Todaro Paradox, Nonhomotheticity and Monopolistic Competition

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Abstract

This paper examines the Todaro paradox by setting up a model à la Harris and Todaro with nonhomothetic technology and monopolistically competitive manufacturing industry. It shows whether the Todaro paradox holds in response to an increase in capital stock depends upon the degree of nonhomotheticity, elasticities of factor substitution and the rate of urban employment.

Key words: Todaro paradox; Nonhomotheticity; Monopolistic competition
JEL classification: F12; J6; O18

1 Introduction

In the literature of unemployment, Harris and Todaro (1970) (henceforth HT) present a well-known model in which a long-run equilibrium is characterized by pervasive urban unemployment. They develop a two-sector model with a minimum wage imposed in the urban manufacturing sector, which in turn leads to rural-urban migration until the actual rural wage equals the expected urban wage, and to urban unemployment in equilibrium. Numerous analyses à la the HT model appeared, e.g., Corden and Findlay (1975), Neary (1981), Raimondos (1993), Brueckner and Kim (2001), Zenou (2005), Chen and Hu (2008), etc.

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Extending the seminal HT model in a standard two-by-two, small open economy, Corden and Findlay (1975) point out a paradoxical phenomenon that an expansion in manufacturing output in response to an increased capital stock brings about the coexistence of the creation of jobs and higher urban unemployment, which is the so-called Todaro paradox. The Todaro paradox has been studied by many theorists, e.g., Raimondos (1993), Brueckner and Kim (2001), Zenou (2005),[1] etc. Raimondos (1993) shows that the Todaro paradox may not hold in response to an increase in manufacturing-specific capital stock under the framework that the agricultural labor market is monopsonistic, whereas Brueckner and Kim (2001) and Zenou (2005) analyze the paradox by considering land market and efficiency wage, respectively.

Since 1980s, the roles of imperfect competition and increasing returns to scale became important in trade theory. These two key elements are incorporated into the HT model as well, e.g., Panagariya and Succar (1986), Lal (1995), Chao and Yu (1997) and Chen and Hu (2008). However, it seems that except for Chen and Hu (2008), the existing HT models have not been analyzed under the framework of monopolistic competition with internal economies of scale. That is the prime objective in this paper. Specifically, we set up a model à la HT by incorporating nonhomothetic technology into Chen and Hu (2008) in which manufacturing sector is monopolistically competitive with internal increasing returns and homothetic technology. We reexamine the Todaro paradox in response to economic expansion under nonhomotheticity in which the size of a typical manufacturing firm is allowed to change, that is not permitted under homothetic technology.

This paper is organized as follows. Section 2 sets up a simple framework with monopolistic competition and nonhomotheticity. Section 3 analyzes the Todaro paradox following an expansion in capital stock. Finally, we make some concluding remarks.

2 The Model

Consider a closed economy with two sectors: rural agriculture (A) and urban manufacturing industry (M) producing a homogeneous good and differentiated products, respectively. All consumers are assumed to have the identical utility function as in Dixit and Stiglitz (1977). Specifically, suppose that $U(D_M, D_A) = D_M^{\alpha} D_A^{1-\alpha}$, ($\alpha < 1$), where D_A and D_M are consumption of the

[1] Several articles along this line are enumerated in Zenou (2005).

agricultural good and a manufactures aggregate, respectively; and α is the share of manufactures in consumer's expenditure. The manufactures aggregate D_M is defined by $D_M = [\sum_j D_j^{\beta}]^{1/\beta}$, ($\beta < 1$), where D_j is the consumption of variety j. As shown in Dixit and Norman (1980), if the number of varieties of manufacturing goods consumed is sufficiently large, the elasticity of demand for a single variety is $1/(1-\beta)$ in absolute value.

On the supply side, agricultural output is produced by utilizing capital and labor with constant returns technology under perfect competition. If the agricultural good is chosen as the numeraire, when the agricultural output, z, is positive, the unit cost, c, satisfies

$$c(w, r) = 1 \tag{1}$$

where w denotes the wage rate in agriculture, r the capital return.

Turn next to the manufacturing sector. Firms in the manufacturing sector produce differentiated goods with identical increasing returns technology under monopolistic competition. Suppose that there is a minimum wage rate, $\bar{w}$, imposed only in the manufacturing industry, as in the HT model. Following Horn (1983), the production function for a representative manufacturing firm is nonhomothetic. The total cost function takes the following form $\phi(\bar{w}, r, x_j)$, where x_j denotes the output of a typical firm j, and ϕ is homogeneous of degree one in factor prices.

The profits of a single manufacturing firm are given $\pi_j = p_j x_j - \phi(\bar{w}, r, x_j)$. Profit maximization for a manufacturing firm facing the constant elasticity of demand, $1/(1-\beta)$, ensures

$$\beta p_j = \phi_x(\bar{w}, r, x_j) \tag{2}$$

Free entry drives profits to zero under monopolistic competition, namely,

$$p_j x_j = \phi(\bar{w}, r, x_j) \tag{3}$$

From (2) and (3), we obtain

$$\Theta(\bar{w}, r, x_j) = \beta \tag{4}$$

By the assumption of symmetry—the technologies adopted by each firm are the same, all firms will have the same constant output level; hence set $x = x_j$ for all j. Given factor prices and the output level x, (3) reveals that prices of all varieties of manufacturing goods are identical; we set $p = p_j$ for all j.

Following the HT hypothesis, pervasive urban unemployment results from the minimum wage imposed in the manufacturing sector. The equilibrium condition for the labor market is that the actual agricultural wage equals the expected wage in the urban sector; namely,

$$w = e\bar{w} \tag{5}$$

where e is the rate of employment in the urban area. Denoting the economy's labor and capital endowments by L and K, the market clearing conditions require

$$L = L_A + L_M + U \tag{6}$$
$$K = K_A + K_M \tag{7}$$

where L_j (K_j), $j = A, M$, denotes labor (capital) demand in industry j and U the level of urban unemployment. By using Shephard's lemma, these equations can be written as

$$L = n\phi_{\bar{w}}(\bar{w}, r, x)/e + zc_w(w, r) \tag{6'}$$
$$K = n\phi_r(\bar{w}, r, x) + zc_r(w, r) \tag{7'}$$

where n denotes the number of manufacturing varieties produced, $e \equiv L_M/(L_M + U)$ and $c_w \equiv \partial c/\partial w$, etc. In addition, recalling that the expenditure on manufacturing goods is α; hence, the equilibrium in the good markets is given by

$$\frac{npx}{z} = \frac{\alpha}{1-\alpha} \tag{8}$$

The equilibrium of the economy can be described by seven equations, (1), (3), (4), (5), (6'), (7') and (8), which in turn determine the equilibrium values of the following variables, w, r, p, e, n, x, and z, given the minimum wage rate and endowments.

3 Economic Expansion and Unemployment

In this section we will examine the impact of an increase in capital stock, in particular, on the levels of urban unemployment and manufacturing employment. Differentiating the equilibrium equations, (1), (3), (4), (6'), (7') (8) and utilizing $\widehat{e} = \widehat{w}$ from (5), with constant minimum wage rate and labor supply, we have the following system:

$$\begin{bmatrix} \theta_{LA} & \theta_{KA} & 0 & 0 & 0 & 0 \\ 0 & \theta_{KM} & -1 & 0 & (\theta-1) & 0 \\ -(\lambda_{LU}+\lambda_{LA}\theta_{KA}\sigma_{A}) & \delta_{L} & 0 & \lambda_{LU} & \lambda_{LU}\mu_{L} & \lambda_{LA} \\ \lambda_{KA}\theta_{LA}\sigma_{A} & -\delta_{K} & 0 & \lambda_{KM} & \lambda_{KM}\mu_{K} & \lambda_{KA} \\ 0 & 0 & 1 & 1 & 1 & -1 \\ 0 & \omega_{K} & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \widehat{w} \\ \widehat{r} \\ \widehat{p} \\ \widehat{n} \\ \widehat{x} \\ \widehat{z} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \widehat{K} \\ 0 \\ 0 \end{bmatrix} \tag{9}$$

where a '$\widehat{\ }$' over a variable denotes the relative change in that variable. λ_{ij} is the fraction of factor i ($i = L, K$) in sector j ($j = A, M$); specifically for example, $\lambda_{LU} \equiv (n\phi_{\bar{w}}/e)/L$ is the fraction of labor force (including urban unemployment) in the urban area. θ_{ij} denotes the distributive share of factor i in industry j. $\delta_{L} \equiv \lambda_{LU}\theta_{KM}\sigma_{M} + \lambda_{LA}\theta_{KA}\sigma_{A}$, $\delta_{K} \equiv \lambda_{KM}\theta_{LM}\sigma_{M} + \lambda_{KA}\theta_{LA}\sigma_{A}$, where σ_j is industry j's elasticity of factor substitution, e.g., $\sigma_{M} \equiv \phi\phi_{wr}/\phi_w\phi_r$. In addition, following the notations of Horn (1983, p. 89), $\omega_{K} \equiv \theta_{KM}(\mu_{K} - \theta)/(\theta\eta)$, where θ ($\equiv x\phi_x/\phi < 1$ under the assumption of increasing returns to scale) is the partial elasticity of scale with respect to output, which measures scale economies; $\eta \equiv (\partial\theta/\partial x)(x/\theta) > 0$ is the partial elasticity of θ with respect to output. $\mu_{K}(\equiv x\phi_{rx}/\phi_r > 0)$ is the partial elasticity of demand for capital with respect to output; similarly for μ_{L}. $(\mu_{K} - \theta)$ is used as a measure of nonhomotheticity; specifically, following the terminology of Horn, there is a 'bias' toward capital (labor) if $\mu_{K} > \theta$ ($\mu_{L} > \theta$), and that the larger $|\mu_{K} - \theta|$ is, the higher the degree of nonhomotheticity.[2]

The primary results in response to an increase in capital stock are given

$$\frac{\widehat{w}}{\widehat{K}} = \frac{\widehat{e}}{\widehat{K}} = \frac{\theta_{KA}}{\Delta} > 0; \quad \frac{\widehat{r}}{\widehat{K}} = \frac{-\theta_{LA}}{\Delta} < 0 \tag{10}$$

$$\frac{\widehat{x}}{\widehat{K}} = \frac{\theta_{LA}\omega_{K}}{\Delta} \tag{11}$$

$$\frac{\widehat{p}}{\widehat{K}} = \frac{-\theta_{LA}}{\Delta}[\theta_{KM} + \omega_{K}(1-\theta)] \tag{12}$$

[2] According to Horn (1983), $\theta_{LM}(\mu_{L} - \theta) = -\theta_{KM}(\mu_{K} - \theta)$ and ω_{K} defined above, ω_{K}, $(\mu_{K} - \theta)$ and $(\theta - \mu_{L})$ have the same sign. In addition, if the technology of a typical manufacturing firm exhibits homothetic, $\theta = \mu_{K} = \mu_{L}$, hence $\omega_{K} = 0$.

$$\frac{\widehat{n}}{\widehat{K}} = \frac{1}{\Delta}[\lambda_{LA}(\theta_{LA}\theta_{KM} + \theta_{KA}\sigma_A) + \lambda_{LU}(\theta_{KA} + \theta_{LA}\theta_{KM}\sigma_M) - \theta_{LA}\omega_K(\lambda_{LA}\theta + \lambda_{LU}\mu_L)] \tag{13}$$

$$\frac{\widehat{z}}{\widehat{K}} = \frac{1}{\Delta}[\lambda_{LU}\theta_{KA} + \lambda_{LA}\theta_{KA}\sigma_A + \lambda_{LU}\theta_{LA}\theta_{KM}(\sigma_M - 1) + \lambda_{LU}\theta_{LA}\omega_K(\theta - \mu_L)] \tag{14}$$

where $\Delta \equiv |\lambda|\theta_{LA}\theta_{KM} + \theta_{LA}(\delta_L + \delta_K) + \theta_{KA}(\lambda_{LU} + \lambda_{KA}\theta_{LA}\sigma_A + \lambda_{LA}\theta_{KA}\sigma_A) + \theta_{LA}\omega_K[\lambda_{KM}(\mu_K - \theta) + \lambda_{LU}(\theta - \mu_L)] > 0$ if the urban sector is relatively capital abundant, i.e., $|\lambda| \equiv \lambda_{KM} - \lambda_{LU} > 0$. The condition of $|\lambda| > 0$ will be assumed throughout this article.[3]

(10) reveals that, given $\bar{w}$ and L, an increase in capital stock gives rise to an increase in agricultural wage rate (w), a rise in the rate of urban employment, and a decline in the return to capital. Furthermore, from (11), as capital stock expands, the output of a typical manufacturing firm is unchanged in the case of homotheticity ($\omega_K = 0$), as expected. However, in the case of nonhomotheticity, the change in output volume is ambiguous. It depends on the degree of nonhomotheticity; specifically, x rises (falls) if the technology is biased toward capital (labor), i.e., $\omega_K > 0$ (< 0).

In addition, (13) shows that changes in the number of manufacturing firms (or varieties) following capital expansion are ambiguous. If manufacturing technology is biased toward labor ($\omega_K < 0$), n will rise, whereas the output volume of a typical firm falls, as shown in (11). In contrast, if manufacturing technology is biased toward capital ($\omega_K > 0$), the change in n is indeterminate, but the size of a typical firm will rise. Furthermore, if the technology is homothetic, capital expansion unambiguously gives rise to an increase in the number of firms, given fixed x. Although it is inappropriate to signify the aggregate manufacturing output as nx in the case of differentiated products, it can be shown easily, from (11) and (13), that $(\widehat{n} + \widehat{x})/\widehat{K}$ to be positive as $\omega_K \geq 0$. It implies that the manufacturing sector will expand following increased capital stock. (12) indicates that if technologies are biased towards capital or homothetic ($\omega_K \geq 0$), p will decline. (14) reveals that if the manufacturing elasticity of factor substitution is sufficiently large (i.e., $\sigma_M \geq 1$), the agricultural output will expand regardless of nonhomotheticity, recalling $\omega_K(\theta - \mu_L) > 0$ under nonhomotheticity. Specifically, labor is relatively easier

[3] As Neary (1988) shows, $|\lambda| > 0$ is the stability condition for a small open economy with constant returns under perfect competition. Following Neary's method and this condition, the stability condition for this model with monopolistic competition and nonhomotheticity in this article ensures $\Delta > 0$. Details of the cumbersome derivation is available on request from the authors.

to replace capital in manufacturing sector, so that some of increased capital flows to the agricultural sector and leads to a higher z and K_A (not shown).

Will an increase in capital stock result in the so called Todaro paradox—a coexistence of job creation in manufacturing and an increase in the level of urban unemployment? Let us begin analyzing the effects on factor demands following capital expansion. According to Shephard's lemma, $L_M = n\phi_{\bar{w}}(\bar{w}, r, x)$ (or $\widehat{L}_M = \widehat{n} + \mu_L \widehat{x} + \theta_{KM}\sigma_M \widehat{r}$). Substituting (10), (11) and (13) yields

$$\frac{\widehat{L}_M}{\widehat{K}} = \frac{1}{\Delta}[\lambda_{LU}\theta_{KA} + \lambda_{LA}\theta_{KA}\sigma_A + \lambda_{LA}\theta_{LA}\theta_{KM}(1-\sigma_M) - \lambda_{LA}\theta_{LA}\omega_K(\theta - \mu_L)] \tag{15}$$

Unfortunately, the change in the level of manufacturing employment, (15), is indeterminate in response to capital expansion. However, when the technology in manufacturing firm is homothetic (i.e., $\omega_K = 0$, or $\mu_L = \theta$ in the last term on the right-hand side of (15)), L_M will rise following capital expansion if the manufacturing elasticity of factor substitution is small enough ($\sigma_M \leq 1$). It implies that if the degree of substitution between capital and labor in manufacturing is sufficiently low, L_M rises following the expansion in K_M.[4] Furthermore, with nonhomothetic technology [$\omega_K(\theta-\mu_L) > 0$ in the last term of (15)], the level of L_M is smaller relative to that in the case of homothetic technology.

Similarly, by using of $L_A = zc_w(w, r)$, (10) and (14), we have

$$\frac{\widehat{L}_A}{\widehat{K}} = \frac{\lambda_{LU}}{\Delta}[\theta_{LA}\theta_{KM}(\sigma_M - 1) + \theta_{KA}(1-\sigma_A) + \theta_{LA}\omega_K(\theta - \mu_L)] \tag{16}$$

(16) indicates that recalling $\omega_K(\theta - \mu_L) > 0$ under nonhomotheticity, if $\sigma_M \geq 1 \geq \sigma_A$, L_A rises in response an increase in capital stock. In addition, (6) ($L = L_A + L_M + U$) shows that when both L_M and U rise simultaneously, L_A must fall. By using of (6), (15) and (16), the change in the level of urban unemployment can be explicitly written as

$$\frac{dU}{\widehat{K}} = \frac{L\lambda_{LU}}{\Delta}\{\lambda_{LA}(1-e)[\theta_{LA}\theta_{KM}(1-\sigma_M)+\theta_{KA}(\sigma_A-1)-\theta_{LA}\omega_K(\theta-\mu_L)]-e\theta_{KA}\} \tag{17}$$

We have the following proposition from (17).

[4] Note that $K_M = n\phi_r(\bar{w}, r, x)$ (or $\widehat{K}_M = \widehat{n} + \mu_K\widehat{x} - \theta_{LM}\sigma_M\widehat{r}$.) Substituting (10), (11) and (13) yields $\frac{\widehat{K}_M}{\widehat{K}} = \frac{1}{\Delta}\{\theta_{LA}\theta_{LM}\sigma_M + \lambda_{LA}(\theta_{LA}\theta_{KM} + \theta_{KA}\sigma_A) + \lambda_{LU}(\theta_{KA} + \theta_{LA}\theta_{KM}\sigma_M) + \theta_{LA}\omega_K[\lambda_{LA}(\mu_K - \theta) + \lambda_{LU}(\mu_K - \mu_L)]\} > 0$. It indicates that an expansion in K leads to a rise in K_M regardless of nonhomotheticity.

Proposition 1. *In an economy characterized by urban unemployment and monopolistically competitive manufacturing sector with nonhomothetic technology, whether the Todaro paradox holds in response to an increase in capital stock depends upon the degree of nonhomotheticity, elasticities of factor substitution and the rate of urban employment; specifically, the Todaro paradox occurs more likely, the higher* σ_A *and, the less* (i) *the degree of nonhomotheticity,* $|\mu_L - \theta|$, (ii) σ_M and (iii) e.

In addition, the paradox may not exist when the rate of urban employed (e) is high, which is similar to the observation of Raimondos (1993) that the level of urban unemployment will be reduced in response to capital expansion if the urban unemployed do not outnumber the urban employed.

Next, according to (15) and (17) and $\omega_K(\theta - \mu_L) > 0$, we have the following lemma.

Lemma 1. *Compared with the case of homothetic case* ($\omega_K = 0$)*, the Todaro paradox occurs less likely under nonhomotheticity in an economy characterized as in proposition 1.*

In addition, by viewing (17), $dU/\widehat{K} < 0$ under the special case of $\sigma_A = \sigma_M = 1$. In general, we have

Lemma 2. *In an economy characterized as in proposition 1, the Todaro paradox never holds if* $\sigma_M \geq 1 \geq \sigma_A$.

Furthermore, Corden and Findlay (1975) illustrate the occurrence of the Todaro paradox in response to an increase in capital stock under the framework of a small open economy. Although substitution between capital and labor in each sector is allowed in their model, the small-country assumption implies that the elasticity of factor substitution in each sector were zero. Turn, now, to the case of $\sigma_A = \sigma_M = 0$ under nonhomotheticity, (16) can be written as

$$\frac{\widehat{L}_A}{\widehat{K}} = \frac{\lambda_{LU}}{\Delta}[\theta_{KA} - \theta_{LA}\theta_{KM} + \theta_{LA}\omega_K(\theta - \mu_L)] \tag{16'}$$

This shows that the change in the level of L_A is ambiguous; hence we have the following lemma.

Lemma 3. *In an economy characterized as proposition 1, if* $\sigma_A = \sigma_M = 0$*, the necessary condition for the Todaro paradox is that in addition to low degree of nonhomotheticity, the distributive share of agricultural capital be sufficiently low* (i.e., $\theta_{KA} < \theta_{KM}\theta_{LA}$).

In contrast to Corden and Findlay (1975) in a small open economy, if $\sigma_A = \sigma_M = 0$, the Todaro paradox does not necessarily hold in our model. By viewing (17), in addition to the necessary conditions in Lemma 3, the Todaro paradox might occur if the rate of urban employment is sufficiently low.

4 Concluding Remarks

This paper incorporates monopolistic competition and nonhomotheticity into the Harris and Todaro model and analyzes the so called Todaro paradox. With nonhomotheticity, the Todaro paradox following an increased capital stock, in general, does not necessarily hold, which depends on (i) the elasticities of factor substitution, (ii) the rate of urban employment and (iii) the degree of nonhomotheticity. Specifically, the Todaro paradox occurs more likely, the higher σ_A and, the less $|\mu_K - \theta|$, σ_M and the rate of urban employment (e). In the special case of $\sigma_A \leq 1 \leq \sigma_M$, the Todaro paradox never occurs. Furthermore, in the Corden and Findlay (1975) model with small open economy where capital-labor ratios remain constant (namely, elasticities of factor substitution are zero), they illustrate that the Todaro paradox takes place in response to an increase in capital stock. However, in our case of $\sigma_A = \sigma_M = 0$, the paradox might not necessarily hold; it depends upon relative distributive shares and the degree of nonhomotheticity.

In addition, the roles of monopolistic competition in manufacturing sector are significantly important. In Raimondos (1993), the Todaro paradox following increased capital does not hold because that the agricultural labor market is monopsonistic, which provides higher agricultural wages and restrains rural-to-urban migration. With monopolistic competition in this article, the wage rate in agriculture rises in response to capital expansion. This provides the same incentive for small rural-to-urban migration; hence, the Todaro paradox may not necessarily hold.

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