac{t^h[\phi\alpha^f + \phi\alpha^h + \beta\gamma^f - (\beta+2\phi)\gamma^h + \beta t^f - (\beta+2\phi)t^h - (\beta+\phi)\tau]}{4\phi(\phi+\beta)} \tag{56}$$

Then, W_2^h is easily computed and expressed as a function of t^h and t^f. As in the Case1, the welfare maximizing tax policy of the home government for the Case2, denoted by t_2^{h*}, is determined as a reaction to the foreign government's tax policy. Then, from $\frac{\partial W_2^h}{\partial t^h} = 0$, the welfare maximizing level of home government's tax policy is derived and given by

$$t_2^{h*} = A' + B' t^f \tag{57}$$

where $A' = \frac{(5\phi^2+4\beta\phi)\alpha^f + (2\beta\phi+3\phi^2)\alpha^h + (5\beta\phi+4\beta^2)\gamma^f - (4\beta^2+11\beta\phi+8\phi^2)\gamma^h - (4\beta^2+9\beta\phi+5\phi^2)\tau}{23\phi\beta+16\phi^2+8\beta^2}$ and $B' = \frac{\beta(5\phi+4\beta)}{23\phi\beta+16\phi^2+8\beta^2}$.

As in the Case1, equation (57) represents a reaction curve of t^h to the t^f and denoted by $R(t^f)$. Let's name this curve HH curve, again. The slope of the HH curve is greater than 1 in $(t^h,\ t^f)$ plane.

4.2.2 Welfare analysis of the Foreign country

As in the Case1, the welfare function of the foreign country is composed of consumer surplus and tax revenue, and given by

$$W_2^f = CS_2^f + GR_2^f \tag{58}$$

From the equations (25)~(30), CS_2^f and GR_2^f are given by

$$CS_2^f = \frac{[(2\beta+\phi)\alpha^f - \phi\alpha^h - \beta\gamma^f - \beta\gamma^h - \beta t^f - \beta t^h - (\beta+\phi)\tau]^2}{32\beta(\phi+\beta)^2} \tag{59}$$

$$GR_2^f = \frac{t^f[\phi\alpha^f + \phi\alpha^h - (\beta+2\phi)\gamma^f + \beta\gamma^h - (\beta+2\phi)t^f + \beta t^h + (\beta+\phi)\tau]}{4\phi(\phi+\beta)} \tag{60}$$

Similarly to the welfare analysis of the home country, from $\frac{\partial W_2^f}{\partial t^f} = 0$, the welfare maximizing level of foreign government's tax policy is derived and given by

$$t_2^{f*} = C^{'} + D^{'}t^h \tag{61}$$

where $C^{'} = \frac{(2\beta\phi+3\phi^2)\alpha^f + (5\phi^2+4\beta\phi)\alpha^h - (4\beta^2+11\beta\phi+8\phi^2)\gamma^f + (5\beta\phi+4\beta^2)\gamma^h + (4\beta^2+9\beta\phi+5\phi^2)\tau}{23\phi\beta+16\phi^2+8\beta^2}$ and $D^{'} = \frac{\beta(5\phi+4\beta)}{23\phi\beta+16\phi^2+8\beta^2}$.

The equation (61) represents a reaction curve of t^f to the t^h and denoted by $R(t^h)$. Let's name this reaction curve FF curve. The slope of the FF curve is less than 1 in (t^h, t^f) plane. We find that both $R(t^f)$ and $R(t^h)$ curves are linear and positively sloped. The slope of $R(t^f)$ is greater than the slope of $R(t^h)$. Thus, the usual stability condition is satisfied. There exists a Nash Equilibium.

As in the Case1, assuming the two countries are identical, we find $A^{'} < C^{'}$, $B^{'} = D^{'}$, $A^{'} < A$ and $C^{'} > C$. This means that the Nash Equilibrium no longer stays on the 45 degree line. Let's name this initial Nash Equilibrium NE^{II}. At the NE^{II}, t^f is greater than t^h. From this fact, the following lemma is obtained and illustrated by Figure1.2.

Lemma 5 *For the two identical host countries, with positive transportation cost, if one-way trade exists from home to foreign country, at Nash equilibrium, the home country's government needs to impose lower tax to the foreign sunsidiary.*

4.2.3 Effects of the country heterogeneity and transportation cost

The following facts of the first order partial derivatives summarize the effects of country heterogeneity and transportation cost between the two host countries.

$$\frac{\partial C^{'}}{\partial \alpha^h} > \frac{\partial A^{'}}{\partial \alpha^h} > 0, \; \frac{\partial A^{'}}{\partial \alpha^f} > \frac{\partial C^{'}}{\partial \alpha^f} > 0 \tag{62}$$

$$\frac{\partial A^{'}}{\partial \gamma^h} < 0 < \frac{\partial C^{'}}{\partial \gamma^h}, \; \frac{\partial C^{'}}{\partial \gamma^f} < 0 < \frac{\partial A^{'}}{\partial \gamma^f} \tag{63}$$

$$\frac{\partial A^{'}}{\partial \tau} < 0 < \frac{\partial C^{'}}{\partial \tau} \tag{64}$$

Slopes of the two reaction curves are fixed. The equation (62) implies that, if market demand size of one country increases, it is always true that both governments impose more tax. Although $\frac{\partial A^{'}}{\partial \alpha^h} = \frac{\partial C^{'}}{\partial \alpha^f}$ and $\frac{\partial C^{'}}{\partial \alpha^h} = \frac{\partial A^{'}}{\partial \alpha^f}$, since, at NE^{II}, $t^h < t^f$, the impact of changes in α^h and α^f are not always symmetric. As α^h increases, it is always true that $t^h < t^f$. This, however, is not always satisfied as α^f increases. The equation (64) implies that, if marginal cost of home (foreign) country increases, home (foreign) government impose lower tax. Again, although $\frac{\partial A^{'}}{\partial \gamma^h} = \frac{\partial C^{'}}{\partial \gamma^f}$ and $\frac{\partial C^{'}}{\partial \gamma^h} = \frac{\partial A^{'}}{\partial \gamma^f}$, the impact of changes in γ^h and γ^f are not always symmetric. As γ^h increases, it is always true that $t^h < t^f$. This, however, is not always true as γ^f increases. Finally, as τ increases, home government imposes lower tax while foreign government raises its tax rate, thus, $t^h < t^f$.

4.3 Case 3. One-way trade from Foreign to Home market ($\tau > 0$, $y^h = 0$ and $y^f > 0$)

As in the Case1 and the Case2, the welfare consists of consumer surplus and government revenues. The government revenue is simply a tax revenue.

4.3.1 Welfare analysis of the Home country

As usual, the welfare function is given by

$$W_3^h = CS_3^h + GR_3^h \tag{65}$$

From the equations (37)~(42), CS_3^h and GR_3^h are given by

$$CS_3^h = \frac{[-\phi\alpha^f + (2\beta+\phi)\alpha^h - \beta\gamma^f - \beta\gamma^h - \beta t^f - \beta t^h - (\beta+\phi)\tau]^2}{32\beta(\phi+\beta)^2} \tag{66}$$

$$GR_3^h = \frac{t^h[\phi\alpha^f + \phi\alpha^h + \beta\gamma^f - (\beta+2\phi)\gamma^h + \beta t^f - (\beta+2\phi)t^h + (\beta+\phi)\tau]}{4\phi(\phi+\beta)} \tag{67}$$

As in the Case1 and the Case2, W_3^h is easily computed and be expressed as a function of t^h and t^f. From $\frac{\partial W_3^h}{\partial t^h} = 0$, the welfare maximizing level of home government's tax policy is derived and given by

$$t_3^{h*} = A^{''} + B^{''} t^f \tag{68}$$

where $A^{''} = \frac{(5\phi^2+4\beta\phi)\alpha^f+(2\beta\phi+3\phi^2)\alpha^h+(5\beta\phi+4\beta^2)\gamma^f-(4\beta^2+11\beta\phi+8\phi^2)\gamma^h+(4\beta^2+9\beta\phi+5\phi^2)\tau}{23\phi\beta+16\phi^2+8\beta^2}$ and $B^{''} = \frac{\beta(5\phi+4\beta)}{23\phi\beta+16\phi^2+8\beta^2}$.

The equation (68) represents a reaction curve of t^h to the t^f and denoted by $R(t^f)$ as usual. As in the Case1 and the Case2, let's name this curve HH curve, again. The slope of the HH curve is greater than 1 in $(t^h,\ t^f)$ plane.

4.3.2 Welfare analysis of the Foreign country

As in the Case1 and the Case2, the welfare function of the foreign country is composed of consumer surplus and tax levied by the government, and given by

$$W_3^f = CS_3^f + GR_3^f \tag{69}$$

From the equations (37)~(42), CS_3^f and GR_3^f are given by

$$CS_3^f = \frac{[(2\beta+\phi)\alpha^f - \phi\alpha^h - \beta\gamma^f - \beta\gamma^h - \beta t^f - \beta t^h + (\beta+\phi)\tau]^2}{32\beta(\phi+\beta)^2} \tag{70}$$

$$GR_3^f = \frac{t^f[\phi\alpha^f + \phi\alpha^h - (\beta+2\phi)\gamma^f + \beta\gamma^h - (\beta+2\phi)t^f + \beta t^h - (\beta+\phi)\tau]}{4\phi(\phi+\beta)} \tag{71}$$

Similarly to the welfare analysis of the home country, from the first order conditon, $\frac{\partial W_3^f}{\partial t^f} = 0$, the welfare maximizing level of foreign government's tax policy is derived and given by

$$t_3^{f*} = C^{''} + D^{''} t^h \tag{72}$$

where $C^{''} = \frac{(2\beta\phi+3\phi^2)\alpha^f+(5\phi^2+4\beta\phi)\alpha^h-(4\beta^2+11\beta\phi+8\phi^2)\gamma^f+(5\beta\phi+4\beta^2)\gamma^h-(4\beta^2+9\beta\phi+5\phi^2)\tau}{23\phi\beta+16\phi^2+8\beta^2}$ and $D^{''} = \frac{\beta(5\phi+4\beta)}{23\phi\beta+16\phi^2+8\beta^2}$.

The equation (72) represents a reaction curve of t^f to the t^h, denoted by $R(t^h)$ as usual, and named FF curve again. The slope of the FF curve is less than 1 in $(t^h,\ t^f)$ plane. We find that both $R(t^f)$ and $R(t^h)$ curves are linear and positively sloped. The slope of $R(t^f)$ is steeper than the slope of $R(t^h)$, satisfying the usual stability condition. There exists a Nash Equilibium.

As in the Case1 and Case2, assuming the two countries are identical, we find $A^{''} > C^{''}$, $B^{''} = D^{''}$, $A^{''} > A$ and $C^{''} < C$. This means that the Nash Equilibrium, this time, called NE^{III}, no longer stays on the 45 degree line as in the Case2. However, in this case, the location of the initial Nash Equilibrium, NE^{III}, is symmetric to NE^{II}. In other words, at the NE^{III}, t^h is greater than t^f. From this fact, the following lemma is obtained and illustrated by Figure1.3.

Lemma 6 *For the two identical host countries, with positive transportation cost, if one-way trade exists from foreign to home country, at Nash equilibrium, the foreign country's government needs to impose lower tax to the foreign sunsidiary*

4.3.3 Effects of the country heterogeneity and transportation cost

From the previous analyses, it is found that, assuming that the two countries are identical, $A^{'} < A < A^{''}$ and $C^{'} > C > C^{''}$. Moreover, $(A^{''} - A) = (A - A^{'}) = (C^{'} - C) = (C - C^{''})$. Then, as seen in Figure1.1~1.3, comparing the three different Nash equilibria - NE^{I}, NE^{II}, and NE^{III}, tax policy combinations at NE^{II} and NE^{III} are symmetric to those at NE^{I}. This means that, at the welfare maximizing level, home government's tax policy in Case2 is equivalent to foreign government's tax policy in Case3. Similarly, welfare maximizing tax policy of home government in Case3 is equivalent to that of foreign government in Case2. Thus, the effects of the country heterogeneity and transportation cost in Case3 are symmetric to those in Case2.

4.4 Switching between Case 2 and Case 3

So far, each of the three cases are separately analyzed. Since both governments, given $\tau > 0$, at the welfare maximizing level, select one of the two cases - Case2 or Case3, it should be analyzed how the MNC switch from Case2 to Case3 or vice versa. In general, if the maximized profit in the Case2 is greater than the one in the Case3, the MNC chooses the trade flow of the Case2, otherwise, chooses the trade flow in the Case3. The magnitude of the maximized profits in both cases are dependent upon values of α^h, α^f, γ^h, γ^f, t^h, t^f, and τ. For instance, if $\alpha^h > \alpha^f$ and $\gamma^h < \gamma^f$, then, the MNC tends to choose the Case2 as long as there is no significant difference between t^h and t^f. In other words, $y^h > 0$, and $y^f = 0$. If the government of the foreign country decides to cut t^f heavily, it might be the situation that the trade flow in the Case3 is more profitable than the one in the Case2. Thus, the MNC switches from the Case2 to the Case3, meaning that $y^f > 0$, and $y^h = 0$.

By definition, for $\tau > 0$, the Case2 exists if and only if

$$y^h = \frac{\phi(\alpha^f - \alpha^h) + \beta(\gamma^f - \gamma^h) + \beta(t^f - t^h) - (\beta + \phi)\tau}{4\beta\phi} > 0 \qquad (73)$$

Likewise, the Case3 exists if and only if

$$y^f = \frac{\phi(\alpha^h - \alpha^f) + \beta(\gamma^h - \gamma^f) + \beta(t^h - t^f) - (\beta + \phi)\tau}{4\beta\phi} > 0 \qquad (74)$$

If the two countries are identical with same tax policies, there is no reason to trade. Mathematically, for $\alpha^h = \alpha^f$, $\gamma^h = \gamma^f, and\ t^h = t^f$, it is found that $y^h = y^f < 0$. This can be interpreted as "no trade flow" between the two countries. From this fact, the following lemma is obtained.

Lemma 7 *Given positive transportation cost, No-Trade can be chosen as an optimal trade policy instead of either Case2 or Case3.*

By equating y^h to zero, we can obtain the linear border line between the Case2 and No-Trade, denoted by

$$\gamma^h = -\frac{\phi}{\beta}\alpha^h + \frac{\phi}{\beta}\alpha^f + \gamma^f - t^h + t^f - \left(\frac{\beta+\phi}{\beta}\right)\tau \tag{75}$$

Likewise, by equating y^f to zero, the linear border line between the No-Trade and the Case3 is obtained and denoted by

$$\gamma^h = -\frac{\phi}{\beta}\alpha^h + \frac{\phi}{\beta}\alpha^f + \gamma^f - t^h + t^f + \left(\frac{\beta+\phi}{\beta}\right)\tau \tag{76}$$

Figure2.1 shows the above linear border lines in $(\alpha^h,\ \gamma^h)$ plane. The equation (75) and (76) is equaivalent to the line "(1)" and the line "(2)" in the Figure2.1, respectively. The lower triangular area and the upper triangular area represent the area where the Case2 exists only and the Case3 exists only, respectively. There is no area in which both of the Case2 and the Case3 exist simultaneously. If initially, the MNC chooses the Case2, given no changes in all other variables, as home market demand size increases, the MNC turns out to quit trade. Then, as home market demand increases further, the MNC resumes its trade, at this time, from the foreign to the home country, that is, Case3. Likewise, given fixed values of all other variables, as marginal cost in home country increases, the MNC switches its choice from its initial choice of Case 2 to the Case3 via No-trade.

Figure2.2 and Figure2.3 show a role of tax polices to MNC's choice of trade flows. As in the Figure2.2, if home government cuts tax rate, assuming no change or a raise in the foreign government's tax rate, both linear border lines move to the upper right-corner at the same rate. This means that there are more chances that the MNC chooses Case2. Likewise, as in the Figure2.3, if foreign government cuts its tax, assuming no change or a raise in the home government's tax rate, both linear border lines move to the lower left-corner at the same rate. This implies that there are less chances that the MNC chooses Case2. It is, instead, more likely that the MNC switches its choice from Case2 to either No-Trade or Case3.

As illustrated above, assume that, initially, the MNC chooses Case2, given $\tau > 0$ and $t^h = t^f$. As t^h increases and reaches at a certain level, denoted by t^h_L, the MNC stops exporting from home to foreign country and chooses not to trade. As t^h increases even further and reaches at a certain level, denoted by t^h_H, the MNC starts to choose Case3 and exports from foreign to home country. From the equation (73), by equating $y^h = 0$, we can obtain t^h_L that is expressed by

$$t^h_L = \frac{\phi}{\beta}(\alpha^f - \alpha^h) + (\gamma^f - \gamma^h) + t^f - \frac{(\beta+\phi)}{\beta}\tau \tag{77}$$

Likewise, from the equation (74), by equating $y^f = 0$, we can obtain t_H^h that is expressed by

$$t_H^h = \frac{\phi}{\beta}(\alpha^f - \alpha^h) + (\gamma^f - \gamma^h) + t^f + \frac{(\beta + \phi)}{\beta}\tau \tag{78}$$

Now, assume that, initially, the MNC chooses Case3 instead of Case2, given $\tau > 0$ and $t^h = t^f$. As t^f increases and reaches at a certain level, denoted by t_L^f, the MNC stops exporting from foreign to home country and chooses not to trade. As t^f increases even further and reaches at a certain level, denoted by t_H^f, the MNC starts to choose Case2 and exports from home to foreign country. From the equation (74), by equating $y^f = 0$, we can obtain t_L^f that is expressed by

$$t_L^f = \frac{\phi}{\beta}(\alpha^h - \alpha^f) + (\gamma^h - \gamma^f) + t^h - \frac{(\beta + \phi)}{\beta}\tau \tag{79}$$

Likewise, from the equation (73), by equating $y^h = 0$, we can obtain t_H^f that is expressed by

$$t_H^f = \frac{\phi}{\beta}(\alpha^h - \alpha^f) + (\gamma^h - \gamma^f) + t^h + \frac{(\beta + \phi)}{\beta}\tau \tag{80}$$

The equation (77)~(80) show that both t_L^h and t_H^h are functions of t^f while both t_L^f and t_H^f are functions of t^h. From the above analysis, the following proposition is obtained.

Proposition 8 *For $\tau > 0$, given a certain level of t^h, the MNC chooses Case2 for $t^f > t_H^f$ and Case3 for $t^f < t_L^f$ but chooses not to trade for $t_L^f \le t^f \le t_H^f$. Similarly for $\tau > 0$, given a certain level of t^f, the MNC chooses Case3 for $t^h > t_H^h$ and Case2 for $t^h < t_L^h$ but chooses not to trade for $t_L^h \le t^h \le t_H^h$.*

Proof. *For $\tau > 0$ and a certain level of t^h, $y^h \le 0$ if $t^f \le t_H^f$ and $y^f \le 0$ if $t^f \ge t_L^f$. This means that $y^h > 0$ if $t^f > t_H^f$ and $y^f > 0$ if $t^f < t_L^f$. In addition, from the equation (79) and (80), $t_H^f > t_L^f$. Likewise, for $\tau > 0$ and a certain level of t^f, $y^h \le 0$ if $t^h \ge t_L^h$ and $y^f \le 0$ if $t^h \le t_H^h$. This means that $y^f > 0$ if $t^h > t_H^h$ and $y^h > 0$ if $t^h < t_L^h$. In addition, from the equation (77) and (78), $t_H^h > t_L^h$.* ■

From the equations (77)~(80), t_L^h and t_H^f are equivalent in (t^h, t^f) plane while t_H^h and t_L^f are equivalent in (t^h, t^f) plane. In addition, t_L^h (or t_H^f) and t_H^h (or t_L^f) are parallel with slope of 1. These facts, with the above proposition, we can obtain sets of possible tax policy combination in which the MNC chooses Case2, No-Trade, and Case3. Figure3 illustrates these policy combination sets.

5 Choice among the three alternatives : Case2,

No-Trade, and Case3

Understanding the mechanism how the MNC switches its choice among Case2, No-Trade, and Case3 and associated tax poicy combination sets, in this section, host government's choice of welfare maximizing tax policy for all three alternatives - Case2, No-Trade, and Case3 are jointly analyzed. If both home and foreign countries have maximum welfare in Case2, at Nash equilibrium level of tax policy combination, both governments choose Case2, denoted by N_2. Likewise, N_3 and N_{NT} represent that both governments choose Case3 and No-Trade at Nash equilibrium level of tax policy combination. For joint welfare analysis, all the reaction functions, t_L^h, t_H^f, t_H^h, and t_L^f need to be considered simultaneously in (t^h, t^f) plane. Figure4.1 illustrates a general situation while Figure4.2 illustrates a special situation where the two countries are identical.

Let's find conditions for the existence of each of the three alternatives one by one.

5.1 Conditions for the existence of Case2 and Case3 at Nash equilibrium

To have a condition for the existence of Case3 at the Nash equilibrium, we need to take a few steps. First, find an intersection point between the two curves of t_3^{h*} and t_H^h. Let's call this intersection point $(t^h, t^f) = (a, b)$. $a = \frac{(6\beta\phi+8\phi^2)(\alpha^h-\gamma^h)}{(18\beta\phi+16\phi^2+4\beta^2)}$ and $b = \frac{\rho_1\alpha^h+\rho_2\alpha^f+\rho_3\gamma^h+\rho_4\gamma^f+\rho_5\tau}{(18\beta^2\phi+16\beta\phi^2+4\beta^3)}$ where $\rho_1 = (10\beta^2\phi + 26\beta\phi^2 + 16\phi^3)$, $\rho_2 = -(4\beta^2\phi + 18\beta\phi^2 + 16\phi^3)$, $\rho_3 = (4\beta^3 + 12\beta^2\phi + 8\beta\phi^2)$, $\rho_4 = -(4\beta^3 + 18\beta^2\phi + 16\beta\phi^2)$, and $\rho_5 = -(4\beta^3 + 22\beta^2\phi + 34\beta\phi^2 + 16\phi^3)$. Since $(\alpha^h - \gamma^h) > 0$, a which is t^h at the intersection point between the two curves of t_3^{h*} and t_H^h is greater than zero. Second, if $t_3^{f*}(t^h = a)$ is less than b, there exists Case3 at the Nash equilibrium. Figure5.1 illustrates this in t^h and t^f space.

Similarly, to have a condition for the existence of Case2 at the Nash equilibrium, we need to find an intersection point between the two curves of t_2^{f*} and t_H^f. Let's call this point $(t^h, t^f) = (b', a')$. $a' = \frac{(6\beta\phi+8\phi^2)(\alpha^f-\gamma^f)}{(18\beta\phi+16\phi^2+4\beta^2)}$ and $b' = \frac{\rho_1'\alpha^h+\rho_2'\alpha^f+\rho_3'\gamma^h+\rho_4'\gamma^f+\rho_5'\tau}{(18\beta^2\phi+16\beta\phi^2+4\beta^3)}$ where $\rho_1' = -(4\beta^2\phi + 18\beta\phi^2 + 16\phi^3)$, $\rho_2' = (10\beta^2\phi+26\beta\phi^2+16\phi^3)$, $\rho_3' = -(4\beta^3+18\beta^2\phi+16\beta\phi^2)$, $\rho_4' = (4\beta^3+12\beta^2\phi+8\beta\phi^2)$, and $\rho_5' = -(4\beta^3+22\beta^2\phi+34\beta\phi^2+16\phi^3)$. Since $(\alpha^f - \gamma^f) > 0$, a' which is t^f at the intersection point between the two curves of t_2^{f*} and t_H^f is greater than zero. Then, if $t_2^{h*}(t^f = a')$ is less than b', there exists Case2 at the Nash equilibrium. Figure5.2 illustrates this in t^h and t^f space.

Mathematical expression of the condition for the existence of N_3 is turned out to be

$$\theta_1(\alpha^f - \alpha^h) + \theta_2(\gamma^f - \gamma^h) + \theta_3\tau < 0 \tag{81}$$

Similarly, a mathematical condition of the condition for the existence of N_2 is turned out to be

$$\theta_1(\alpha^h - \alpha^f) + \theta_2(\gamma^h - \gamma^f) + \theta_3\tau < 0 \tag{82}$$

where $\theta_1 = 10\beta^4\phi + 71\beta^3\phi^2 + 173\beta^2\phi^3 + 176\beta\phi^4 + 64\phi^5$, $\theta_2 = 4\beta^5 + 30\beta^4\phi + 78\beta^3\phi^2 + 84\beta^2\phi^3 + 32\beta\phi^4$, and $\theta_3 = 4\beta^5 + 40\beta^4\phi + 149\beta^3\phi^2 + 257\beta^2\phi^3 + 208\beta\phi^4 + 64\phi^5$. Thus, conditions of N_2 and N_3 are described in $(\alpha^h - \alpha^f)$ and $(\gamma^h - \gamma^f)$ space and illustrated by Figure6.

5.2 Condition for the existence of No-Trade at Nash equilibrium

To have a condition for the existence of No-Trade at the Nash equilibrium, we need to find profit maximizing consumptions, market prices, and FDI amounts and welfare maximizing tax policies first.

5.2.1 Stage 2 : when MNC chooses No-Trade

In the following two sections, including this section, profit maximizing level of outputs and prices, and welfare analysis when MNC chooses not to trade are evaluated. We know that t^f between t^f_L and t^f_H as well as t^h between t^h_L and t^h_H, the MNC chooses not to trade, meaning that $y^h = 0$ and $y^f = 0$. Then, from the first order conditions, we obtain the following computations.

$$\frac{\partial\pi}{\partial x^h} = p^{h'} x^h + p^h - [(\gamma^h + t^h) + \phi 2x^h] = 0 \tag{83}$$

$$\frac{\partial\pi}{\partial x^f} = p^{f'} x^f + p^f - [(\gamma^f + t^f) + \phi 2x^f] = 0 \tag{84}$$

From the equation (83) and (84), outputs for domestic sales and market prices are obtained and given by

$$x^h = \frac{\alpha^h - \gamma^h - t^h}{2(\beta + \phi)} = q^h = z^h \tag{85}$$

$$x^f = \frac{\alpha^f - \gamma^f - t^f}{2(\beta + \phi)} = q^f = z^f \tag{86}$$

$$p^h = \frac{(\beta + 2\phi)\alpha^h + \beta\gamma^h + \beta t^h}{2(\beta + \phi)} \tag{87}$$

$$p^f = \frac{(\beta + 2\phi)\alpha^f + \beta\gamma^f + \beta t^f}{2(\beta + \phi)} \tag{88}$$

5.2.2 Stage 1 : when MNC chooses No-Trade

For t^f between t^f_L and t^f_H as well as t^h between t^h_L and t^h_H, welfare function of each country when the MNC chooses not to trade consists of consumer surplus and government's tax ravenue. They are denoted by $W^h_{NT} = CS^h_{NT} + GR^h_{NT}$

and $W_{NT}^f = CS_{NT}^f + GR_{NT}^f$. It is found that W_{NT}^h is a function of α^h, γ^h, and t^h while W_{NT}^f is a function of α^f, γ^f, and t^f.

Then, from $\frac{\partial W_{NT}^h}{\partial t^h} = 0$, the welfare maximizing tax rate chosen by home government is obtained and given by

$$t_{NT}^{h*} = \left(\frac{\beta + 2\phi}{3\beta + 4\phi}\right)(\alpha^h - \gamma^h) \tag{89}$$

Likewise, from $\frac{\partial W_{NT}^f}{\partial t^f} = 0$, the welfare maximizing tax rate chosen by foreign government is obtained and given by

$$t_{NT}^{f*} = \left(\frac{\beta + 2\phi}{3\beta + 4\phi}\right)(\alpha^f - \gamma^f) \tag{90}$$

By plugging the equation (89) and (90) into (85) and (86), respectively, we obtain welfare maximizing outputs for domestic sales.

$$x_{NT}^{h*} = \frac{1}{(3\beta + 4\phi)}(\alpha^h - \gamma^h) \tag{91}$$

$$x_{NT}^{f*} = \frac{1}{(3\beta + 4\phi)}(\alpha^f - \gamma^f) \tag{92}$$

Since $x_{NT}^{h*} > 0$ and $x_{NT}^{f*} > 0$, it is true that $\alpha^h > \gamma^h$ and $\alpha^f > \gamma^f$, respectively. Then, immediately, it is also true that $t_{NT}^{h*} > 0$ and $t_{NT}^{f*} > 0$. From this fact, the following proposition is obtained.This means that, if there is no trade, given positive transportation costs, both Home and Foreign government impose a positive tax on output productions for domestic sales, instead of providing subsidies.

Proposition 9 *When the two countries choose not to trade, given positive transportation cost, for both host countries, the welfare maximizing fiscal policy is to impose positive tax to each output produced by MNCs for domestic sales, instead of providing subsidy.*

Associated welfare maximizing domestic market prices are obtained and given by

$$p_{NT}^{h*} = \frac{1}{(3\beta + 4\phi)}[2(\beta + 2\phi)\alpha^h + \beta\gamma^h] \tag{93}$$

$$p_{NT}^{f*} = \frac{1}{(3\beta + 4\phi)}[2(\beta + 2\phi)\alpha^f + \beta\gamma^f] \tag{94}$$

Finally, Then, the maximized economic welfare level of the host countries are obtained and given by

$$W_{NT}^{h*} = \frac{(\alpha^h - \gamma^h)^2}{2(3\beta + 4\phi)} \quad (95)$$

$$W_{NT}^{f*} = \frac{(\alpha^f - \gamma^f)^2}{2(3\beta + 4\phi)} \quad (96)$$

From the above analysis, the following lemma is obtained.

Lemma 10 *The conjectures of W_{NT}^h and W_{NT}^f are not affected by changes in t^f and t^h, respectively. Moreover, given values of α^h and γ^h, and parameters of β and ϕ, both t_{NT}^{h*} and W_{NT}^{h*} are fixed values and not affected by changes in t^f. Likewise, given values of α^f and γ^f, and parameters of β and ϕ, both t_{NT}^{f*} and W_{NT}^{f*} are fixed values and not affected by changes in t^h.*

From above analysis, we can build up a logic to find condition for existence of No-Trade at Nash equilibeirum. When foreign government decides to block trade, for any given optimal tax policy of the home government, $t_{NT}^{h*} > 0$, optimal tax policy of the foreign government is positive and must be within $t_L^f(t_{NT}^{h*})$ and $t_H^f(t_{NT}^{h*})$, shortly, $t_{NT}^{f*} \in [t_l^f, t_u^f]$ where $t_l^f = t_L^f(t_{NT}^{h*})$ and $t_u^f = t_H^f(t_{NT}^{h*})$. Following this logic, the condition for existence of No-Trade can be derived and mathematically expressed by

$$-\theta_3^{'}\tau \leq \theta_1^{'}(\alpha^f - \alpha^h) + \theta_2^{'}(\gamma^f - \gamma^h) \leq \theta_3^{'}\tau \quad (97)$$

where $\theta_1^{'} = \frac{(\beta+\phi)(\beta+4\phi)}{\beta(3\beta+4\phi)}$, $\theta_2^{'} = \frac{2(\beta+\phi)}{(3\beta+4\phi)}$, and $\theta_3^{'} = \frac{(\beta+\phi)}{\beta}$. Figure6 illustrates this condition and shows the No-Trade area between the two parallel lines of $\theta_1^{'}(\alpha^f - \alpha^h) + \theta_2^{'}(\gamma^f - \gamma^h) = \theta_3^{'}\tau$ and $\theta_1^{'}(\alpha^f - \alpha^h) + \theta_2^{'}(\gamma^f - \gamma^h) = -\theta_3^{'}\tau$ in $(\alpha^h - \alpha^f)$ and $(\gamma^h - \gamma^f)$ space. Combining all three conditions, Figure6 describes feasible policy choices at Nash equilibrium in terms of degree of heterogeneity of the two host countries.

From the conditions for existence of Case2, Case3, and No-Trade at the Nash equilibrium with associated equation (81), (82), and (97), following series of lemma and proposition are obtained.

Lemma 11 *If home country's market demand size and marginal production cost is at least foreign country's market demand size and marginal production cost, respectively, there is no one-way trade from home to foreign country. By symmetry, if foreign country's market demand size and marginal production cost is at least home country's market demand size and marginal production cost, respectively, there is no one-way trade from foreign to home country.*

Proof. *From the equation (82), for $\alpha^h \geq \alpha^f$ and $\gamma^h \geq \gamma^f$, $\theta_1(\alpha^h - \alpha^f) + \theta_2(\gamma^h - \gamma^f) + \theta_3\tau > 0$. Thus, Case2 is no longer considered as a Nash equilibrium. Likewise, from the equation (81), for $\alpha^h \leq \alpha^f$ and $\gamma^h \leq \gamma^f$, $\theta_1(\alpha^f - \alpha^h) + \theta_2(\gamma^f - \gamma^h) + \theta_3\tau > 0$. Therefore, Case3 is no longer possible candidate for Nash equilibrium.* ■

Proposition 12 *There must be situations in which both governments have two choices - block trade and one-way trade either from home to foreign or foreign to home country simultaneously. This means that, depending on market demand sizes and marginal costs, there exist multiple Nash equilibria - Case2 and No-Trade or Case3 and No-Trade.*

Proof. *The slopes of the lines* $\theta_1(\alpha^h - \alpha^f) + \theta_2(\gamma^h - \gamma^f) + \theta_3\tau = 0$ *and* $\theta_1(\alpha^f - \alpha^h) + \theta_2(\gamma^f - \gamma^h) + \theta_3\tau = 0$ *are flatter than* $\theta_1^{'}(\alpha^f - \alpha^h) + \theta_2^{'}(\gamma^f - \gamma^h) = \theta_3^{'}\tau$ *and* $\theta_1^{'}(\alpha^f - \alpha^h) + \theta_2^{'}(\gamma^f - \gamma^h) = -\theta_3^{'}\tau$ *in* $\alpha^h - \alpha^f$ *and* $\gamma^h - \gamma^f$ *space since* $\frac{\theta_3}{\theta_1}\tau > \frac{\theta_3^{'}}{\theta_1^{'}}\tau$ *and* $\frac{\theta_3}{\theta_2}\tau < \frac{\theta_3^{'}}{\theta_2^{'}}\tau$. *Thus, there exist overlapped area between* N_{NT} *and* N_2 *as well as* N_{NT} *and* N_3. *Figure6 illustrates this and shows area where* N_{NT} *and* N_2 *exist simultaneously and so does* N_{NT} *and* N_3. ■

Proposition 13 *Depending on values of market demand sizes and marginal costs, there are situations where no Nash equilibrium exists.*

Lemma 14 *When marginal production cost of the two countries are close enough, one-way trade from home to foreign country happens only if foreign market demand is sufficiently greater than home market demand. Otherwise, either No-Trade is chosen or no Nash equilibrium exists. However, under the same marginal cost condition, one-way trade from foreign to home country happens only if home market demand is sufficiently greater than foreign market demand. Otherwise, again, either No-Trade is chosen or no Nash equilibrium exists.*

Proof. *For* $\gamma^h = \gamma^f$, *the equation (82) is true if* $\alpha^f > \alpha^h + \frac{\theta_3}{\theta_1}\tau$ *where* θ_1 *and* θ_3 *are positive. However, for* $\gamma^h = \gamma^f$, *the equation (81) is true if* $\alpha^h > \alpha^f + \frac{\theta_3}{\theta_1}\tau$.

Lemma 15 *When the two countries market demand sizes are close enough, one-way trade from home to foreign country happens only if foreign country's marginal production cost is sufficiently greater than that of home country. Otherwise, No-trade is chosen. Under the same market demand condition, one-way trade from foreign to home country happens only if home country's marginal production cost is sufficiently greater than that of foreign country. Otherwise, No-Trade is chosen.*

Proof. *For* $\alpha^h = \alpha^f$, *the equation (82) is true if* $\gamma^f > \gamma^h + \frac{\theta_3}{\theta_2}\tau$ *where* θ_2 *and* θ_3 *are positive. However, for* $\alpha^h = \alpha^f$, *the equation (81) is true if* $\gamma^h > \gamma^f + \frac{\theta_3}{\theta_2}\tau$.
■

Lemma 16 *When the two countries market demand sizes are close enough and marginal production cost of foreign country is greater than that of home country, if the difference between the two marginal costs are less than or equal to transportation cost between the two countries, one-way trade from home to foreign country no longer exists and, instead, No-Trade is chosen. Under the same market demand condition, when marginal production cost of home country is greater than that of foreign country, if the difference between the two marginal costs are less than or equal to transportation cost between the two countries, one-way trade from foreign to home country is no longer possible and, instead, No-Trade is chosen.*

Proof. *From the equation (82), for* $\alpha^h = \alpha^f$, $\theta_2(\gamma^h - \gamma^f) + \theta_3\tau < 0$ *or, by rearrainging,* $\theta_3\tau < \theta_2(\gamma^f - \gamma^h)$. *Since* $\theta_3 > \theta_2$, *if* $(\gamma^f - \gamma^h) \leq \tau$, *then,* $\theta_2(\gamma^h - \gamma^f) + \theta_3\tau > 0$, *that is, the equation (82) is no longer valid. Similarly, from the equation (81), for* $\alpha^h = \alpha^f$, $\theta_2(\gamma^f - \gamma^h) + \theta_3\tau < 0$ *or, by rearrainging,* $\theta_3\tau < \theta_2(\gamma^h - \gamma^f)$. *Again, since* $\theta_3 > \theta_2$, *if* $(\gamma^h - \gamma^f) \leq \tau$, *then,* $\theta_2(\gamma^f - \gamma^h) + \theta_3\tau > 0$, *that is, the equation (81) is not binding.* ■

Proposition 17 *Export from home to foreign country and export from foreign to home country cannot be chosen simultaneously at the welfare maximizing level. This means, assuming homogeneous products, there exist no multiple Nash equilibria of Case2 and Case3.*

Proof. *By rearraing the equation (82), we have* $\theta_1(\alpha^h - \alpha^f) + \theta_2(\gamma^h - \gamma^f) < -\theta_3\tau$. *Likewise, by rearraining the equation (81), we have* $\theta_1(\alpha^h - \alpha^f) + \theta_2(\gamma^h - \gamma^f) > \theta_3\tau$. *These two inequalities contradict to each other. Moreover, the Figure14 shows a clear cut of this lemma.* N_2 *area and* N_3 *area are not overlapped at all in* $\alpha^h - \alpha^f$ *and* $\gamma^h - \gamma^f$ *space since the two lines of* $\theta_1(\alpha^h - \alpha^f) + \theta_2(\gamma^h - \gamma^f) + \theta_3\tau = 0$ *and* $\theta_1(\alpha^f - \alpha^h) + \theta_2(\gamma^f - \gamma^h) + \theta_3\tau = 0$ *are parallel to each other.* ■

■

As illustrated in Figure6, overlapped area of N_{NT} and N_2 does not exist for $\gamma^h > \gamma^f$. The overlapped area of N_{NT} and N_3 does not exist for $\gamma^h < \gamma^f$. Thus, the following lemma is obtained.

Lemma 18 *If home country's marginal production cost is greater than that of foreign country, multiple Nash equilibria of Case2 and No-Trade is no longer possible. If foreign country's marginal production cost is greater than that of home country, multiple Nash equilibria of Case3 and No-Trade is no longer possible.*

From the conditions for existence of Case2, Case3, and No-Trade at the Nash equilibrium with associated equations of (81), (82), (97), and Figure6, we can observe the following facts. At the intersection point between $\theta_1(\alpha^h - \alpha^f) + \theta_2(\gamma^h - \gamma^f) + \theta_3\tau = 0$ and $\theta_1^{'}(\alpha^f - \alpha^h) + \theta_2^{'}(\gamma^f - \gamma^h) = \theta_3^{'}\tau$, we have $\alpha^h - \alpha^f = \left(\frac{\theta_2\theta_3^{'} - \theta_2^{'}\theta_3}{\theta_1\theta_2^{'} - \theta_1^{'}\theta_2}\right)\tau < 0$ and $\gamma^h - \gamma^f = -\left(\frac{\theta_1\theta_3^{'} - \theta_1^{'}\theta_3}{\theta_1\theta_2^{'} - \theta_1^{'}\theta_2}\right)\tau < 0$. Since $\theta_2\theta_3^{'} - \theta_2^{'}\theta_3 = \frac{(\beta+\phi)(4\beta^6+26\beta^5\phi+56\beta^4\phi^2+50\beta^3\phi^3+16\beta^2\phi^4)}{\beta(3\beta+4\phi)}$, $\theta_1\theta_3^{'} - \theta_1^{'}\theta_3 = \frac{(\beta+\phi)(-4\beta^6-26\beta^5\phi-56\beta^4\phi^2-50\beta^3\phi^3-16\beta^2\phi^4)}{\beta(3\beta+4\phi)}$, and $\theta_1\theta_2^{'} - \theta_1^{'}\theta_2 = \frac{(\beta+\phi)(-4\beta^6-26\beta^5\phi-56\beta^4\phi^2-50\beta^3\phi^3-16\beta^2\phi^4)}{\beta(3\beta+4\phi)}$ we have $(\theta_1\theta_3^{'} - \theta_1^{'}\theta_3) = (\theta_1\theta_2^{'} - \theta_1^{'}\theta_2) = -(\theta_2\theta_3^{'} - \theta_2^{'}\theta_3)$. Thus, at the intersection point, we have $\alpha^h - \alpha^f = -1$ and $\gamma^h - \gamma^f = -1$. Similarly, the two curves of $\theta_1(\alpha^f - \alpha^h) + \theta_2(\gamma^f - \gamma^h) + \theta_3\tau = 0$ and $\theta_1^{'}(\alpha^f - \alpha^h) + \theta_2^{'}(\gamma^f - \gamma^h) = -\theta_3^{'}\tau$ intersect at $\alpha^h - \alpha^f = 1$ and $\gamma^h - \gamma^f = 1$. This means that the two intersection points locate themselves on the 45° line from the origin in the $\alpha^h - \alpha^f$ and $\gamma^h - \gamma^f$ space. Thus, as illustrated by Figure6, if $\alpha^h - \alpha^f > \gamma^h - \gamma^f$, that is, the right-hand-side of the 45° line, there exists overlapped area of No-Trade and Case2 but not that of No-Trade and Case3. Similarly, if $\alpha^h - \alpha^f < \gamma^h - \gamma^f$, that is, the left-hand-side of the 45° line, there exists overlapped area of No-Trade and Case3 but not that of No-Trade and Case2. From this observation, the following lemma is obtained.

Lemma 19 *If $\alpha^h - \alpha^f > \gamma^h - \gamma^f$, depending on values of $\alpha^h - \alpha^f$ and $\gamma^h - \gamma^f$, both governments might have multiple Nash equilibria of Case2 and No-Trade but not that of Case3 and No-Trade. If $\alpha^h - \alpha^f < \gamma^h - \gamma^f$, depending on values of $\alpha^h - \alpha^f$ and $\gamma^h - \gamma^f$, both governments might have both governments might have multiple Nash equilibria of Case3 and No-Trade but not that of Case2 and No-Trade.*

Moreover, following lemmas are obtained and also illustrated by Figure6.

Lemma 20 *If $\alpha^h > \alpha^f + 1$ and $\gamma^h > \gamma^f + 1$ happen simultaneously, both governments can only choose Case3 at the Nash equilibrium. Likewise, if $\alpha^h < \alpha^f - 1$ and $\gamma^h < \gamma^f - 1$ happen simultaneously, both governments can only choose Case2 at the Nash equilibrium.*

Lemma 21 *If $\alpha^h > \alpha^f + 1$ and $\gamma^h < \gamma^f + 1$ happen simultaneously, there can be no multiple Nash equilibria of No-Trade and Case3. If $\alpha^h < \alpha^f - 1$ and $\gamma^h > \gamma^f - 1$ happen simultaneously, there can be no multiple Nash equilibria of No-Trade and Case2.*

Lemma 22 *If $-1 < \alpha^h - \alpha^f < 1$ and $-1 < \gamma^h - \gamma^f < 1$ happen simultaneously, both governments can only choose No-Trade.*

As shown by Figure6, at the origin of $(\alpha^h - \alpha^f)$ and $(\gamma^h - \gamma^f)$ plane, the two host governments have no chance to accept either Case2 or Case3. They can choose No-Trade only. Thus, the following lemma is obtained.

Proposition 23 *For a special case where the two host countries are identical in terms of market demand size and marginal production cost, at the welfare maximizing level, both host governments should choose to block trade and impose same level of tax rates to the foreign subsidiaries instead of providing subsidies.*
Proof. *For $\alpha^f = \alpha^h$ and $\gamma^f = \gamma^h$, the equation (81) and (82) do not hold. Thus, no one-way trade at all exists. However, given $\alpha^f = \alpha^h$ and $\gamma^f = \gamma^h$, the equation (97) is binding. This means that No-Trade is the only choice remains for the two governments. Moreover, from the equations (89) and (90), if $\alpha^f = \alpha^h = \alpha$ and $\gamma^f = \gamma^h = \gamma$, $t_{NT}^{h*} = t_{NT}^{f*} = \left(\frac{\beta+2\phi}{3\beta+4\phi}\right)(\alpha - \gamma) > 0$.* ■

5.3 Joint analysis of welfare of all three alternatives - Case2, No-Trade, and Case3

In this section, for the choices of all three alternatives - Case2, No-Trade, and Case3, at the welfare maximizing level, feasible tax policy combinations of the two host countries are evaluated. To proceed this task, some useful findings are shown in the following section.

5.3.1 Some preliminary works

From the equations of (61), (72), (79), and (80), the following lemma is obtained.

Lemma 24 *As long as $t_L^f \geq t_3^{f*}$, it is true that $t_H^f > t_2^{f*}$. In addition, as long as $t_H^f \leq t_2^{f*}$, it is true that $t_L^f < t_3^{f*}$.*

Proof. *From the equation (79) and (80), we find that $t_H^f - t_L^f = \frac{2(\beta+\phi)}{\phi}\tau = \left(2 + \frac{2\phi}{\beta}\right)\tau$. From the equation (61) and (72), we find that $t_2^{f*} - t_3^{f*} = \left(\frac{18\phi\beta+10\phi^2+8\beta^2}{23\phi\beta+16\phi^2+8\beta^2}\right)\tau$. Then, $t_H^f - t_L^f > t_2^{f*} - t_3^{f*}$. This means that $t_H^f - t_2^{f*} > t_L^f - t_3^{f*}$ and $t_3^{f*} - t_L^f > t_2^{f*} - t_H^f$.* ■

Likewise, from the equations of (57), (68), (77), and (78), the following lemma is obtained.

Lemma 25 *As long as $t_L^h \geq t_2^{h*}$, it is true that $t_H^h > t_3^{h*}$. In addition, as long as $t_H^h \leq t_3^{h*}$, it is true that $t_L^h < t_2^{h*}$.*

Proof. *From the equation (77) and (78), we find that $t_H^h - t_L^h = \frac{2(\beta+\phi)}{\phi}\tau = \left(2 + \frac{2\phi}{\beta}\right)\tau$. From the equation (57) and (68), we find that $t_3^{h*} - t_2^{h*} = \left(\frac{18\phi\beta+10\phi^2+8\beta^2}{23\phi\beta+16\phi^2+8\beta^2}\right)\tau$. Then, $t_H^h - t_L^h > t_3^{h*} - t_2^{h*}$. This means that $t_H^h - t_3^{h*} > t_L^h - t_2^{h*}$ and $t_2^{h*} - t_L^h > t_3^{h*} - t_H^h$.* ■

Moreover, from the equations (61), (72), (79), and (80), we get the following lemma.

Lemma 26 *If $t_2^{f*} > t_H^f$, then, as t^h increases, t_H^f catches up t_2^{f*} and t_L^f catches up t_3^{f*}. However, as t^h decreases, t_2^{f*} and t_3^{f*} never catch up t_H^f and t_L^f, respectively. If $t_3^{f*} < t_L^f$, then, as t^h increases, t_3^{f*} and t_2^{f*} never catch up t_L^f and t_H^f, respectively. However, as t^h decreases, t_L^f catches up t_3^{f*} and t_H^f catches up t_2^{f*}.*

Proof. *From the equations (61), (72), (79), and (80), by differentiating t_L^f, t_H^f, t_2^{f*}, and t_3^{f*} with respect to t^h, we get $\frac{\partial t_L^f}{\partial t^h} = \frac{\partial t_H^f}{\partial t^h} = 1$ and $\frac{\partial t_2^{f*}}{\partial t^h} = \frac{\partial t_3^{f*}}{\partial t^h} = \frac{5\phi\beta+4\beta^2}{23\phi\beta+16\phi^2+8\beta^2} < 1$. Also, from Proposition 8, $t_H^f > t_L^f$, $t_2^{f*} > t_3^{f*}$, and $t_H^f - t_L^f > t_2^{f*} - t_3^{f*}$.* ■

Likewise, from the equations (57), (68), (77), and (78), we get the following lemma.

Lemma 27 *If $t_3^{h*} > t_H^h$, then, as t^f increases, t_H^h catches up t_3^{h*} and t_L^h catches up t_2^{h*}. However, as t^f decreases, t_3^{h*} and t_2^{h*} never catch up t_H^h and t_L^h, respectively. If $t_2^{h*} < t_L^h$, then, as t^f increases, t_2^{h*} and t_3^{h*} never catch up t_L^h and t_H^h, respectively. However, as t^f decreases, t_L^h catches up t_2^{h*} and t_H^h catches up t_3^{h*}.*

Proof. *From the equations (57), (68), (77), and (78), by differentiating t_L^h, t_H^h, t_2^{h*}, and t_3^{h*} with respect to t^f, we get $\frac{\partial t_L^h}{\partial t^f} = \frac{\partial t_H^h}{\partial t^f} = 1$ and $\frac{\partial t_2^{h*}}{\partial t^f} = \frac{\partial t_3^{h*}}{\partial t^f} = \frac{5\phi\beta+4\beta^2}{23\phi\beta+16\phi^2+8\beta^2} < 1$. Also, from Proposition 8, $t_H^h > t_L^h$, $t_3^{h*} > t_2^{h*}$, and $t_H^h - t_L^h > t_3^{h*} - t_2^{h*}$.* ■

Since it is known that the conjectures of W^h_{NT} and W^f_{NT} are not affected by changes in t^f and t^h, respectively, it is useful to find any relationships between W^h_{NT} and either W^h_2 or W^h_3 in terms of t^h. Likewise, it is also useful to find any connections between W^f_{NT} and eitehr W^f_2 or W^f_3 in terms of t^f. Based on this logic, the following facts are obtained.

$$W^f_{NT}(t^f_L) = W^f_3(t^f_L),\ W^f_{NT}(t^f_H) = W^f_2(t^f_H) \tag{98}$$

$$W^h_{NT}(t^h_L) = W^h_2(t^h_L),\ W^h_{NT}(t^h_H) = W^h_3(t^h_H) \tag{99}$$

From the above equations, the following lemma is obtained and illustrated by Figure7.1 and Figure7.2.

Lemma 28 *Given any level of t^h, the welfare function W^f_{NT} intersects W^f_3 and W^f_2 at t^f_L and t^f_H, respectively. Likewise, given any level of t^f, the welfare function W^h_{NT} intersects W^h_2 and W^h_3 at t^h_L and t^h_H, respectively.*

Having all the previous analyses, a general framework of the joint analysis of all three alternatives - Case2, No-Trade, and Case3 is shown below.

For joint analysis of welfare for home country, the following framework shows how home country chooses its maximum welfare among the three alternatives - Case2, No-Trade, and Case3.

Given a certain level of t^f,
$\hat{W}^h_{NT} = \max W^h_{NT}(t^h)$ for $t^h \in [t^h_L,\ t^h_H]$
$\hat{W}^h_2 = \max W^h_2(t^h)$ for $t^h \in (-\infty,\ t^h_L]$
$\hat{W}^h_3 = \max W^h_3(t^h)$ for $t^h \in [t^h_H,\ \infty)$
$W^{h*} = \max\{\hat{W}^h_{NT},\ \hat{W}^h_2,\ \hat{W}^h_3\}$
As described above, W^{h*} represents maximized welfare chosen by home country among the three alternatives. The associated tax with W^{h*} is welfare maximizing level of tax imposed by home government.

Likewise, for joint analysis of welfare for foreign country, the following framework shows how foreign country chooses its maximum welfare among the three alternatives - Case2, No-Trade, and Case3.

Given a certain level of t^h,
$\hat{W}^f_{NT} = \max W^f_{NT}(t^f)$ for $t^f \in [t^f_L,\ t^f_H]$
$\hat{W}^f_2 = \max W^f_2(t^f)$ for $t^f \in [t^f_H,\ \infty)$
$\hat{W}^f_3 = \max W^f_3(t^f)$ for $t^f \in (-\infty, t^f_L]$
$W^{f*} = \max\{\hat{W}^f_{NT},\ \hat{W}^f_2,\ \hat{W}^f_3\}$
Again, as described above, W^{f*} represents maximized welfare chosen by foreign country among the three alternatives. The associated tax with W^{f*} is welfare maximizing level of tax imposed by foreign government.

Then, for No-Trade to be selected as an optimal trade policy in this economy, at the Nash equilibrium level of tax policy combination, $(W^{h*},\ W^{f*})$ should be $(\hat{W}^h_{NT},\ \hat{W}^f_{NT})$. Similarly, for Case2 (Case3) to be chosen, at the Nash equilibrium, $(W^{h*},\ W^{f*})$ should be $(\ \hat{W}^h_2,\ \hat{W}^f_2)$ $((\hat{W}^h_3,\ \hat{W}^f_3))$.

Having this framework, for each one of the three alternatives chosen at the Nash equilibrium, we need to find associated tax policy combinations of the two host countries. Let's start with a special situation where the two host countries are identical, then, evaluate general situations where the host countries are different in terms of market demand size and marginal production cost.

5.3.2 Joint analysis of welfare when the two countries are identical

According to the previous sections, it is known that, for the two identical host countries, No-Trade should be chosen as an optimal trade policy. Assuming $\alpha^h = \alpha^f = \alpha$ and $\gamma^h = \gamma^f = \gamma$, the following equations are derived.

$$x_{NT}^{h*} = x_{NT}^{f*} = \left(\frac{\alpha-\gamma}{3\beta+4\phi}\right) \tag{100}$$

$$p_{NT}^{h*} = p_{NT}^{f*} = \frac{2(\beta+2\phi)\alpha+\beta\gamma}{3\beta+4\phi} \tag{101}$$

$$t_L^h = t^f - \frac{(\beta+\phi)}{\beta}\tau, \quad t_H^h = t^f + \frac{(\beta+\phi)}{\beta}\tau \tag{102}$$

$$t_L^f = t^h - \frac{(\beta+\phi)}{\beta}\tau, \quad t_H^f = t^h + \frac{(\beta+\phi)}{\beta}\tau \tag{103}$$

$$t_{NT}^{h*} = t_{NT}^{f*} = \left(\frac{\beta+2\phi}{3\beta+4\phi}\right)(\alpha-\gamma) > 0 \tag{104}$$

$$W_{NT}^h(t_{NT}^{h*}) = W_{NT}^f(t_{NT}^{f*}) = \frac{(\alpha-\gamma)^2}{2(3\beta+4\phi)} \tag{105}$$

The next step is to evaluate associated tax policy combinations when No-Trade is selected as an optimal trade policy in this economy. Let's consider associated foreign tax policy first. For foreign government to choose No-Trade, given t^h, the following three possible scenarios, depending on magnitude of t_L^f, t_H^f, and t_{NT}^{f*}, need to be considered - $t_L^f \geq t_{NT}^{f*}; t_L^f < t_{NT}^{f*} < t_H^f$; $t_H^f \leq t_{NT}^{f*}$.

For $t_H^f \leq t_{NT}^{f*}$, No-Trade cannot be definitely better off both Case2 and Case3 at the same time. Similarly, for $t_L^f \geq t_{NT}^{f*}$, No-Trade cannot be definitely better off both Case2 and Case3 simultanesouly.

Then, for $t_L^f < t_{NT}^{f*} < t_H^f$, No-Trade can be definitely better off Case2 and Case3 if some additional conditions are satisfied. This is summarized by Table1 and Table2 below.

Table1: For $t_L^f < t_{NT}^{f*} < t_H^f$ and $t_3^{f*} \geq t_L^f$ and $t_2^{f*} \leq t_H^f$

$t_3^{f*} < t_{NT}^{f*} < t_2^{f*}$	$t_3^{f*} \geq t_{NT}^{f*}$	$t_2^{f*} \leq t_{NT}^{f*}$
$\hat{W}_{NT}^f = W_{NT}^f(t_{NT}^{f*})$ $\hat{W}_2^f = W_2^f(t_H^f)$ $\hat{W}_3^f = W_3^f(t_L^f)$	$\hat{W}_{NT}^f = W_{NT}^f(t_{NT}^{f*})$ $\hat{W}_2^f = W_2^f(t_H^f)$ $\hat{W}_3^f = W_3^f(t_L^f)$	$\hat{W}_{NT}^f = W_{NT}^f(t_{NT}^{f*})$ $\hat{W}_2^f = W_2^f(t_H^f)$ $\hat{W}_3^f = W_3^f(t_L^f)$
$\hat{W}_{NT}^f > \hat{W}_2^f$ $\hat{W}_{NT}^f > \hat{W}_3^f$	$\hat{W}_{NT}^f > \hat{W}_2^f$ $\hat{W}_{NT}^f > \hat{W}_3^f$	$\hat{W}_{NT}^f > \hat{W}_2^f$ $\hat{W}_{NT}^f > \hat{W}_3^f$
NT $\succ$ 2 and NT $\succ$ 3	NT $\succ$ 2 and NT $\succ$ 3	NT $\succ$ 2 and NT $\succ$ 3

From Table1, NT, 2, and 3 represent No-Trade, Case2, and Case3, respectively. In addition, '$\succ$' represents the choice comes to the left-hand-side of '$\succ$' gives better welfare than the choice comes to the right-hand-side of '$\succ$' does. Thus, host country prefers the choice comes to the left-hand-side of '$\succ$' to the choice comes to the right-hand-side. Table1 shows that, for $t_L^f < t_{NT}^{f*} < t_H^f$, as long as $t_L^f \leq t_3^{f*} < t_2^{f*} \leq t_H^f$ is true, the two identical host countries prefer NT to Case2 and Case3 simultaneously. Thus, for $t_L^f < t_{NT}^{f*} < t_H^f$, additional condition for No-Trade to be definitely better off both Case2 and Case3 is $t_L^f \leq t_3^{f*} < t_2^{f*} \leq t_H^f$.

Table2: For $t_L^f < t_{NT}^{f*} < t_H^f$

$t_3^{f*} < t_L^f$	$t_3^{f*} < t_L^f$	$t_2^{f*} > t_H^f$	$t_2^{f*} > t_H^f$
$\hat{W}_{NT}^f = W_{NT}^f(t_{NT}^{f*})$ $\hat{W}_2^f = W_2^f(t_H^f)$ $\hat{W}_3^f = W_3^f(t_3^{f*})$	$\hat{W}_{NT}^f = W_{NT}^f(t_{NT}^{f*})$ $\hat{W}_2^f = W_2^f(t_H^f)$ $\hat{W}_3^f = W_3^f(t_3^{f*})$	$\hat{W}_{NT}^f = W_{NT}^f(t_{NT}^{f*})$ $\hat{W}_2^f = W_2^f(t_2^{f*})$ $\hat{W}_3^f = W_3^f(t_L^f)$	$\hat{W}_{NT}^f = W_{NT}^f(t_{NT}^{f*})$ $\hat{W}_2^f = W_2^f(t_2^{f*})$ $\hat{W}_3^f = W_3^f(t_L^f)$
$\hat{W}_3^f > \hat{W}_{NT}^f > \hat{W}_2^f$	$\hat{W}_{NT}^f > \hat{W}_2^f$ $\hat{W}_{NT}^f > \hat{W}_3^f$	$\hat{W}_2^f > \hat{W}_{NT}^f > \hat{W}_3^f$	$\hat{W}_{NT}^f > \hat{W}_2^f$ $\hat{W}_{NT}^f > \hat{W}_3^f$
3 $\succ$ NT $\succ$ 2	NT $\succ$ 2 and NT $\succ$ 3	2 $\succ$ NT $\succ$ 3	NT $\succ$ 2 and NT $\succ$ 3

From the first and second column of the Table2, for $t_L^f < t_{NT}^{f*} < t_H^f$, if $t_3^{f*} < t_L^f$, No-Trade is prefered to Case2. However, depending on $\hat{W}_{NT}^f$ and $\hat{W}_3^f$, No-Trade might not be prefered to Case3. Thus, for $t_L^f < t_{NT}^{f*} < t_H^f$, if $t_3^{f*} < t_L^f$, additional condition for No-Trade to be definitely better off both Case2 and Case3 is $\hat{W}_{NT}^f > \hat{W}_3^f$. This result is also true if $t_2^{f*} \leq t_L^f$. From the third and fouth column of the Table2, for $t_L^f < t_{NT}^{f*} < t_H^f$, if $t_2^{f*} > t_H^f$, No-Trade is prefered to Case3 but might not be prefered to Case2 depending on $\hat{W}_{NT}^f$ and $\hat{W}_2^f$. Thus, for $t_L^f < t_{NT}^{f*} < t_H^f$, if $t_2^{f*} > t_H^f$, additional condition for No-Trade to be definitely better off both Case2 and Case3 simultaneously is $\hat{W}_{NT}^f > \hat{W}_2^f$. This result is also true if $t_3^{f*} \geq t_H^f$.

Likewise, by symmetry, for Home government to choose No-Trade, $t_L^h < t_{NT}^{h*} < t_H^h$ must be met with some additional conditions. From the results above, the following lemma is obtained.

Proposition 29 *For $t_L^f < t_{NT}^{f*} < t_H^f$, if $t_L^f \leq t_3^{f*} < t_2^{f*} \leq t_H^f$, Foreign government chooses to block trade which is definitely better off Case2 and Case3. That is $\hat{W}_{NT}^f = W_{NT}^f(t_{NT}^{f*}) > \max\{\hat{W}_2^f, \hat{W}_3^f\}$ where $\hat{W}_2^f = W_2^f(t_H^f)$ and $\hat{W}_3^f = W_3^f(t_L^f)$. For $t_L^h < t_{NT}^{h*} < t_H^h$, if $t_L^h \leq t_2^{h*} < t_3^{h*} \leq t_H^h$, Home government chooses to block trade which is definitely better off Case2 and Case3. That is $\hat{W}_{NT}^h = W_{NT}^h(t_{NT}^{h*}) > \max\{\hat{W}_2^h, \hat{W}_3^h\}$ where $\hat{W}_2^h = W_2^h(t_L^h)$ and $\hat{W}_3^h = W_3^h(t_H^h)$.*

On top of the above proposition, from the equation (104), the two reaction functions of t_{NT}^{h*} and t_{NT}^{f*} intersect each other at the 45 degree line in t^h and t^f plane. Thus, the following lemma is obtained.

Lemma 30 *If the conditions mentioned in the above lemma are satisfied, both home and foreign governments choose not to trade at Nash equilibrium and* *impose same amount of positive tax by t_{NT}^{h*} and t_{NT}^{f*}, respectively.*

5.3.3 Joint analysis of welfare when the two countries are different

In this section, main consideration is to find associated tax policy combinations when Case2, Case3, and No-Trade are selected as an optimal trade policy in this economy assuming the two host countries are not identical. First of all, let's denote $t_3^{h*}(t^f = 0) = t^{h'}$ and $t_H^h(t^f = 0) = t^{h''}$. Similarly, denote $t_3^{f*}(t^h = 0) = t^{f'}$ and $t_L^f(t^h = 0) = t^{f''}$. Then, Figure8.1 illustrates a condition for the choice of Case3 at Nash equilibrium.

Likewise, let's denote $t_2^{h*}(t^f = 0) = T^{h'}$ and $t_L^h(t^f = 0) = T^{h''}$. Similarly, denote $t_2^{f*}(t^h = 0) = T^{f'}$ and $t_H^f(t^h = 0) = T^{f''}$. Then, Figure8.2 illustrates a condition for the choice of Case2 at Nash equilibrium.

As shown in Figure8.1, for $a > 0$, depending on the range of $t^{h'}$ and $t^{h''}$, the all possible outcomes of tax policies chosen by the two host governments when they accept one-way trade from foreign to home country at the Nash equilibrium are summarized in the following table.

Table3.

$0 < t^{h''} < t^{h'}$	$0 < t^{h'} \leq t^{h''}$	$t^{h''} \leq 0 < t^{h'}$	$t^{h''} < t^{h'} = 0$	$t^{h''} < t^{h'} < 0$
$t^h = 0, t^f < 0$	$t^h > 0, t^f < 0$	$t^h > 0, t^f > 0$	$t^h > 0, t^f > 0$	$t^h > 0, t^f > 0$
$t^h > 0, t^f = 0$	$t^h < 0, t^f < 0$	$t^h > 0, t^f < 0$	$t^h = 0, t^f = 0$	$t^h = 0, t^f > 0$
$t^h > 0, t^f > 0$	$t^h = 0, t^f < 0$	$t^h < 0, t^f < 0$	$t^h < 0, t^f < 0$	$t^h < 0, t^f = 0$
$t^h > 0, t^f < 0$		$t^h > 0, t^f = 0$		$t^h < 0, t^f > 0$
$t^h < 0, t^f < 0$		$t^h = 0, t^f < 0$		$t^h < 0, t^f < 0$

For $0 < t^{h'} \leq t^{h''}$, the value b is less than or equal to zero. Thus, from the second column of the Table3, the following proposition is obtained.

Proposition 31 *When both governments choose to accept one-way trade from foreign to home country, if $b \leq 0$, it is impossible that both governments levy tax. In this case, the foreign government should provide subsidy.*

Since $b = \frac{\rho_1 \alpha^h + \rho_2 \alpha^f + \rho_3 \gamma^h + \rho_4 \gamma^f + \rho_5 \tau}{(18\beta^2\phi + 16\beta\phi^2 + 4\beta^3)}$, the sufficient condition that the foreign government provides subsidy when both governments accept export from foreign to home country is to be

$$\rho_1 \alpha^h + \rho_2 \alpha^f + \rho_3 \gamma^h + \rho_4 \gamma^f + \rho_5 \tau \leq 0 \tag{106}$$

where $\rho_1 = (10\beta^2\phi + 26\beta\phi^2 + 16\phi^3)$, $\rho_2 = -(4\beta^2\phi + 18\beta\phi^2 + 16\phi^3)$, $\rho_3 = (4\beta^3 + 12\beta^2\phi + 8\beta\phi^2)$, $\rho_4 = -(4\beta^3 + 18\beta^2\phi + 16\beta\phi^2)$, and $\rho_5 = -(4\beta^3 + 22\beta^2\phi + 34\beta\phi^2 + 16\phi^3)$.

Moreover, from the fourth column of the Table3, the following proposition is obtained.

Proposition 32 *When both governments choose to accept one-way trade from foreign to home country, for $t^{h''} < t^{h'} = 0$, if one government levy tax, the other should not provide subsidy. Likewise, if one government provide subsidy, the other should not levy tax. In other words, the direction of tax policy of both host countries are same.*

From the first and the third column of the Table3, it is found that when t^h is less than or equal to zero, t^f is less than zero. Thus, the following proposition is obtained.

Proposition 33 *When both governments choose to accept one-way trade from foreign to home country, for $0 < t^{h''} < t^{h'}$ and $t^{h''} \leq 0 < t^{h'}$, if home government does not levy tax, the foreign government should provide subsidy.*

From the last column of the Table3, it is found that when t^f is less than or equal to zero, t^h is less than zero. Thus, the following proposition is obtained.

Proposition 34 *When both governments choose to accept one-way trade from foreign to home country, for $t^{h''} < t^{h'} < 0$, if foreign government does not levy tax, the home government should provide subsidy.*

Likewise, as shown in Figure8.2, for $a^{'} > 0$, depending on the range of $T^{f'}$ and $T^{f''}$, all possible outcomes of tax policies chosen by the two host governments when they accept one-way trade from home to foreign country at the Nash equilibrium are summarized in the following table.

Table4.

$0 < T^{f''} < T^{f'}$	$0 < T^{f'} \leq T^{f''}$	$T^{f''} \leq 0 < T^{f'}$	$T^{f''} < T^{f'} = 0$	$T^{f''} < T^{f'} < 0$
$t^h > 0, t^f > 0$	$t^h < 0, t^f > 0$	$t^h > 0, t^f > 0$	$t^h > 0, t^f > 0$	$t^h > 0, t^f > 0$
$t^h = 0, t^f > 0$	$t^h < 0, t^f = 0$	$t^h = 0, t^f > 0$	$t^h = 0, t^f = 0$	$t^h > 0, t^f = 0$
$t^h < 0, t^f > 0$	$t^h < 0, t^f < 0$	$t^h < 0, t^f > 0$	$t^h < 0, t^f < 0$	$t^h > 0, t^f < 0$
$t^h < 0, t^f = 0$		$t^h < 0, t^f = 0$		$t^h = 0, t^f < 0$
$t^h < 0, t^f < 0$		$t^h < 0, t^f < 0$		$t^h < 0, t^f < 0$

For $0 < T^{f'} \leq T^{f''}$, the value $b^{'}$ is less than or equal to zero. Thus, from the second column of the Table4, the following proposition is obtained.

Proposition 35 *When both governments choose to accept one-way trade from home to foreign country, if $b^{'} \leq 0$, it is impossible that both governments levy tax. In this case, home government should provide subsidy.*

Since $b^{'} = \frac{\rho_1^{'}\alpha^h + \rho_2^{'}\alpha^f + \rho_3^{'}\gamma^h + \rho_4^{'}\gamma^f + \rho_5^{'}\tau}{(18\beta^2\phi + 16\beta\phi^2 + 4\beta^3)}$, the sufficient condition that home government provides subsidy when both governments accept export from home to foreign country is to be

$$\rho_1^{'}\alpha^h + \rho_2^{'}\alpha^f + \rho_3^{'}\gamma^h + \rho_4^{'}\gamma^f + \rho_5^{'}\tau \leq 0 \tag{107}$$

where $\rho_1^{'} = -(4\beta^2\phi + 18\beta\phi^2 + 16\phi^3)$, $\rho_2^{'} = (10\beta^2\phi + 26\beta\phi^2 + 16\phi^3)$, $\rho_3^{'} = -(4\beta^3 + 18\beta^2\phi + 16\beta\phi^2)$, $\rho_4^{'} = (4\beta^3 + 12\beta^2\phi + 8\beta\phi^2)$, and $\rho_5 = -(4\beta^3 + 22\beta^2\phi + 34\beta\phi^2 + 16\phi^3)$.

Moreover, from the fourth column of the Table6 and associated Figure13F, the following lemma is obtained.

Lemma 36 *When both governments choose to accept one-way trade from home to foreign country, for $T^{f''} < T^{f'} = 0$, if one government levy tax, the other should not provide subsidy. Likewise, if one government provide subsidy, the other should not levy tax.*

From the first and the third column of the Table4, it is found that when t^f is less than or equal to zero, t^h is less than zero. Thus, the following proposition is obtained.

Proposition 37 *When both governments choose to accept one-way trade from home to foreign country, for $0 < T^{f''} < T^{f'}$ and $T^{f''} \leq 0 < T^{f'}$, if foreign government does not levy tax, the home government should provide subsidy.*

From the last column of the Table4, it is found that when t^h is less than or equal to zero, t^f is less than zero. Thus, the following proposition is obtained.

Proposition 38 *When both governments choose to accept one-way trade from home to foreign country, for $T^{f''} < T^{f'} < 0$, if home government does not levy tax, the foreign government should provide subsidy.*

From the equation (89) and (90) in the section 5.2.2, when both home and foreign governments accept No-Trade instead of one-way trade as an optimal trade policy, the associated tax policy at the Nash equilibrium is to impose positive tax. Thus, the following proposition is also true.

Proposition 39 *When both governments choose to block trade, at the Nash equilibrium level, they always levy tax instead of offerring subsidy. Moreover, each host government's tax policy is independently processed.*

Lemma 40 ***Proof.*** *When No-Trade is chosen at the Nash equilibrium, home and foreign governments independently provides $t_{NT}^{h*} = \left(\frac{\beta+2\phi}{3\beta+4\phi}\right)(\alpha^h - \gamma^h)$ and $t_{NT}^{f*} = \left(\frac{\beta+2\phi}{3\beta+4\phi}\right)(\alpha^f - \gamma^f)$, respectively. Since $(\alpha^h - \gamma^h) > 0$ and $(\alpha^f - \gamma^f) > 0$, both t_{NT}^{h*} and t_{NT}^{f*} are greater than zero. This means that both governments levy tax when they choose No-Trade. Moreover, t_{NT}^{h*} is not a function of t^f while t_{NT}^{f*} is not a function of t^h.* ■

6 Concluding Remarks

This paper shows a development of a partial equilibrium model where two FDI host countries, called home and foreign country, maximize economic welfare through tax policy competition in bilateral FTA environment. A multinational corporation (MNC) directly invests in both host countries and becomes monopolist in each market. A foreign subsidiary in one country can access the other market by exporting. Profit maximizing trade strategies of the foreign multinationals are analyzed assuming that the output schedules are coordinated by the MNC's headquarter. With no tariffs, the role of transportation cost is significant. If transportation cost is negligible, there exists intra-industry trade. Otherwise, only one-way trade remains.

Depending on tax policies of the two host countries, with positive transportation cost, profit maximizing trade strategy of the MNC consists of three kinds - one-way trade from home to foreign country, one-way trade from foreign to home country, and no-trade. This way, although there are no tariffs, using tax policies on FDI, the two host governments control trade flows to maximize their economic welfares. In the two-stage game framework, in each of the three trade strategies, profit maximizing outputs and market prices and welfare maximizing level of tax policies are separately analyzed. Moreover, impacts of country heterogeneity on the optimal tax policies are evaluated.

Then, combining all the separate analyses, all three alternatives - no-trade and two kinds of one-way trades - are jointly analyzed. The joint welfare analysis, with positive transportation cost, shows the following important results - i) there is no multiple Nash equilibrium with one-way trade from home to foreign country and one-way trade from foreign to home country at the same time; ii) there are two kinds of multiple Nash equilibrium - 1) one-way trade from home to foreign country and no-trade; 2) one-way trade from foreign to home country and no-trade; iii) depending on values of market demand sizes and marginal production costs of the two host countries, there are situations where no Nash equilibrium exists; iv) for a special case where the two host countries are identical in terms of market demand size and marginal production cost, both host governments choose not to trade and impose same level of tax rates to the foreign subsidiaries; v) if the two host governments choose no-trade at the welfare maximizing level, both governments impose positive tax instead of offering subsidies; vi) If the two host governments choose one-way trade either from home to foreign country or foreign to home country at the welfare maximizing level, there are various kinds of tax (or subsidy) policy combinations are possible. In addition, for iv), v), and vi), the associated tax policy combinations, at the Nash equilibrium level, are presented.

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