Optimal Bank Regulation in an Economic Boom and Recession

Yu-Hsiu Lin*

Department of Finance, National Kaohsiung University of Applied Sciences, Taiwan

Len-Kuo Hu**

Department of International Business, National Chengchi University, Taiwan

Abstract

This paper utilizes a representative agent model to address how the regulation policy for banks should respond to different general economic conditions. With the consideration of a fair deposit insurance and sufficient bank reserve to meet the expected need of liquidity shock, our model suggests counter-cyclical capital standards in a near-competitive loan market. The exception occurs when the moral hazard problem becomes very unwieldy and the representative individual is rather risk averse. With regard to the closure policy, we find that it closely related to the individual's degree of risk aversion. A counter-cyclical closure policy is recommended when the individual is highly risk averse. Otherwise, a pro-cyclical closure policy is preferred.

Key words: liquidity shock, closure policy, capital adequacy requirement

*Corresponding author. Tel: 886-7-3814526-6311. Fax: 886-7-3831544. 415 Chien-kung Rd., Kaohsiung 807, Taiwan. E-mail: yushiu@cc.kuas.edu.tw

** 64, Sec. 2, Zhi-nan Rd., Wenshan, Taipei 116, Taiwan. E-mail: lkhu@nccu.edu.tw

Optimal Bank Regulation in an Economic Boom and Recession

1. Introduction

Credit crunch beleaguers an economy and, to be worse, exacerbates the deterioration when the general economic environment turns sour. To comply with Basle II requirements banks are obliged to constrain their lending policy in the presence of an economic downturn as they perceive the plunging of market value and worsening of credit as well as liquidity risk in their bank assets. The mushrooming financial intermediaries in the world are thereby condemned to be the culprit that aggravates the degree and frequency of business cycle. Our study is intended to examine the prudence of banking regulation especially at the moment when an unprecedented financial storm is striking the Wall Street and the global financial system.

Bank regulation has a long history and addresses diverse issues, especially those related to the safety and soundness of the banking system.[1] The New Basel Accord (Basel II) proposes three pillars for the supervision of banking industry, among which the adoption of more risk-sensitive capital standards generates most debates. One major concern arises that this higher risk-sensitivity may cause additional volatility in general economic cycles, or the so called pro-cyclicality problem.[2] In this article, we do not deal with the risk attributes of capital standards. Instead, we focus on examining how the minimum capital requirements should respond to the changes of general economic condition. Should it be pro-cyclical (tightening the requirements when the economy falls into recession) or counter-cyclical to stimulate the economy during the downswing?

Moreover, in the literature the bank's deposit capital and equity capital are often assumed to be determined by the bank's financing decisions. Our study considers the other extreme case on the spectrum that the deposit and equity capital are solely determined by a representative agent's inter-temporal consumption-saving decisions. More specifically, we consider in this model three key players: a representative individual who plays multiple roles as consumer, depositor, and bank's equity holder;

---

[1] Contemporary banking theory offers a series of explanations for the emergence of banks and the need of their regulation (Diamond and Dybvig (1980), Diamond (1984), Dewatripont and Tirole (1994), Bhattacharya, Boot and Thakor (1998), Bhattacharya and Thakor (1993) etc). Freixas and Rochet (2008) provides an overview of different theories of bank regulation. In the banking industry, the regulatory instruments in general can be classified into six broad types, which are (1) deposit interest rate ceilings; (2) entry, branching, network, and merger restrictions; (3) portfolio restrictions such as reserve requirements; (4) deposit insurance; (5) capital requirements; and (6) regulatory monitoring and supervision (including closure policy).

[2] Some empirical researches confirm this concern (Gordy and Howells (2006), Catarineu-Rabell et al.,(2005)) . For a literature survey, please refer to Allen and Saunders (2004). On the contrary, Zhu (2007) develops a stochastic dynamic model to examine the impact of capital regulation on bank's financial decisions. He finds that in equilibrium, the capital constraint is not binding; which implies that the adoption of a more risk-sensitive capital requirement does not necessarily lead to a reinforcement of the credit cycle.

a banker who receives deposit and equity injection from the individual and lends a loan to facilitate project investment; and a regulator who acts on behalf of the best interest of the representative individual by structuring a prudent policy. To capture the special features of a bank, our model has built in two kinds of inherent uncertainties: a mid-term liquidity shock and a risk associated with the ultimate investment outcome. Further, we assume that, to effectively cope with these uncertainties, the bank has developed a self-fulfilling deposit insurance fund (DIF) and a profound reserve requirement that meets the expected need of liquidity shock.

Under these model specifications, we find that the appropriate capital regulation corresponding to different economic conditions hinges on the loan project's return and the seriousness of the moral hazard problem (for monitoring the loan). Specifically, we suggest that the capital adequacy ratio should be *counter-cyclical*, i.e., lowering the required ratio in recession and strengthening it in boom. The reasoning is as follows. When the regulator imposes a higher capital adequacy ratio, the representative individual will be induced to allocate more of his savings in the bank equity instead of deposit. A greater fund will then be available for lending as a result of the reduction in the provision of deposit insurance. The representative individual will benefit from the expansion of bank loan while suffer from the reduced bankrupt protection due to the shrinkage of deposit. As the general economic condition improves, both bank loan and bank deposit will increase, lowering the marginal impact of aforementioned benefit and cost of capital requirement on the individual. Our study shows that the marginal cost will decrease to a greater extent than the marginal benefit when the loan market is competitive enough so that the expected excess return of the loan project does not significantly differ from zero. Consequently, a higher capital adequacy ratio should be recommended in an economic boom. However, if the moral hazard problem is very severe and the individual is highly risk averse, the capital adequacy ratio would become *pro-cyclical*.

In addition to capital regulation, we also consider the issue of closure policy. The regulator must set up an acceptable rescuing threshold (for the liquidity shock) beyond which the bank is closed. We find that a higher threshold value will result in the increase of bank deposit, enhance the probability of bank solvency, but lower the bank loan. The optimal threshold value turns out to be closely related to the individual's degree of risk aversion. Specifically, if the individual has a low degree of risk aversion, the effect of an enlarged bank loan and deposit from an improved economic condition outweighs the associated effect of lower marginal utilities. As a result, the marginal benefit and cost of a softer closure policy (i.e., a higher threshold value) increase as the economic condition improves. Our study shows that the increased marginal benefit will dominate the increased marginal cost when the loan project's expected excess return does not significantly differ from zero (i.e., when the project's expected return is near the competitive level). Consequently, a more soft closure policy should be adopted in a booming economy if the individual is less risk-averse. However, if the individual is substantially risk averse, the marginal benefit and cost of a softer closure policy will decrease with an improved economic condition. Hence, as the economy booms, a stricter closure policy is recommended if the individual is highly risk-averse.

This paper is organized as follows. Section 2 establishes the basic model. The bank's lending policy and the individual's consumption-saving decisions are examined in section 3. Section 4 investigates how the regulator sets the appropriate regulation policy by assuming that the minimum adequacy requirement is binding. This primitive analysis is then extended in section 5 to explore the impact of moral hazard on the regulator's policy. Finally, section 6 provides some concluding remarks.

2. The Model

A representative individual with initial income Y_0 at time 0 can consume C_0 and save the remaining by either depositing D_0 as a depositor or investing K_0 as a shareholder in the representative bank. Let the savings be allocated between D_0 and K_0 as $D_0 = \mu(Y_0 - C_0)$ and $K_0 = (1-\mu)(Y_0 - C_0)$. The individual chooses the current consumption C_0 and deposit share μ in order to maximize his intertemporal utility.

Once receiving the deposit and equity injection from the individual, the banker is in charge of a prudent lending policy so as to maximize the welfare of its shareholder. The loan L is used in an investment project which will generate an output $L\tilde{R}$ at time 1, where $\tilde{R} = R$ with probability p and $\tilde{R} = 0$ with probability $1-p$. The quality of the investment is affected by an unobservable monitoring of the banker. The latter can either monitor the loan, in which case it has a high probability of success, $p = p_H$, or shirk, in which case the loan has a probability of success of only $p = p_L = p_H - \Delta p$, with $\Delta p > 0$. Shirking provides the banker with a private benefit with monetary equivalent B (per unit of investment).

This loan from the bank is confronted with the liquidity shock at an interim date ($t = 1/2$). The macroeconomic shock, as in Holmstrom and Tirole (1998), will take place with some probability q. When it occurs, the bank needs an injection ρL to meet its liquidity needs. We assume ρ is unknown ex ante (at t = 0) and the representative individual perceives that ρ is distributed according to a continuous distribution function $F(\rho)$ (with the corresponding density function $f(\rho)$).Whether the bank survives the liquidity shock relies on the regulator's closure policy. Specifically, the regulator's closure policy is a function $x(\rho)$ conditional on the verifiable ρ at $t = 1/2$. If the exposure to the macroeconomic shock is very large, the regulator opts to close the bank and $x(\rho) = 0$. Otherwise, the regulator provides

any liquidity shortage and $x(\rho) = 1$. Hence, the regulator must determine the optimal threshold value of ρ, denoted by ρ^*, in establishing the standard for rescuing the bank. A higher $\rho *$ represents a more soft closure policy. When the bank is closed, its assets are liquidated (with zero value) and the depositor is compensated by the deposit insurance fund (DIF). We assume that q is sufficiently small for bank lending to remain profitable even if the risk of macroeconomic shock is taken into account.
Assumption 1: $(1-q)pR > 1$.

To sum up, there are three players in our model: the representative "individual" who acts as the sole depositor as well as the shareholder of the bank, the "banker" who represents the interests of the bank's manager and shareholder, and the "regulator" who considers the situation of the banking system as a whole with an aim to maximize the welfare of the representative individual.

We formulate the sequence of the action of each player as follows. At $t = 0$, the regulator decides his closure policy ρ^* and imposes a minimum capital requirement k on the bank. Then, the individual determines his optimal choices of C_0 and μ. With capital funds D_0 and K_0 on hand, the banker chooses the loan amount L and decides whether or not to monitor the loan. At the intermediate date $t = 1/2$, the bank will continue in operation if there is no liquidity shock, or if the liquidity shock happens with a realized value of $\rho \le \rho^*$. Or else the bank is closed (liquidated), and the DIF pays off the deposits. At $t = 1$, the project return is realized for the surviving bank. When the project fails, the bank goes bankrupt and the DIF steps in to compensate the depositor. When the project succeeds, the bank pays the deposits and any remaining to its shareholder. Overall, the probability of bank solvency is $p[1-q+qF(\rho^*)]$ while that of bank insolvency is $1-p[1-q+qF(\rho^*)]$.

3. Optimal Choices of the Bank and the Individual

3.1 Lending Policy of the Bank

By backward induction, we initially examine the bank's lending policy. Given ρ^*, k, C_0 and μ, the banker (risk neutral) will choose the optimal loan size so as to maximize the expected share value of its shareholder subject to the bank's budget constraint and the incentive compatible constraint (for monitoring the loan). Denote v as the bank's reserve per unit of loan and K_1 as its share value at $t = 1$. When the bank is insolvent, K_1 equals zero. When the bank is solvent, K_1 either equals $L(R+v)-D_0$ (if the bank does not encounter liquidity shock), or equals $L(R-\rho+v)-D_0$ (if the bank encounters and survives the liquidity shock).[3] As a result,

[3] Once the liquidity shock occurs, the bank's reserve covers the required fund injection at $t = 1/2$ for $\rho \le v$. In this case, K_1 equals $LR - D_0$ plus the reserve surplus $(v-\rho)L$ when the project succeeds. For $\rho > v$, the bank needs external liquidity injection $(\rho - v)L$ from the regulator to meet the shortage of funds. We presume that the bank must repay $(\rho - v)L$ later when the project succeeds.

the expected value of K_1 is $[p(1-q+qF(\rho^*))]\times[L(R+v)-D_0]-qpL\int_0^{\rho^*}\rho f(\rho)d\rho$. We further assume that the bank, in anticipation of a liquidity shock, retains a reserve ratio of $v(\rho^*)\equiv q\int_0^{\rho^*}\rho\ f(\rho)d\rho$ which equals its expected requirements for the liquidity fund.[4] The banker's problem can be formulated as follows.

$$\text{Max}\quad L\times[p(1-q+qF(\rho^*))]\times[R+v(\rho^*)]-pLv(\rho^*)-D_0\times[p(1-q+qF(\rho^*))]$$

$$\text{Subject to}\quad L[1+v(\rho^*)]\le p[1-q+qF(\rho^*)]D_0+K_0, \qquad \text{(IR)}$$

$$\text{and}\qquad D_0\le L(R-\frac{B}{\Delta p}). \qquad \text{(IC)}$$

In the IR constraint (or budget constraint), the available fund after the consideration of deposit insurance premium is $p(1-q+qF)D_0+K_0$ because the bank must pay the DIF a fair premium that is equal to $D_0[1-p(1-q+qF(\rho^*))]$ (note that $1-p(1-q+qF(\rho^*))$ is the probability of bank insolvency). As to the construction of IC constraint, for simplicity, we follow Holmstrom and Tirole (1998) by assuming that moral hazard occurs after the liquidity shock. The banker will do his best to monitor the performance of investment project (so that the probability of success becomes $p=p_H$) only if the following IC constraint is satisfied.[5]

$$p_H(LR-D_0)\ge p_L(LR-D_0)+BL, \qquad \text{or} \qquad D_0\le L(R-\frac{B}{\Delta p}).$$

The p in the problem setting will become p_H when the incentive constraint is satisfied for the banker. Otherwise, p_H is replaced by p_L.

Since the expected share value is increasing in L,[6] the banker will choose L until the IR constraint (budget constraint) is binding or

$$L(\rho^*, C_0, \mu)=\frac{(1-\mu)+p[1-q+qF(\rho^*)]\mu}{1+v(\rho^*)}\times(Y_0-C_0). \qquad (1)$$

In equation 1 the capital adequacy ratio chosen by the bank is

$$k^b(\rho^*, \mu) = \frac{K}{L(\rho^*, C_0, \mu)}=\frac{(1-\mu)(Y_0-C_0)}{L(\rho^*, C_0, \mu)}=\frac{1+v(\rho^*)}{1+p(1-q+qF(\rho^*))(\mu/1-\mu)}. \qquad (2)$$

If this capital ratio is smaller than the required ratio k imposed by the regulator (i.e., the minimum capital adequacy ratio is binding), the loan amount L provided by the bank will become $(1-\mu)(Y_0-C_0)/k$ instead of that shown by equation1.

Consequently, K_1 equals $L(R-\rho+v)-D_0$ for $\rho\le\rho^*$.

4 For simplicity, we do not endogenously determine the reserve ratio v in this model.

5 We assume that, in deciding whether or not to monitor the loan, the banker compares the incremental net return (i.e. $\Delta p\times(LR-D_0)$) from the loan project itself to his private benefit without considering the part of return from bank reserve.

6 We derive that $\frac{\partial E(K_1)}{\partial L}=p(1-q+qF)\times R-pq(1-F)\times v$. This term is positive because the reserve ratio v should be smaller than 1 and $p(1-q+qF)\times R>1$ according to assumption 1.

For the time being, we implicitly assume that the IC constraint is slack. In other words, we assume $B/\Delta p$ is relatively small such that IC is automatically satisfied for the loan amount either determined by equation 1 or by the binding required capital ratio. The scenario of binding IC constraint (i.e., when $B/\Delta p$ is sufficiently large relative to R such that the moral hazard problem becomes severe) will be discussed later on.

3.2. Consumption-investment Decision of the Individual

In this subsection, we analyze the individual's optimal choices of C_0 and μ. The individual is intended to maximize his intertemporal utility as follows.

$$\underset{C_0,\,\mu}{Max} \quad U(C_0) + \delta\, EU(\tilde{C}_1)$$

$$\text{Subject to} \qquad Y_0 = C_0 + D_0 + K_0 .$$

Since the individual plays the role of depositor as well as shareholder in the representative bank, he will receive D_0 from DIF once the bank is insolvent. Alternatively, when the bank is solvent, $\tilde{C}_1$ equals either $L(R+v(\rho^*))$ or $L(R-\rho+v(\rho^*))$. Therefore, the individual's problem above can be rephrased as

$$Max \;\; U(C_0) + \delta\; p\{\int_0^{\rho^*} qU[L(R-\rho+v(\rho^*))]f(\rho)d\rho + (1-q)U[L(R+v(\rho^*))]\} + \delta U(D_0)[1-p(1-q+qF(\rho^*))]$$

$$\text{Subject to} \quad D_0 = \mu(Y_0 - C_0) \quad \text{and} \quad K_0 = (1-\mu)(Y_0 - C_0) .$$

We temporarily presume that the bank's capital ratio k^b (as shown in equation 2), which is endogenously determined by the individual's optimal choice of deposit share μ and the bank's loan policy, is larger than the regulator's required capital ratio k. Thus, the individual will form his conjecture about bank loan L according to equation 1 in making his optimal decision. The first order conditions with respect to C_0 and μ are derived respectively as

$$U^{'}(C_0) + \delta \frac{\partial L}{\partial C_0} \times p\{\int_0^{\rho^*} qU^{'}[L(R-\rho+v(\rho^*))]\times(R-\rho+v)f(\rho)d\rho + (1-q)U^{'}[L(R+v(\rho^*))]\times(R+v)\}$$
$$- \delta\mu U^{'}(D_0)[1-p(1-q+qF(\rho^*))] = 0, \qquad (3)$$

$$\frac{\partial L}{\partial \mu} \times p\{\int_0^{\rho^*} qU^{'}[L(R-\rho+v(\rho^*))]\times(R-\rho+v)f(\rho)d\rho + (1-q)U^{'}[L(R+v(\rho^*))]\times(R+v)\}$$
$$+ U^{'}(D_0)(Y_0 - C_0)[1-p(1-q+qF(\rho^*))] = 0. \qquad (4)$$

where $\frac{\partial L}{\partial C_0} = -\frac{(1-\mu)+p(1-q+qF)\mu}{1+v} < 0$ and $\frac{\partial L}{\partial \mu} = \frac{[1-p(1-q+qF)](C_0-Y_0)}{1+v} < 0$ according to equation 1.

The second term in equation 3 can be visualized as the marginal cost of additional current consumption C_0 by way of its impact on the reduction of bank loan and the last term reflects the marginal cost from the reduced deposit. The enhancement of C_0 reduces both the loan fund and the amount of deposit, which in turn decreases the utility at t = 1. Similarly, the first term in equation 4 represents the marginal cost of deposit share μ. As μ increases, the loan fund will decrease because bank's available fund is increased by $p(1-q+qF)$ unit through the injection of one additional unit of deposit but decreased by one whole unit through the accompanying one-unit reduction of equity capital. The second term in equation 4 reflects the marginal benefit of higher μ because with probability $1-p(1-q+qF)$ the depositor can obtain D_0 from the DIF. A higher deposit share leads to a greater compensation from the DIF. The optimal C_0 and μ, which are solved simultaneously by these two first order conditions, are then the functions of the regulator's decision ρ^*, i.e., $C_0(\rho^*)$ and $\mu(\rho^*)$.

The following lemma addresses the necessary and sufficient condition for μ to have an interior solution.

Lemma 1: When the degree of risk aversion of the individual is high enough so that $U'(C)\times C$ is non-increasing in C, the individual will not invest any of his savings in the bank equity and the optimal deposit share is $\mu=1$. On the contrary, when the individual is less risk averse such that $U'(C)\times C$ is increasing, he will invest some savings in the bank equity capital. In this circumstance, μ will have an interior solution.

Proof: Appendix 1.

In Appendix 1 we show that when $U'(C)\times C$ is non-increasing, the marginal benefit of one more unit of μ (from higher deposit) is always greater than its marginal cost (from lower bank loan). The individual with a high degree of risk aversion would not invest any fund in the bank equity capital; hence the bank's capital ratio is zero in this occasion. In other words, our temporary presumption of $k^b > k$ would be violated when $U'(C)\times C$ is non-increasing. Henceforth, in analyzing the regulator's policy we will focus on examining the scenario of binding capital adequacy ratio.[7]

Assumption 2: $p[(R+v)(1-q+qF)-v] > 1+v$

Parallel to assumption 1, assumption 2 requires that the bank's loan project return after considering the liquidity shock and reserve requirement, i.e., $p[(R+v)(1-q+qF)-v]$, should be larger than the safe return $1+v$. Otherwise, even a risk-neutral individual will consume all his income and leave no saving in the bank.

---

[7] We have also solved the non-binding scenario for the case of a less risk-averse individual (i.e., $U'(C)\times C$ is increasing). We find that the impact of general economic conditions on the regulator's closure policy and the bank's capital ratio are similar to those found under the scenario of binding capital adequacy ratio.

We exclude this situation in this article.

4. Regulations of the Bank

Assume that the capital adequacy ratio k imposed by the regulator is binding (either when $U^{'}(C)\times C$ is non-increasing and $k^{b}=0$, or when $U^{'}(C)\times C$ is increasing but the bank's positive capital ratio k^{b} is less than k). In either case, the individual's deposit share will not be determined by equation 4. Instead, to satisfy the minimum capital adequacy requirement, i.e., $k^{b}=k$, the individual should select his deposit share as shown below in equation 5

$$\mu(\rho^{*}, k)=\frac{1+v-k}{1+v-k[1-p(1-q+qF)]}\,. \tag{5}$$

And the bank loan is

$$L\,(\rho^{*},k,C_0)=\frac{p(1-q+qF)}{1+v-k[1-p(1-q+qF)]}(Y_0-C_0)\,. \tag{6}$$

Lemma 2: Consider the scenario that the regulator's required capital ratio k is binding. Under this circumstance, the individual's deposit share must satisfy $k^{b}=k$. *This deposit share becomes smaller and the resulting bank loan expands as the regulator asks a higher capital adequacy ratio, other things equal. Moreover, the deposit share hinges on the regulator's closure policy* ρ^{*} *as follows: for* $\rho^{*}\geq\frac{1+v-k}{1-q+qF}$, $\partial\mu/\partial\rho^{*}$ *is positive and* $\partial L/\partial\rho^{*}$ *is negative; for* $\rho^{*}<\frac{1+v-k}{1-q+qF}$, $\partial\mu/\partial\rho^{*}$ *is negative and* $\partial L/\partial\rho^{*}$ *is positive.*

With regard to the overall influences of regulator's policy on the bank loan, we must further consider the changes of the individual's consumption (which can be determined by equation 3 after taking equations 5 and 6 into account). In general, we discover that when the regulator imposes a strict regulation, the individual may lower or increase his consumption depending on his degree of risk aversion.[8] Specifically, if the individual is highly risk averse, both his consumption level and deposit share will be reduced as the required capital ratio k increases. Thus the bank loan enlarges when the capital regulation becomes stricter. If the individual is less risk averse, the bank loan may decrease with a higher k because the positive impact of a lower deposit share on the bank loan is now counterbalanced by the increases of consumption. As for the impact of closure policy, we find that when the regulator adopts a more lenient closure policy, a highly risk-averse individual will raise his consumption level along with a bigger deposit share. This will result in the shrinkage

[8] To obtain concrete results, we assume that the individual is limitedly rational as he comprehends the impacts of the regulator's policy on his consumption. The indirect impacts of ρ^{*} and k on C_0 through the bank's lending policy is ignored.

of bank loans. If the individual is less risk-averse, he would consume less as ρ^* increases. Bank loan may expand in this case because the individual saves more capital into the bank which counterbalances the negative impact of a higher deposit share on the bank loan. We conclude that if the regulator intends to stimulate bank credits, a strict regulation policy is effective when the individual is highly risk averse. If the individual's degree of risk aversion is low, we are unsure it's a soft or strict policy that can result in greater bank loans.

By taking into account the individual's choices of C_0, μ (equations 3 and 5) and the bank's optimal choice of loan L (equation 6), the regulator will then determine the optimal closure policy ρ^* and the minimum capital adequacy ratio k with an objective of maximizing the representative individual's welfare. In other words, the regulator faces the following problem.

$$\underset{\rho^*,\ k}{Max}\quad U(C_0)+\delta\left(p\{\int_0^{\rho^*} qU[L(R-\rho+v(\rho^*))]f(\rho)d\rho+(1-q)U[L(R+v(\rho^*))]\}+U(D_0)[1-p(1-q+qF(\rho^*))]\right)$$

subject to $C_0(\rho^*, k)$ and $\mu(\rho^*, k)$ that satisfy equations 3 and 5, and L that satisfies equation 6. The first order conditions are

$$\begin{aligned}0=&\ p\,q\,f(\rho^*)[U(L(R-\rho^*+v))-U(D_0)]+\frac{\partial v}{\partial\rho^*}\times p\times\{\int_0^{\rho^*} qU'(L(R-\rho+v))\times L f(\rho)d\rho\\&+(1-q)U'(L(R+v))\times L\}\\&+\frac{\partial L}{\partial\rho^*}p\{\int_0^{\rho^*} qU'(L(R-\rho+v))\times(R-\rho+v)f(\rho)\,d\rho+p(1-q)U'(L(R+v))\times(R+v)\}\\&+\frac{\partial\mu}{\partial\rho^*}\times[U'(D_0)(Y_0-C_0)\times(1-p(1-q+qF))]\end{aligned}\qquad(7)$$

$$\begin{aligned}0=&\ \frac{\partial L}{\partial k}\times p[\int_0^{\rho^*} qU'(L(R-\rho+v))\times(R-\rho+v)f(\rho)d\rho+(1-q)U'(L(R+v))\times(R+v)]\\&+\frac{\partial\mu}{\partial k}\times U'(D_0)\times(Y_0-C_0)\times[1-p(1-q+qF)]\\=&\ \frac{\partial L}{\partial k}\times\{p[\int_0^{\rho^*} qU'(L(R-\rho+v))\times(R-\rho+v)f(\rho)d\rho+(1-q)U'(L(R+v))\times(R+v)]-(1+v)U'(D_0)\}\end{aligned}\qquad(8)$$

The regulator's optimal choices of ρ^* and k are simultaneously determined by equations 7 and 8. Based on these equations, we find that the regulator's optimal choice of ρ^* would be larger than $(1+v-k)/(1-q+qF)$.[9] Conditional on k, the regulator needs to consider the direct impact of ρ^* and the indirect impact via the

[9] Let the regulator's objective function be H, equations 7 and 8 can be rewritten as $\frac{\partial H}{\partial\rho^*}+\frac{\partial L}{\partial\rho^*}\{\frac{\partial H}{\partial L}-kU'(D_0)[1-p(1-q+qF)]\}=0$ and $\frac{\partial L}{\partial k}\{\frac{\partial H}{\partial L}-(1+v)U'(D_0)\}=0$, respectively. Thus, we obtain that $\partial L/\partial\rho^*$ should be negative at the optimal ρ^* (i.e. $\rho^*>(1+v-k)/(1-q+qF)$ by Lemma 2).

individual's deposit share μ and the bank's lending L when contemplating the effect of its closure policy on the individual's welfare.[10] The first two terms of equation 7 address the (direct) marginal benefit of higher ρ^*. Intuitively, individual's welfare increases with higher ρ^* since a more lenient closure policy will provide the bank more chance to forestall the liquidity shock (i.e., uplifting the probability of solvency). Besides, the bank will keep a higher reserve ratio $v(\rho^*)$ in anticipation of a more lenient closure policy. The resulting increase in reserve will enhance the individual's terminal wealth and welfare when ignoring the indirect impact of $v(\rho^*)$ on bank loan *L*.

The third and fourth terms in equation 7 represent the marginal effects of ρ^* through its indirect impact on bank loan *L* and the individual's binding deposit share μ. From Lemma 2, we know that the fourth term is positive, which represents the (indirect) marginal benefit of ρ^* from the enlarged deposit; while the third term is negative which denotes the marginal cost of ρ^* from the lowered bank loan. The impact of capital adequacy requirement *k* comes from its influences on bank loans and deposits. Equation 8 show that, conditional on ρ^*, the optimal capital ratio is determined based on a tradeoff between the marginal benefit from greater bank loans and the marginal cost from lower deposits.

Proposition 1 Consider the scenario that the minimum capital requirement k is binding. Besides, assume the bank operates in such a competitive environment that the loan project return R is around $\hat{R}$ *that satisfies* $p[(R+v)(1-q+qF)-v]=1+v$. *Then, the impact of the current income* Y_0 *on the regulator's policy can be characterized as follows.*

(1)When the individual is less risk averse such that $U'(C)\times C$ *is increasing and the degree of increase remains constant (i.e.* $U''(C)\times C+U'(C)\equiv\varphi>0$*), the regulator should employ a soft closure policy accompanied with a tight (high) capital adequacy ratio in a good economic condition (in terms of a high* Y_0*).*

(2)When the individual is relatively highly risk-averse such that $U'(C)\times C$ *is decreasing and the degree of decrease remains constant (i.e.* $U''(C)\times C+U'(C)\equiv\varphi<0$*), the regulator should adopt a stringent closure policy and requires a high capital adequacy ratio when the economy is booming.*

Proof: Appendix 2.

Other things equal, the bank loan and deposit are enlarged as Y_0 (the proxy for general economic conditions) expands. This positive effect of larger loans and deposits associated with a higher Y_0 impacts the marginal influences of ρ^* and k

[10] The indirect impact of ρ^* (and k) through the individual's C_0 can be ignored according to the implication of envelope theorem.

differently. In Appendix 2 we show that the impact of Y_0 on the marginal influences of k mainly hinges on the changes of marginal utilities derived from bank loans and deposits. As Y_0 increases, the marginal benefit (the first term in equation 8) and marginal cost (the second term in equation 8) of k all decrease. Further, the marginal cost decreases in a larger degree when the project's return R is near the competitive level of $\hat{R}$.[11] Therefore, the model suggests a tight capital regulation with the improvements in the economy.

With regard to the closure policy, the impact of Y_0 becomes more complex. Owing to the direct effects of ρ^* on the individual's welfare (the first two terms in equation 7), we show that, in addition to the changes of marginal utility derived from bank loans and deposits, the impact of Y_0 on the marginal influences of ρ^* also depends on the level of bank loans and deposits. If the individual is less risk-averse (case 1 of Proposition 1), the positive effect of larger loans and deposits associated with a higher Y_0 dominates the negative effect of lower marginal utilities. As a result, the marginal benefit and cost of ρ^* all increase as the general economic condition improves. On the contrary, the marginal benefit and cost of ρ^* all decrease in Y_0 if the individual is highly risk-averse (case 2 of Proposition 1). Besides, the effect of increased (or decreased) marginal benefit outweighs that of increased (or decreased) marginal cost when the project's return R is around the competitive level of $\hat{R}$. Hence, the model predicts a soft closure policy (a higher ρ^*) in an economic boom when the individual's degree of risk aversion is low, while a stringent closure policy is preferred when the individual has a high degree of risk aversion.

We interpret $\hat{R}$ as the break-even loan return at which the loan project generates zero excess return. The more competitive the banking environment is, the more likely the loan return approaches $\hat{R}$. The results imply that the appropriate regulation is closely related to the competitive status of the banking industry. The optimal regulation would become more delicate when the bank operates less competitively. To sum, Proposition 1 suggests a *counter-cyclical* capital regulation. The closure policy could be *pro-cyclica*l or *counter-cyclical* depending on whether the individual has a low or high degree of risk aversion. At last, we claim that the major attribute of case 1 is not affected under the scenario of non-binding capital adequacy requirement.

---

[11] If the project's return R is sufficiently larger than $\hat{R}$, the impacts of Y_0 on the marginal influences of k and ρ^* are indeterminate. So we focus on discussing those project returns that are around the upper neighborhood of $\hat{R}$.

5. The Scenario of Binding Incentive Constraint

Thus far, the analysis proceeds by implicitly assuming that the banker's IC constraint is fulfilled. The banker monitors the loan project and the probability of success p is p_H. To verify that this presumption is consistent, the deposit share μ must satisfy $\mu \le \frac{(R-B/\Delta p)}{1+v+(R-B/\Delta p)(1-p(1-q+qF))}$ according to the IC constraint. When the moral hazard problem becomes severe (i.e. high $B/\Delta p$ relative to R), it is very likely that the IC constraint will be violated. Once the constraint is violated, the regulator must then impose a capital adequacy requirement equal to $1+v-(R-B/\Delta p)\times p(1-q+qF)$ so as to assure the satisfaction of IC constraint, i.e., the best effort exerted by the banker to monitor the loan (otherwise the probability of success would be p_L).[12] The resulting deposit share and bank loan from the binding IC and IR constraints are

$$\mu = \frac{R-B/\Delta p}{1+v+(R-B/\Delta p)(1-p(1-q+qF))} \tag{9}$$

$$L = \frac{Y_0 - C_0}{1+v+(R-B/\Delta p)(1-p(1-q+qF))} \quad . \tag{10}$$

Lemma 3 Assume that the incentive compatibility constraint for monitoring the bank loan is binding. To prompt the monitoring effort from the banker, the imposed capital adequacy ratio should be equal to $1+v-(R-B/\Delta p)\times p(1-q+qF)$. *Conditional on* ρ^*, *the required capital adequacy ratio becomes higher and the deposit share lower when the moral hazard problem worsens. Moreover, the impact of* ρ^* *on deposit share and bank loan is as follows. For* $\rho^* > p(R-B/\Delta p)$, $\partial L/\partial\rho^*$ *and* $\partial\mu/\partial\rho^*$ *are negative;* $\partial L/\partial\rho^*$ *and* $\partial\mu/\partial\rho^*$ *are positive for* $\rho^* < p(R-B/\Delta p)$.

Analogous to equation 7, the regulator determines his optimal closure policy based on the following first order condition.

$$\begin{aligned}
0 = {} & p\,q\,f(\rho^*)[U(L(R-\rho^*+v))-U(D_0)] + \frac{\partial v}{\partial \rho^*}\times p\times\{\int_0^{\rho^*} qU'(L(R-\rho+v))\times L\,f(\rho)d\rho \\
& +(1-q)U'(L(R+v))\times L\} \\
& +\frac{\partial L}{\partial \rho^*}\,p\{\int_0^{\rho^*} qU'(L(R-\rho+v))\times(R-\rho+v)\,f(\rho)\,d\rho + p(1-q)U'(L(R+v))\times(R+v)\} \\
& +\frac{\partial L}{\partial \rho^*}\times(R-B/\Delta p)\times[U'(D_0)\times(1-p(1-q+qF))]
\end{aligned} \tag{11}$$

where we use the relationship that $(\partial L/\partial\rho^*)\times(R-B/\Delta p) = (\partial\mu/\partial\rho^*)\times(Y_0-C_0)$. Based

[12] This critical capital ratio is obtained by substituting the inequality of μ (for the satisfaction of the IC constraint) into equation 5.

on equation 11, we immediately derive that the optimal ρ^* should be greater than $p\,(R-B/\Delta p)$. The relationship between ρ^* and *k* can be described as given below in Lemma 4.

Lemma 4 Assume that the incentive compatibility constraint for monitoring the bank loan is binding. The regulator will construct its closure policy ρ^* *such that* $\rho^* > p(R-B/\Delta p)$. *Besides, the regulator's required capital adequacy ratio is positively related with his closure policy* ρ^* for those ρ^*'s that are greater than $p(R-B/\Delta p)$. *Thus, a more (less) soft closure policy is accompanied with a higher (lower) capital adequacy ratio.*

Comparable to proposition 1, the impact of Y_0 on the regulator's regulation policy is characterized in Proposition 2.

Proposition 2 Assume the incentive compatibility constraint is binding and the regulator decides to induce the monitoring effort from the banker. Then, for those loan project returns that are around $\hat{R}$, *the impact of the current income* Y_0 *on the regulator policy can be characterized as follows.*
(1)When the individual is less risk averse such that $U^{'}(C)\times C$ *is increasing and the degree of increase remains constant (i.e.*$U''(C)\times C+U'(C)\equiv\varphi>0$*), the regulator should employ a soft closure policy accompanied with a tight (high) capital adequacy ratio in a good economic condition, akin to proposition1.*
(2) On the contrary, when the individual is more risk averse such that $U^{'}(C)\times C$ *is decreasing and the degree of decrease remains constant (i.e.*$U''(C)\times C+U'(C)\equiv\varphi<0$*), the regulator should employ a stringent closure policy accompanied with a soft (lower) capital adequacy ratio in a good economic condition.*

Proposition 2 reveals that if the individual is less risk averse, the desirable regulation policy corresponding to different economic conditions is unchanged after the moral hazard problem is taken into account. The situation changes if the individual is more risk averse. Now the capital regulation becomes *pro-cyclical*. This is because as Y_0 increases, a stringent closure policy will reinforce the IC constraint for $\rho^* > p(R-B/\Delta p)$. The required capital ratio for stimulating the monitoring effort can thereby be lowered. To summarize, the preferred closure policy could be *pro-* or *counter-cyclical* contingent on the individual's degree of risk aversion (i.e., contingent on whether $U^{''}(C)\times C$ is increasing or decreasing). The required capital adequacy ratio should be *counter-cyclical* which is unrelated to the individual's degree of risk aversion. The only exception occurs when the individual is highly risk averse and the moral hazard problem is binding. In this circumstance, the minimum capital requirements would become *pro-cyclical*.

Before ending this section, we briefly discuss the impact of $B/\Delta p$ (a proxy for the significance of moral hazard problem) on the regulator's policy. When moral hazard problem worsens (i.e., $B/\Delta p$ becomes larger), in order to induce the monitoring effort, the deposit D_0 is reduced (bank equity K_0 enhanced) which results in an enlarged bank loan. As long as the degree of risk aversion is not too high (i.e., $U''(C)\times C+U'(C)=\varphi$ is positive or not too much negative), the marginal benefit and marginal cost of ρ^* would increase in $B/\Delta p$ owing to the effect of enlarged bank loan and reduced bank deposit. Unfortunately, we cannot determine their relative size and hence the definite impact of $B/\Delta p$ on ρ^*. What can be proved is that as the degree of individual's risk aversion strengthens, it will become more likely for the regulator to adopt a stringent closure policy (and lower capital adequacy ratio) when the binding moral hazard problem gets worse.

6. Conclusion

Bank's regulation policy (closure policy and capital adequacy requirements) should respond to the general economic condition. Depending on the complex interaction of the agent's risk preferences, the loan project's return, and the seriousness of the moral hazard problem, our study suggests an opposite relationship between the two regulation policies for the representative agent who has a low degree of risk aversion. More specifically, a soft closure policy should be accompanied with a strict capital requirement when the economy flourishes, and vice versa in an economic recession. On the other hand, for the representative agent who has a high degree of risk aversion, the two regulation policies should be implemented in the same direction. That is to tighten the regulation of closure policy and capital adequacy requirement simultaneously in a prosperous economic condition while relaxing the regulation when the economic condition is deteriorating. In other words, this model proposes a *counter-cyclical* minimum capital adequacy requirement whatever the individual's degree of risk aversion is. The only exception would occur only when the moral hazard problem becomes very unwieldy (so that the banker's incentive compatibility constraint for monitoring the bank loan becomes binding) and the individual is substantially risk averse. Under this circumstance, the appropriate policy for capital adequacy requirement should become *pro-cyclical*, i.e., strengthening the required ratio in recession and loosening it in boom.

The general economic conditions are exogenously given in our model. To thoroughly explore the interaction between bank regulation and general economic performance, we need an inter-temporal business cycle model in which both the two regulation policies and overall economic output are endogenously determined. It merits future research.

Appendix 1

Proof Lemma 1: We rewrite the marginal benefit of additional deposit share μ (i.e., $U^{'}(D_0)(Y_0 - C_0)[1-p(1-q+qF)]$ in equation 4) as $U^{'}(D_0)D_0 \times [1-p(1-q+qF)]/\mu$. By substituting $\partial L/\partial\mu$ into the marginal cost term, the marginal cost equals

$$\frac{[1-p(1-q+qF)]}{1+v}\times\frac{D_0}{\mu}\times p\times\{\int_0^{\rho^*} qU^{'}(L(R-\rho+v))\times(R-\rho+v)f(\rho)d\rho+(1-q)U^{'}(R+v)\times(R+v)\}.$$

Therefore, the relative size of marginal cost and marginal benefit is determined by comparing $U^{'}(D_0)\times D_0$ with

$$Z\equiv\frac{D_0}{(1+v)L}\times p\times\{\int_0^{\rho^*} qU^{'}(L(R-\rho+v))\times L(R-\rho+v)f(\rho)d\rho+(1-q)U^{'}(R+v)\times L(R+v)\}.$$

When $U^{'}(C)\times C$ is non-increasing in C (assuming $L(R-\rho+v)>D_0$), we know that

$$Z\le\frac{D_0}{(1+v)L}\,p\times\{\int_0^{\rho^*} qU'(D_0)D_0\,f(\rho)d\rho+(1-q)U'(D_0)\,D_0\}=\frac{\mu\,p(1-q+qF)}{(1-\mu)+p(1-q+qF)\mu}\times U^{'}(D_0)D_0.$$
$$<U'(D_0)D_0$$

Therefore, the marginal cost is always less than the marginal benefit and the optimal μ =1 when $U^{'}(C)\times C$ is non-increasing. On the contrary, when $U^{'}(C)\times C$ is increasing in C,

$$Z>\frac{D_0}{(1+v)L}\,p\times\{\int_0^{\rho^*} qU'(D_0)D_0\,f(\rho)d\rho+(1-q)U'(D_0)\,D_0\}=\frac{\mu\,p(1-q+qF)}{(1-\mu)+p(1-q+qF)\mu}\times U^{'}(D_0)D_0.$$

At μ =1 the marginal cost is larger than the marginal benefit. Since the marginal cost is increasing in μ while marginal benefit is decreasing in μ, there would exist an interior solution of optimal μ, i.e. $0<\mu<1$. Q.E.D

Appendix 2

Proof of Proposition 1: Denote the first order conditions of equations 8 and 9 as h =0 and j = 0, respectively. By totally differentiating h and j, we obtain that $h_{\rho^*}\frac{\partial\rho^*}{\partial Y_0}+h_k\frac{\partial k}{\partial Y_0}+h_{Y_0}=0$ and $j_{\rho^*}\frac{\partial\rho^*}{\partial Y_0}+j_k\frac{\partial k}{\partial Y_0}+j_{Y_0}=0$, where h_{ρ^*} and j_k are negative by the second order condition. Using Cramer rule,

$$\begin{bmatrix}\frac{\partial\rho^*}{\partial Y_0}\\ \frac{\partial k}{\partial Y_0}\end{bmatrix}=\frac{1}{h_{\rho^*}j_k-h_k j_{\rho^*}}\times\begin{bmatrix}h_k j_{Y_0}-j_k h_{Y_0}\\ j_{\rho^*}h_{Y_0}-h_{\rho^*}j_{Y_0}\end{bmatrix}.$$

First we calculate h_{Y_0} .

$$\frac{dh}{dY_0} = pq f(\rho^*)\{U'(L(R-\rho^*+v))\times(R-\rho^*+v)\frac{dL}{dY_0} - U'(D_0)\frac{dD_0}{dY_0}\}$$

$$+\frac{\partial v}{\partial \rho^*} p\{ q\int_0^{\rho^*}[U''(L(R-\rho+v))\times L(R-\rho+v)+U'(L(R-\rho+v))]f(\rho)d\rho$$

$$+(1-q)[U''(L(R+v))\times L(R+v)+U'(L(R+v))]\}\times\frac{dL}{dY_0}$$

$$+\frac{\partial^2 L}{\partial \rho^* \partial Y_0} A + \frac{\partial L}{\partial \rho^*}\times\frac{dA}{dY_0}+\frac{\partial \mu}{\partial \rho^*}\times[U''(D_0)D_0+U'(D_0)]\times[1-p(1-q+qF)](1-\frac{\partial C_0}{\partial Y_0}).$$

where

$$A \equiv p\{\int_0^{\rho^*} qU'[L(R-\rho+v)]\times(R-\rho+v)f(\rho)d\rho + (1-q)U'[L(R+v)]\times(R+v)\},$$

$$\frac{dL}{dY_0} = \frac{p(1-q+qF)}{1+v-k[1-p(1-q+qF)]}[1-\frac{\partial C_0}{\partial Y_0}] = \frac{L}{Y_0-C_0}\times(1-\frac{\partial C_0}{\partial Y_0}),$$

$$\frac{dD_0}{dY_0} = \mu[1-\frac{\partial C_0}{\partial Y_0}],$$

$$\frac{\partial^2 L}{\partial \rho^* \partial Y_0} = \frac{pq f(\rho^*)\times[1+v-k-\rho^*(1-q+qF)]}{\{1+v-k[1-p(1-q+qF)]\}^2}\times(1-\frac{\partial C_0}{\partial Y_0}) = \frac{\partial L}{\partial \rho^*}\times\frac{1}{Y_0-C_0}\times(1-\frac{\partial C_0}{\partial Y_0}),$$

$$\frac{dA}{dY_0} = p\{\int_0^{\rho^*} qU''(L(R-\rho+v))\times(R-\rho+v)^2 f(\rho)d\rho + (1-q)U''(L(R+v))\times(R+v)^2\}\times\frac{dL}{aY_0}\ .$$

Hence we yield

$$\frac{\partial^2 L}{\partial \rho^* \partial Y_0} A + \frac{\partial L}{\partial \rho^*}\times\frac{dA}{dY_0} = \frac{\partial L}{\partial \rho^*}\times\frac{1}{Y_0-C_0}[1-\frac{\partial C_0}{\partial Y_0}]\times$$

$$p\{\int_0^{\rho^*} q(R-\rho+v)\times[U''(\mathrm{L}(R-\rho+v))\times L(R-\rho+v)+U'(L(R-\rho+v))]f(\rho)d\rho$$

$$+(1-q)(R+v)\times[U''(L(R+v))\times L(R+v)+U'(L(R+v))]\}$$

$$= \frac{\partial L}{\partial \rho^*}\times\frac{1}{Y_0-C_0}\times(1-\frac{\partial C_0}{\partial Y_0})\times[(R+v)\times p(1-q+qF)-pv]\times\varphi\ .$$

where we assume $\varphi \equiv U''(C)\times C + U'(C)$ is a constant. Since $\frac{\partial L}{\partial \rho^*} = \frac{\partial \mu}{\partial \rho^*}\times\frac{-1}{k}\times(Y_0-C_0)$,

we can rewrite h_{Y_0} as follows.

$$h_{Y_0} = pq f(\rho^*)\{U'(L(R-\rho^*+v))\times L(R-\rho^*+v) - U'(D_0)\times D_0\}\times\frac{1}{Y_0-C_0}\times(1-\frac{\partial C_0}{\partial Y_0})$$

$$+\frac{\partial v}{\partial \rho^*} p\times(1-q+qF)\times\varphi\times\frac{L}{Y_0-C_0}\times(1-\frac{\partial C_0}{dY_0}) \qquad \text{(A1)}$$

$$+\frac{\partial \mu}{\partial \rho^*}\times(1-\frac{\partial C_0}{\partial Y_0})\times\varphi\{[1-p(1-q+qF)]-\frac{1}{k}[(R+v)p(1-q+qF)-pv]\}\ .$$

Note that $[1-\partial C_0/\partial Y_0] > 0$ and $[1-p(1-q+qF)]-(1/k)\times[(R+v)p(1-q+qF)-pv] < 0$ based on assumption 1. The sign of equation A1 is primarily determined by the individual's risk-aversion attitude φ and the project's return *R*. Consider the case of $\varphi > 0$. Given $\varphi > 0$, we know that $U'(L(R-\rho^* + v)) \times L(R-\rho^* + v) > U'(D_0) \times D_0$ by the presumption that $L(R-\rho^* + v) > D_0$. Hence, the first two terms are positive, reflecting the increased marginal benefit (of ρ^*) associated with a higher Y_0. The last term (after combing the indirect marginal benefit from larger deposits and the marginal cost from lower bank loans) is negative because $\partial\mu / \partial\rho^*$ is positive at the optimal ρ^* and $\varphi > 0$. This combined term represents the increased marginal cost (of ρ^*) associated with a higher Y_0. On the contrary, when $\varphi < 0$, the first two terms are negative and the combined term is positive, reflecting the decreased marginal benefit and decreased marginal cost (of ρ^*), respectively.

To explore the sign of h_{Y_0}, we further combine the last two terms of equation A1 and obtain

$$h_{Y_0} = pq f(\rho^*)[U'(L(R-\rho^* + v)) \times L(R-\rho^* + v) - U'(D_0) \times D_0] \times \frac{1}{Y_0 - C_0} \times (1 - \frac{\partial C_0}{\partial Y_0})$$
$$+ (1 - \frac{\partial C_0}{\partial Y_0}) \times \frac{pq f}{1 + v - k(1 - p(1 - q + qF))} \times \varphi \times$$
$$\{ \rho^* \times p(1-q+qF)^2 + k \times \frac{\rho^*(1-q+qF) - (1+v-k)}{1+v-k(1-p(1-q+qF))} \times [(1-p(1-q+qF)) - \frac{1}{k}[(R+v)p(1-q+qF) - pv]] \}.$$

At $R = \hat{R}$, the term in {.} equals

$$= \rho^* \times p(1-q+qF)^2 + k \times \frac{\rho^*(1-q+qF) - (1+v-k)}{1+v-k(1-p(1-q+qF))} \times [(1-p(1-q+qF)) - \frac{1+v}{k}]$$
$$= \rho^* \times p(1-q+qF)^2 + k \times \frac{\rho^*(1-q+qF)}{1+v-k(1-p(1-q+qF))} \times [\frac{k(1-p(1-q+qF)) - (1+v)}{k}]$$
$$+ \frac{(1+v-k) \times [(1+v) - k(1-p(1-q+qF))]}{1+v-k(1-p(1-q+qF))}$$
$$= \rho^*(1-q+qF) \times [p(1-q+qF) - \frac{1+v-k(1-p(1-q+qF))}{1+v-k(1-p(1-q+qF))}] + (1+v-k)$$
$$= \rho^*(1-q+qF) \times [p(1-q+qF) - 1] + (1+v-k) \le (1+v-k) \times p(1-q+qF).$$

The last inequality is derived from the condition that the optimal ρ^* must satisfy $\rho^* > (1+v-k)/(1-q+qF)$. We obtain that the term in {.} is positive (negative) when $\rho^* < (>)\ (1+v-k)/(1-q+qF)[1-p(1-q+qF)]$. As a result, at $R = \hat{R}$ the sign of h_{Y_0} is certain when $\rho^* < (1+v-k)/(1-q+qF)[1-p(1-q+qF)]$: h_{Y_0} is positive (negative) given $\varphi > (<)\ 0$. However, when $\rho^* > (1+v-k)/(1-q+qF)[1-p(1-q+qF)]$ the sign of h_{Y_0} is indeterminate even at $R = \hat{R}$. In this article, we only discuss the former case based on two reasons. First, only in this case the relative impact of Y_0 on the marginal influences of ρ^* can be determined around $R=\hat{R}$ (when ignoring the indirect impact of k). Second, rescuing banks is not a free lunch for the government. The related costs (not explicitly considered in this model) might prevent the government from setting a too high endurance level of ρ^*.

Next, we calculate j_{Y_0} as follows.

$$\frac{dj}{dY_0}=\frac{\partial^2 L}{\partial k\,\partial Y_0}[A-(1+v)U^{'}(D_0)]$$
$$+\frac{\partial L}{\partial k}\{\frac{dL}{dY_0}\times p\,[\int_0^{\rho^*} q\,U^{"}(L(R-\rho+v))\times(R-\rho+v)^2 f(\rho)d\rho+(1-q)U^{"}(L(R+v))\times(R+v)^2]$$
$$-\frac{dD_0}{dY_0}\times(1+v)U^{"}(D_0)\}$$

The first term is zero according to the first order condition equation 8. The second term can be rewritten as

$$\frac{dj}{dY_0}=\frac{\partial L}{\partial k}(1-\frac{\partial C_0}{\partial Y_0})\times\frac{1}{Y_0-C_0}\times$$
$$\{p\,[\int_0^{\rho^*} q\,(R-\rho+v)\times U^{"}(L(R-\rho+v))\times L(R-\rho+v)f(\rho)d\rho+(1-q)\times(R+v)U^{"}(L(R+v))\times L(R+v)]$$
$$-(1+v)U^{"}(D_0)\times D_0\}$$

(A2)

Since $U^{'}$ is decreasing, we know that $U^{"}(C)\times C=\varphi-U^{'}(C)$ is increasing. Hence it follows that

$$p\,[\int_0^{\rho^*} q\,(R-\rho+v)\times U^{"}(L(R-\rho+v))\times L(R-\rho+v)f(\rho)d\rho+(1-q)\times(R+v)U^{"}(L(R+v))\times L(R+v)]$$
$$>U^{"}(D_0)\times D_0\times p[(R+v)(1-q+qF)-v]$$

At $R=\hat{R}$, $U^{"}(D_0)\times D_0\times p[(R+v)(1-q+qF)-v]=U^{"}(D_0)\times D_0\times(1+v)$. Therefore, j_{Y_0} is positive for those project's returns that are around $\hat{R}$ according to equation A2. Note that this outcome does not rely on the sign of φ.

Theoretically, we should explore the cross-impact term ($h_k=j_{\rho^*}$) and follow the Cramer rule to determine the overall impact of Y_0 on the regulation policy. However, the calculation is too tedious to derive any concrete result. For the sake of simplicity, we only contemplate the circumstance that the own-effect dominates and ignore the cross-impact in examining Proposition 1.

References

Allen, Linda and Anthony Saunders. 2004. Incorporating systematic influence into risk measurement: A survey of the literature. *Journal of Financial Services Research*, Vol.26, 161-192.

Bhattacharya, S., A. Boot, and A. V. Thakor. 1998. The economics of bank regulation. *Journal of Money, Credit and Banking* 3, 745-770.

Bhattacharya, S., and A. V. Thakor. 1993. Contemporary banking theory. *Journal of Financial Intermediation* 3, 2-50.

Catarineu-Rabell, Eva, Patricia Jackson, and Dimitrios Tsomocos. 2005. Procyclicality and the new Basel Accod-bank’s choice of loan rating system. *Economic Theory*, Vol 26, 537-557.

Dewatripont, M., and J. Tirole. 1994. *The prudential regulation of banks*. Cambridge, MA: MIT Press.

Diamond, D. W. 1984. Financial Intermediation and delegated monitoring. *Review of Economic Studies* 51.3, 393-414.

Diamond, D.W., and P. Dybvig. 1980. Bank runs, deposit insurance, and liquidity. *Journal of Political Economy* 91.3, 401-419.

Freixas, X., and Rochet, J.C. 2008. *Microeconomics of banking*. Cambridge, MA; MIT Press,

Gordy, M. B., and B. Howells. 2006. Procyclicality in Basel II: Can we treat the disease without killing the patient? *Journal of Financial Intermedation* 15, 395-417.

Holmstrom, B., and J. Tirole. 1998. Private and public supply of liquidity. *Journal of Political Economy* 106, 1-40.

Rochet, J.C. 2008. Macroeconomic shocks and banking supervision. In *Why are there so many banking crises*? Princeton University Press.

Zhu Haibin. 2007. Capital regualtion and banks’ financing decisions. *BIS working paper*.