Bearing the Cost of Population Aging: Familial vs. Public Old-Age Support.
(Rough and preliminary draft)

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June 13^{th}, 2009

Abstract

Consumption after retirement can be financed through assets, public transfers, and private transfers. However, private transfers are generally missed in the literature. In order to fill this gap I compare the macroeconomic outcomes of an economy where consumption after retirement is financed through assets and public assets with those where consumption is financed through assets and private transfers. By doing this comparison I find two major results. First, in the steady state an age-related payroll tax that mimics familial old-age support is preferable to a flat payroll tax, since this policy may guarantee the same pension benefit with a higher aggregate consumption. Second, simulation results, using a computable general equilibrium model with realistic demography, show that this policy cannot be implemented when population is aging.

[*]I particularly thank the extremely helpful comments and challenging ideas received from Ronald Lee. This research is funded by the Fulbright Commission and Secretaría de Estado de Universidades e Investigación del Ministerio de Educación y Ciencia (SPAIN), reference number 2007-0445. Of course, any remaining errors are my own.

1 Introduction

The continuing increase in the number of retirees in relation to the number of workers raises concerns about the sustainability of pay-as-you-go (PAYG) pension systems. Several policies are suggested in order to guarantee its sustainability; for example, postpone the normal age of retirement above age 65, reduce the pension benefits and hold payroll taxes constant, switch the unfunded system to a funded system, among many others. Unfortunately, postponing the age of retirement reduces the time for leisure and reductions in the pension benefits decreases the utility of consuming goods and services. Most of these policies therefore imply a lost of welfare that make them difficult to be accepted by all age groups. More comprehensive models, however, in which the main problem is to finance the consumption after retirement can help to have a better understanding. Indeed, the goal of the social security pension system is not to maintain a specific replacement rate (the proportion of the average pension benefit relative to the average salary), which is usually debated, but to finance the consumption after retirement.

Consumption after retirement not only can be financed through assets and public transfers but also through familial transfers.However, public and familial transfers are generally PAYG, which crowd out assets held by individuals when they are selfish (Feldstein, 1974). As a consequence, the crowding-out effect will yield a lost of competence in the future. In addition, both systems are almost perfect substitutes and thus higher public transfers tend to eliminate previous private transfers and vice versa (Cox, 1987, Cox and Jimenez, 1990, Gale and Scholz, 1994). Nonetheless, each system has different characteristics. For example, familial transfers usually depends on the relationship between parents' and offspring's earnings, which involves that parents can also finance their offspring when they need it. In contrast, in public PAYG systems current workers pay the benefits of current retirees regardless the economic situation of her parents relative to their own.[1] Second, public pension systems are comprised by a larger number of people than a family, which reduces not only the risk of not receiving support but also inequality within families. Third, whereas familial old-age support is age related (i.e it depends on parents' ages), payroll taxes are generally constant across ages. In short, its advantages and disadvantages range from more flexibility and high risk to less flexibility and lower risk.

The crowding-out effect produced either by public or familial old-age

[1] A downward transfer could be possible levying taxes on old people and giving benefits to workers.

support relies on the extent to which each system modifies the age-savings profile. On the one side, the life cycle theory of saving predicts that young workers have positive saving, while old people have negative savings (Ando and Modigliani, 1963, Deaton and Paxson, 1997). On the other side, a monetary unit saved and invested during a longer period of time yields a greater capital or asset income than that invested for a shorter period of time. Consequently, we might expect that the transfer system that imposes more heavily the old-age support burden on the youngest workers might cause a greater crowding-out effect. Hence, given that public PAYG pension systems levy the same tax rate regardless the age of the taxpayer, familial old-age support should lead to a higher stock of capital, since the probability of having a retired parent when individuals first enter into the labor market is small. Nevertheless, as it was mentioned previously, familial old-age support also creates a greater inequality. For this reason, familial old-age support should not be implemented. Instead, the theory would suggest that payroll taxes should be age-related, so that the crowding-out effect would be reduced.

In this paper, I prove under a steady state equilibrium and using a three overlapping generations model that familial old-age support Pareto dominates public old-age support with a flat payroll tax.[2] Afterwards, in order to compare the main macroeconomic outcomes that each transfer system yields during the demographic transition, I create a general equilibrium overlapping generations model based on the Taiwanese demography. The simulation results however show that this policy cannot be implemented until the population is stable. This is because familial old-age support reduces more heavily aggregate savings when population is aging.

The remainder of the paper proceeds as follow: Section 2 introduces the general equilibrium three overlapping generations model. In Section 3 I prove that under steady state equilibrium familial old-age support Pareto dominates public old-age support with a flat payroll tax. Afterwards, based on a OLG model with realistic demography, I compare in Section 4 the evolution of savings and capital deepening over time for the two transfer systems. Section 5 concludes. An Appendix at the end of the paper includes the programming of the OLG model.

[2]A simple two overlapping generations model is not suitable for this analysis since this model does not capture the fact it is likely that young workers do not have retired parents. Specifically, if we assume that each generation differs twenty years and that the first generation are twenty years old, then there is no young worker who have a retired parent at the end of the period.

2 The Model

Time is discrete and is denoted by letter t. The population at time t consists of three adult generations: young workers (N_t^0), senior workers (N_t^1), and retirees (N_t^2). All generations face risk of mortality. Let the proportion of young workers that survive at the end of their first period of life be $p_1 \in [0,1]$ and the proportion of senior workers who survive up to retirement be $p_2 \in [0,1]$. Finally, all retirees die at the end of the period. For convenience, let p_1 be strictly greater than p_2; i.e. the probability of surviving to the next period decreases as the individual ages. The relationship between young workers at time t and young workers at time $t+1$ is given by $n > 0$, which is a measure of the net reproduction ratio. Therefore, the population evolves over time according to the dynamic equation:

$$\begin{pmatrix} N_{t+1}^0 \\ N_{t+1}^1 \\ N_{t+1}^2 \end{pmatrix} = \begin{pmatrix} n & 0 & 0 \\ p_1 & 0 & 0 \\ 0 & p_2 & 0 \end{pmatrix} \cdot \begin{pmatrix} N_t^0 \\ N_t^1 \\ N_t^2 \end{pmatrix}. \tag{1}$$

Note that age-specific rates of giving birth and dying do not change over time, which implies that for a given initial distribution, the population will be stable and will grow at a rate equal to n after a few periods. Consequently, the following relationships are satisfied: $N_t^1/N_t^0 = p_1/n$ and $N_t^2/N_t^0 = p_1 p_2/n^2$ for any $t \geq t^*$ (Preston et al., 2001). For simplicity and in order to be able to study an economy that grows steadily I assume from now on that the population is stable.[3]

2.1 Consumption/Savings' problem with old-age support

Individuals are assumed to have perfect foresight and do not have bequest motive, so that they just derive utility from their lifetime consumption. At time t every worker is endowed with one unit of labor that is supplied inelastically to a firm in exchange for a salary (w).[4] There is no annuity market and, as a consequence, individuals both face liquidity constraints (Yaari, 1965) and leave accidental bequests to their offspring (h). Let the

[3] According to Demographic textbooks and Paul A. Samuelson a stationary population is that which its population growth rate is zero, whereas a stable population is that which grows at a constant rate.

[4] The results presented in this section do not change with a more realistic income profile, and thus, for simplicity, the salary in each period is going to be the same for all workers regardless to which cohort they belong.

bequests inherited by generation i in period t be

$$h_t^i = \frac{q_{i+1}}{n} R_t a_t^{i+1} \text{ with } i = \{0, 1\}, \text{ for all } t, \tag{2}$$

where q_{i+1} is the probability that an individual of generation $i+1$ will die at the end of period, $R_t - 1$ is the rate of return to capital in period t, and a_t^{i+1} is the assets held by generation $i+1$ in period t. Hence, bequests inherited increases when parents held more assets and their probability of dying is higher, but it also decreases according to the number of siblings alive. Retired individuals receive old-age support in the form of pension benefits. Depending upon the scenario, in period t pension benefits could be financed either by her offspring or by the social security, being b^o and b^{ss} the replacement rates for familial and public old-age support respectively. In the particular case that old-age support is managed within the family, her offspring will pay a proportion (ϕ) of their salaries,

$$\phi = b^o \frac{p_2}{n}. \tag{3}$$

In contrast, if the social security system manages the pension benefits for retirees using a balanced budget, the cost will be financed through a mandatory and proportional payroll tax (τ) levied to all workers, equal to the replacement rate times the old-dependency ratio; that is,

$$\tau = b^{ss} \frac{\frac{p_1 p_2}{n^2}}{1 + \frac{p_1}{n}}. \tag{4}$$

Comparing equations (3) and (4) we can see that, if both replacement rates are the same ($b^o = b^{ss}$), senior workers will pay under familial old-age support n/p_1 times more than under public old-age support. Although it is also possible that for the same pension benefit $\phi < \tau$ if b^o/b^{ss} is lower than $\frac{p_1/n}{1+p_1/n}$. In particular, this will happen when the capital-to-labor ratio will be greater under familial old-age than under public old-age support.

Each period individuals allocate their resources (salary, benefits, assets income, and bequests) between consumption (c) and savings/investment. Individuals' preferences are assumed to be additively separable and they are represented by a utility function u that is bounded, concave, continuously differentiable, and satisfies $\lim_{c \to 0} u'(c) = \infty$ (Inada condition). Thus, the decision problem of a young individual in period t is to choose a stream of consumption $\{c_{t+i}^i\}_{i=0}^2$ and assets holding $\{a_{t+i}^i\}_{i=1}^2$,[5] such that her expected

[5]For convenience, I assume that the assets held at the end of the last period is equal to zero.

utility function is maximized:

$$V(R,\omega,\pi) = Max_{\{c^i_{t+i}\}^2_{i=0},\{a^i_{t+i}\}^2_{i=1}} \left\{u(c^0_t) + \beta p_1 u(c^1_{t+1}) + \beta^2 p_1 p_2 u(c^2_{t+2})\right\}$$
$$\text{subject to:}$$
$$a^1_{t+1} = h^0_t + w_t(1-\pi\tau) - c^0_t,$$
$$a^2_{t+2} = R_{t+1}a^1_{t+1} + h^1_{t+1} + w_{t+1}(1-\pi\tau-(1-\pi)\phi) - c^1_{t+1},$$
$$0 = R_{t+2}a^2_{t+2} + \pi b^{ss} w_{t+2} + (1-\pi)b^o w_{t+2} - c^2_{t+2},$$
$$a^0_t = 0, a^1_{t+1} \geq 0, a^2_{t+2} \geq 0, c^0_t \geq 0, c^1_{t+1} \geq 0, c^2_{t+2} \geq 0, \tag{5}$$

for all $t = \{0, 1, 2, \ldots\}$. Where $\beta \in (0,1)$ is the subjective discount factor, R_t is the real interest real in period t, w_t is the salary in period t, $\pi = \{0,1\}$ is an index function that takes the value of one when old-age support is managed by the social security and zero otherwise.

The first-order condition of the maximization problem (5) with respect to consumption is

$$u'(c^0_t) \geq R_{t+1}\beta p_1 u'(c^1_{t+1}) \geq R_{t+2}R_{t+1}\beta^2 p_1 p_2 u'(c^2_{t+2}), \tag{6}$$

with equality if the solution is an interior point.

2.2 Technology

There is only one firm that combines labor and capital to produce a single good that can be either consumed or stored. The firm produces under constant returns to scale on capital (K) and labor (L) and it is assumed that there is no technological progress, $F(K,L)$. Under this setup the output per worker in period t is $y_t = f(k_t)$, where $k_t = K_t/L_t$ is the stock of physical capital per worker, or capital deepening. The technology satisfies the usual assumptions: f is at least twice differentiable, positive, strictly concave and satisfies the Inada conditions, i.e. $\lim_{k\to 0} f'(k) = \infty$ and $\lim_{k\to\infty} f'(k) = 0$.

The firm produces under competitive conditions demanding capital and labor from households, and thus production factors are paid their marginal products:

$$R_t = 1 + f'(k_t) - \delta, \tag{7}$$

$$w_t = f(k_t) - k_t f'(k_t), \tag{8}$$

where $\delta \in [0,1]$ is the capital depreciation rate.

2.3 Steady-state equilibrium

In this subsection I characterize the steady state of the economy. I proceed to exclude time subindex.

Definition 1. *An economy under a competitive steady-state equilibrium is a stable population, a set of factor prices (R, w), and a policy function for consumption and assets holding $\{c^0(\cdot), c^1(\cdot), c^2(\cdot), a^1(\cdot), a^2(\cdot)\}$ such that*

(i) Given (R, w) the indirect utility function solves (5).

(ii) The indirect utility function is maximized.

(iii) Factor prices are competitive, so that (7) and (8) are satisfied.

(iv) Factor market clearance,

$$K_t = a^1 N_{t-1}^0 + a^2 N_{t-1}^1 \quad \text{and} \quad L_t = N_t^0 + N_t^1. \tag{9}$$

(v) The good market clears

$$K_{t+1} - (1-\delta)K_t + \sum_{i=0}^{2} c^i N_t^i = (R - 1 + \delta)K_t + wL_t. \tag{10}$$

3 Familial vs. public old-age support

The idea pursued in this section is to show, assuming a steady-state equilibrium, that a payroll tax scheme that does not impose the burden on young workers is preferable to a flat payroll tax levied to all workers. The intuition is that, in an economy with physical capital, a decrease in the average age of savings boosts capital and production per capita (Fry and Mason, 1982). Then, if young workers are exempted from paying payroll taxes, they will marginally save more and the average age of savings will decrease. As a result, the higher capital deepening will lessen the burden to taxpayers necessary to keep pension benefits constant.

The idea of suggesting a different payroll tax scheme rather than going back to an economy where familial old-age support will be more important is to avoid a higher inequality. On the one side, this is so because not all retirees will have offspring, either because their offspring will die or because individuals decide not to have children. On the other side, we cannot assure that in all families offspring will be able to support their parents. In contrast, a social insurer is preferable because it guarantees old-age support with a lower risk.

To tackle this problem, I derive the condition under which a taxation scheme that mimics familial old-age support is preferred to a flat payroll tax levied to all wokers. To do this, I first calculate the combination of replacement rates (b^{ss}, b^{o}) for which the output per worker is the same. Second, I show that if an increase in any of the replacement rates crowds out the stock of physical capital, the higher replacement rate will dominate, in the sense of Pareto, the other one. That is, if both systems with a similar replacement rate achieve the same capital per worker in the steady state, then they will be equally desirable. This is because both systems have the same age consumption profile and pension benefit. However, if the steady-state achieved is the same in both systems but with different replacement rates, the system with the highest replacement rate will be preferred, because although aggregate consumption is the same the pension benefit is higher.[6] The second step is needed in order to guarantee that there is no inflection point from which the latter relationship can change. And thus, we can find a lower replacement rate that yields a new steady-state equilibrium, where the pension benefit is the same and aggregate consumption is higher.

In order to simplify the exposition, I consider the simplest case in which the utility function is logarithmic, the economy is in a steady-state equilibrium, and the solution of the household's problem (5) is an interior point. Taking all these assumptions into account, the good market clearance condition (10) can be rewritten as

$$\sum_{i=0}^{2} c^i n^i = (R-n)k + w, \tag{11}$$

which is equivalent to

$$c^0 \frac{1 + \frac{R}{n}\beta p_1^2 + \left(\frac{R}{n}\beta\right)^2 p_1^2 p_2^2}{1 + \frac{p_1}{n}} = f(k) - (n - 1 + \delta)k; \tag{12}$$

i.e. total consumption per worker is equal to production per worker minus the break-even investment line. Where n^i is the proportion number of individual of age i relative to the total number of workers. From Equation (12) we know that the steady-state capital per worker can be derived calculating the initial consumption and that

[6] Many times the debate on the sustainability of the public pension system is centered (incorrectly) not in maintaining the same consumption level, but to maintain the same pension benefits.

Definition 2. *Given capital per worker, both familial and public old-age support systems achieve the same steady-state equilibrium when their initial consumptions are the same ($\tilde{c}^0 = \hat{c}^0$).*

Where symbols '$\tilde{}$' and '$\hat{}$' represent consumption under familial and public old-age support, respectively. Using (2)-(6) the consumption of a young worker under familial old-age support ($\tilde{c}^0$) is

$$w\frac{1+\left(\frac{1}{R}-\frac{q_1}{n}\right)+b^o\left[\left(\frac{1}{R}-\frac{q_1}{n}\right)\left(\frac{1}{R}-\frac{1}{n}\right)\right]}{1+R\beta p_1\left(\frac{1}{R}-\frac{q_1}{n}\right)+R^2\beta^2 p_1 p_2\left(\frac{1}{R}-\frac{q_1}{n}\right)\left(\frac{1}{R}-\frac{q_2}{n}\right)}, \tag{13}$$

and under public old-age support $\hat{c}^0$ is equal to

$$w\frac{1+\left(\frac{1}{R}-\frac{q_1}{n}\right)+b^{ss}\left[\left(\frac{1}{R}-\frac{q_1}{n}\right)\left(\frac{1}{R}-\frac{q_2}{n}\right)-\frac{p_1p_2}{n(n+p_1)}\left(1+\left(\frac{1}{R}-\frac{q_1}{n}\right)\right)\right]}{1+R\beta p_1\left(\frac{1}{R}-\frac{q_1}{n}\right)+R^2\beta^2 p_1 p_2\left(\frac{1}{R}-\frac{q_1}{n}\right)\left(\frac{1}{R}-\frac{q_2}{n}\right)}. \tag{14}$$

In words, consumption is equal to the sum of human and transfer wealth times the propensity to consume.[7] Therefore the difference between equations (13) and (14) depends upon the size of transfer wealth. And thus, Definition 2 is satisfied when both transfer wealths are equal

$$\begin{aligned} &b^o\left(\frac{1}{R}-\frac{q_1}{n}\right)\left(\frac{1}{R}-\frac{1}{n}\right) \\ &= b^{ss}\left[\left(\frac{1}{R}-\frac{q_1}{n}\right)\left(\frac{1}{R}-\frac{q_2}{n}\right)-\frac{p_1p_2}{n(n+p_1)}\left(1+\left(\frac{1}{R}-\frac{q_1}{n}\right)\right)\right]. \end{aligned} \tag{15}$$

We can derived from Equation (15) the set of replacement rates (b^{ss}, b^o) that yields the same production per worker (isoquant). Before, it is important to remember that we are in a decentralized economy with capital and that this economy is populated by selfish individuals. Hence, the stock of capital will be below the golden rule stock of capital, which implies that $R > n$.[8] In addition, because individuals do not oversave, it is necessary that $R < \frac{n}{q_1}$ in order to satisfy Equation (15); otherwise the equality does not hold. After some algebra we have that:

Proposition 1. *Given the set (n, p_1) of population characteristics, the familial old-age support replacement rate (b^o) is greater than the public old-age support replacement rate (b^{ss}) when $R < \frac{n}{q_1}$.*

[7] In this particular case, human wealth is defined as the present value of the stream of salaries, while transfer wealth is the present value of the benefits received minus the money transferred to support old people.

[8] In a decentralized economy, real interest rates can also be lower or equal than the population growth rate if there is no capital and there is fiat money (Samuelson, 1958), or parents transfer substantial amounts of money to their offspring (Fuster, 2000).

Proposition 1 shows that when $R < \frac{n}{q_1}$ the pension benefit is greater when the burden is imposed just on senior workers than when it is imposed on all workers. However, this does not mean that welfare is greater under any particular system. Indeed, this circumstance only happens if a marginal increase in each replacement rates lead to a new steady state where aggregate consumption and the stock of capital per worker is lower. In other words, an increase in each replacement rate causes a crowding-out effect. Differentiating with respect to the replacement rate in equations (13) and (14) we have that both partial derivatives are negative ($\partial\tilde{c}^0/\partial b^o, \partial\hat{c}^0/\partial b^{ss} < 0$). But although the increase in the replacement rate reduces consumption at age 0, aggregate consumption can either increase or decrease depending on the interest rate of the new steady-state equilibrium, see Equation (12). I will prove by contradiction that when $R \in (n, \frac{n}{q_1})$ an increase in both replacement rates causes a crowding-out effect. Thus, if an increase in b^o yields a new steady state with a higher capital per worker then, on the one hand, the interest rate will be lower ($f''(k) < 0$), reducing even more the lef-hand side of Equation (12). On the other hand, how the initial steady state was in an efficient region, a higher capital per worker will increase the right-hand side of (12). As a result, we cannot find a new equilibrium in which the new steady state has a lower capital per worker. A similar proof can be done for an increase in b^{ss}. Thus,

Proposition 2. *Given the set (n, p_1) of population characteristics. If an economy is in a steady-state equilibrium with a real interest rate $R \in (n, \frac{n}{q_1})$ then, an increase in both familial and public old-age support replacement rates (b^o, b^{ss}) will cause a crowding-out effect.*

Finally, let consider an initial steady-state equilibrium where capital per capita is k_1^* and its associated factor prices are (R_1^*, w_1^*) with $R_1^* > n$. From Proposition 1 we have that $b_1^o w_1^* > b_1^{ss} w_1^*$. Using Proposition 2 we know that we can find a new steady-state equilibrium $k_2^* > k_1^*$ such that $b_2^o < b_1^o$ and the pension benefit equals the initial public pension benefit, $b_2^o w_2^* = b_1^{ss} w_1^*$.

4 Taxes and Capital Deepening with a N age groups model

In the previous section I studied the economic consequences of two old-age support systems using a simple three overlapping generations model, where the population was stable and the economy was in a steady-state equilibrium. The concern about the sustainability of the social security system however arises because future retirees, who belong to the baby-boom generation, will be supported by their children, who belong to the baby-bust generation, which is a characteristic of the last stage of the demographic transition. Therefore, both old-age support systems should also be tested in a population that is not stable. Moreover, although three age groups models can support steady states that are impossible in two age groups models. In the same fashion, a N age groups model gives more realistic results and support a wider range of consumption/savings profiles. Unfortunately, the drawback of introducing more realism is that solutions are more complex and, usually, cannot be expressed, as in the last section, in simple terms.

In this section in order to introduce more realism and compare both old-age support systems and its capital deepening during the demographic transition I extend the model in several ways. Now, the optimal allocation of resources across the life cycle is modeled using an extended version of the life cycle theory of saving proposed by Lee et al. (2001) and used to compute a general equilibrium model in Sánchez-Romero (2009). The novelties of this model with respect to the three age groups presented in the previous section are: i) individuals can live for a maximum of 101 periods (Ω); ii) individuals start working at age 21 (T_w) and retire at age 65 (T_r); iii) the salary earned by an individual of age x in year t, denoted by y_t^x, depends on an age-specific labor productivity index (ε_x) and on the salary in units of effective labor in year t, or w_t; iv) parents finance the consumption of their children (childrearing cost) up to they are 21 years old;[9] and v) preferences are represented by a time-separable constant relative risk aversion (CRRA) utility function. Then, the new optimal consumption along the life-cycle for the cohort born in year t is given by the following maximization problem:

$$\max_{\{c_{t+x}^x \geq 0, a_{t+x}^x \geq 0\}_{x=T_w}^{\Omega-1}} E\left[\sum_{x=T_w}^{\Omega-1} \beta^{x-T_w} \lambda_{t+x}^x u(c_{t+x}^x)\right], \tag{16}$$

subject to an age-dependent flow budget constraint, which is equal to

$$a_{t+x+1}^{x+1} = R_{t+x} a_{t+x}^x + h_{t+x}^x + (1 - \pi\tau_{t+x} - (1-\pi)\phi_{t+x}^x) y_{t+x}^x - \lambda_{t+x}^x c_{t+x}^x, \tag{17}$$

when the individual is in the labor market, and

$$a_{t+x+1}^{x+1} = R_{t+x} a_{t+x}^x + h_{t+x}^x + \pi b^{ss} E y_{t+x} + (1-\pi) b^o \bar{y}_{t+x}^x - \lambda_{t+x}^x c_{t+x}^x, \tag{18}$$

when the individual is retired. Where λ_{t+x}^x is the number of equivalent adult consumers within a household whose head is x years old in year $t+x$, λc

[9] As a consequence, the individual's allocation problem is transformed into a household's problem.

is the household consumption, Ey_{t+x} is the average labor income in year $t+x$, and $\bar{y}$ is the average labor income of the offspring. Note that when π equals zero the pension benefit received depends directly on the average age of the offspring, so that the pension benefit will vary across ages. This is important because as time goes by the probability that individuals will survive to age 80 will be greater, to the extent that very old people could receive less under familial old-age support.

The population used to model the demographic transition is a variation of the Taiwanese population, which from now on I will name pseudo-Taiwan. Taiwan is of special interest since we have access to reliable data from before its demographic transition started. Pseudo-Taiwan differs from the actual Taiwanese population in that the former is a closed population. Similar population projections of Taiwan were previously done by Lee et al. (2000), Lee et al. (2001), Lee et al. (2003) to study the demographic transition and its macroeconomic consequences assuming an open economy.[10] Nevertheless, here I will use pseudo-Taiwan as an exogenous factor that drives production prices in a closed economy.

4.1 Old-age support during the demographic transition

During the demographic transition the dependency ratio (number of people aged 0 to 15 and over 65 divided by the number of people aged 16 to 64) first increases, because mortality increases but fertility remains high, second decreases and finally increases again, because people live longer and fertility is lower. This implies that, holding the replacement rate constant, old-age support will rise with population aging. This will be reflected in continuous increases in the payroll tax, if the old-age support is public, or in the proportion that working family members will pay to their retired parents, if the old-age support is handled within the family. The difference between both systems is that workers aged 21 to 30 do not pay much, while workers between age 40 and age 60 face most of the burden. Looking at Figure 1, the former age group will pay less under familial old-age support than under public old-age support. But the opposite result is also true for the latter age group. Another important characteristic is that familial old-age support is very sensitive to changes in the age structure, which ultimately might imply that an age-related payroll tax is probably not the most efficient tax when the demographic transition is very fast.

[10]Pseudo-Taiwan was calculated using demographic data from Lee et al. (2000, 2001, 2003), Human Mortality Database, and the Statistical Yearbook of the Republic of China 1975, 1989, 1995, 2005, 2008.

[Figure 1 about here.]

From Section 3 we should expect that if both replacement rates are equal, capital deepening and pension benefits will be higher under familial old-age support. However this is only guarantee when the economy is in a steady-state equilibrium. Figures 2 and 3 show that while in the steady-state pension benefits and capital deepening are higher under familial old-age support, it is not so during the demographic transition. After 2025 simulations show that the average pension benefit received from familial old-age support decreases until 2080, whereas public pension benefits increases during this period. And from 2075 to 2085 public pension benefits are higher.

[Figure 2 about here.]

There are two reasons for this pattern during the demographic transition. First, neither familial old-age support nor public old-age support cause a strong crowding-out effect at the beginning of the demographic transition. Therefore, both systems begin with a very similar capital deepening before population aging starts. Second, the average age of savings not only depends on the age-savings profile but also on the mean age of the population. When population aging starts and the old-dependency ratio rises, the average age of savings also increases regardless the system that is implemented (at least this is the case in pseudo-Taiwan because of its rapid demographic transition). Hence, how the cost of supporting old people is higher for senior workers in the familial old-age support than in the public old-age support, in the former system aggregate savings are smaller during population aging; and thus capital deepening growth is lower under familial old-age support, see Figure 3. Nonetheless, the policy studied in this paper is to substitute the current flat payroll tax with an aged-related payroll tax, holding pension benefits constant (in real terms). Consequently, the difference between the benefits depicted in Figure 2 will be similar to those in Figure 3.

[Figure 3 about here.]

Finally, the theory suggests that whenever $R \in (n, \frac{n}{q_1})$, we can find a b^o lower than b^{ss} that guarantees both the same pension benefit than that of the public pension system and a higher aggregate consumption. Unfortunately, during the demographic transition this result does not hold. Figure 3 shows that in pseudo-Taiwan familial old-age support leads to a lower capital deepening until 2040. But after 2040 an implementation of this policy could boost capital deepening.

5 Conclusions and future research

The life cycle theory of savings suggest that if the average age of savings is tilted toward younger ages, the rate of savings will increase. However, the ongoing population aging process coupled with generous public transfers to old people are expected to reduce aggregate savings.

A taxation scheme that would mimic familial old-age support might boost savings because it tilts the average age of savings toward younger ages. This is because familial old-age support levies taxes more heavily on senior workers, while younger workers are likely to be exempted from paying taxes. Nevertheless, although this policy works under a stable population and with the economy in steady-state, using a computable general equilibrium model I show that this taxation scheme is not convenient when population is aging.

Nonetheless, alternatives ideas and models analyzing private transfers are worth developing. Thus, the next step would be to extend the theoretical result to a N age groups model. Second, it is also worth studying the average age of savings and its characteristics depending upon how fast the population ages.

6 Appendix

6.1 Calibration

In the model economy the value of γ is set to 0.60, which is the average value estimated for Taiwan by Reinhart et al. (1996). Usual values for this parameter range between 0.5 and 1, so that γ is within the range frequently used in the literature. Age-specific labor productivity indexes are calculated using data from the National Transfers Account Database. The subjective discount factor is set at 0.99. With the National Accounts of the Republic of China from CEIC Global Database, I calculated the stock of capital for Taiwan by using the time series of "Gross Fixed Capital Formation" (GFCF) and the "Consumption of Fixed Capital" from 1953 to 2004. I discarded the information from 1953 to 1960. I obtained an average the depreciation of capital (δ) from 1975 to 2003 of 3.25%. Based on a Cobb-Douglas production function with constant returns to scale and under the assumption that the economy is closed, the value of capital share ($\hat{\alpha} = 0.32$) was estimated to be the average value from 1951 to 2004 as follows:

$$\hat{\alpha} = \frac{1}{T} \sum_{t=1}^{T} \frac{\mathrm{O.S}_t - \mathrm{E.I}_t}{\mathrm{N.I}_t - \mathrm{E.I}_t - \mathrm{N.I.T}_t}, \tag{19}$$

where O.S. is "Operating Surplus", E.I. is "Entrepreneurial Income", N.I. is "National Income", and N.I.T. is "Net Indirect Taxes".

Table 1: Parameters in the OLG Model

Parameters	values
β: subjective time discount factor	0.99
$\frac{1}{\gamma}$: relative risk aversion	1.67
δ: capital depreciation rate	3.25%
α: capital share	0.32
T_w: age upon first entry into the labor market	21
T_r: age of retirement	65
Ω: maximum longevity	100

6.2 Computational Details

To simplify notation, I remove the time indexes when they are not absolutely necessary, I denote the next period with "$'$", and I use the notation $\mathbf{R}_+$ to represent $\mathbf{R}^+ \cup \{0\}$.

6.2.1 Household Problem

The aim of the head of the household of age $x \in \mathcal{X} = \{T_w, \ldots, \Omega\}$ in year $t \in \mathcal{T} = \{t_0, t_0+1, \ldots, T\}$ is to maximize her expected utility by choosing the optimal consumption and assets in period $t+1$. The Bellman equation for the head of the household reads as

$$v(t, x, a) = \max_{c, a'} \left\{ \lambda_t^x u(c) + \beta p_t^x v(t+1, x+1, a') \right\} \tag{20}$$

subject to

$$a' = \begin{cases} (1+r)a + h + (1 - \pi\tau - (1-\pi)\phi_t^x) w\epsilon_x - \lambda_t^x c & T_w \le x < T_r \\ (1+r)a + h + \pi b^{ss} w E[\epsilon_x] + (1-\pi) b^o w \tilde{\epsilon}_x - \lambda_t^x c & T_r \le x < \Omega \end{cases} \tag{21}$$

$$c, a \ge 0, \text{ with } a_{\cdot, T_w} = a_{\cdot, \Omega} = 0.$$

where $\beta \in (0,1)$ is the subjective discount factor, λ_t^x is the number of equivalent adult consumers within a household whose head is x years old in year t, $p_t^x \in [0,1)$ is the probability of surviving to age $x+1$ in year $t+1$ conditional on being alive at age x in year t, h is the unintentional bequest received, τ is the proportion of the salary spent to support her elderly parent under public old-age support, ϕ is the proportion of the salary spent to support her elderly parent under familial old-age support, ϵ is the age-specific labor productivity index, (b^o, b^{ss}) are the replacement rates for familial and public old-age support respectively, π is an index that takes the value of one when old-age support is managed by the social security and zero otherwise, and a denotes asset holdings.

I define $G(a, a'|I)$ as the function of total amount of consumption goods obtainable for any combination of assets held at any t and $t+1$, given the information I at t, which depends upon the age of the individual and other time dependent variables $\{r, w, \lambda, \epsilon, \tilde{\epsilon}, h, \tau, \phi, \pi, b^o, b^{ss}\} \in \mathbf{I}$, then $\mathbf{I} \subset \mathcal{X} \times \mathbf{R}^{+4} \times \mathbf{R}^5_+$. That is,

$$c = G(a, a'|I) = \begin{cases} \frac{1}{\lambda}\left((1+r)a - a' + h + (1 - \pi\tau - (1-\pi)\phi_t^x)w\epsilon_x\right) & T_w \leq x < T_r \\ \frac{1}{\lambda}\left((1+r)a - a' + h + \pi b^{ss} w E[\epsilon_x] + (1-\pi)b^o w\tilde{\epsilon}_x\right) & T_r \leq x < \Omega \end{cases}. \tag{22}$$

Let us define the set $C \subset \mathbf{R}_+ \times \mathbf{R}_+$ as the region of pairs $(a, a') \in \mathbf{R}^2_+$ where consumption is nonnegative; that is, $C = \{(a, a') \in \mathbf{R}^2_+ : G(a, a'|I) \geq 0, \text{ for any given } I\}$. It is easy to prove that C is a convex set. Now, using (22) let us rewrite the Bellman equation as

$$v(t, x, a) = \max_{a'} \left\{\lambda_t^x u(G(a, a'|I)) + \beta p_t^x v(t+1, x+1, a')\right\}. \tag{23}$$

The algorithm operates on (23) for all individuals in each year up to the model converges. The algorithm involves the following steps:

1. Define a time-independent grid for assets, with $||a_{i+1} - a_i||$ sufficiently small,
$$G^a = \{a_1 = 0, a_2, a_3, \ldots, a_n\}, \tag{24}$$
where a_n is the maximum realization of assets weighted by units of effective labor.

2. Define the correspondence $f : G^a \to G^a$ of optimal combinations of assets in t and $t+1$ at age x.
$$f(a_k|I) = a_j^* = \arg\max_{a_j} \left\{\lambda_t^x u(G(a_k, a_j|I)) + \beta p_t^x v(t+1, x+1, a_j)\right\}, \tag{25}$$
where $I \in \mathbf{I}$ and $A' \in \mathbf{R}^+$ is the productivity in year $t+1$. Note that f has a one-to-one correspondence given that C is a convex set and $u(\cdot)$ is strictly concave.

3. Calculate the set $\{(a_k, f(a_k|I))\}_{k=1}^n \in C^{2n}$ of all possible optimal asset pairs for the household head in year t. Evaluate Equation (23) by introducing all optimal asset pairs. Repeat this process for the entire life cycle of the head of the household.

4. Repeat step 3 for all households.

5. Given the initial boundary conditions, we know that wealth at the beginning of adulthood is zero, or $a_{t,T_w} = a_1 \in G^a$, $\forall t \in \mathcal{T}$. Therefore, given all information sets over the life cycle of the head of the household and Equation (25), I iterate forward on age and time to get the optimal path of asset holdings.

6. Repeat step 5 for all individuals.

6.2.2 Aggregate Model

In this model the equilibrium price vector is numerically obtained using the Gauss-Seidel algorithm, see Auerbach and Kotlikoff (1987) and Börsch-Supan et al. (2006) among others.

The simulation strategy was to calculate the demand and supply of capital at all times for a given vector of interest rates $\{r_t^i\}_{t=0}^T$, with T sufficiently large and i denoting the i-th iteration, such that there is no excess of demand of capital at any time. The information set prior to the simulation is a vector of time-invariant parameters and demographic characteristics for $t \in \mathcal{T}$. In order to guarantee the existence of an equilibrium, the phase-in (out) period begins (finishes) with a stable population 200 years before (after) the period being analyzed, so that the economy before and after the demographic transition is in a steady-state equilibrium. The algorithm is divided into the following seven steps:

1. Choose a dumping factor of $\xi = 0.05$ and a tolerance ϵ equal to 0.02.

2. Make an initial guess $\{R_t^0\}_{t=0}^T$, where R_t^i is equal to $r_t^i + \delta$ for all $i \in \mathbf{N}$, in which the initial and final steady-state interest rates are included.

3. Given the initial guess, use a Cobb-Douglas production function to calculate its associated salary over time in units of effective labor

$$w_t^i = (1-\alpha)\left(\frac{R_t^i}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}, \forall t \in \mathcal{T}. \tag{26}$$

4. Compute the household problem and aggregate assets across all household heads to determine the capital stock per units of effective labor

$$\kappa_t = \frac{\sum_{x=T_w}^{\Omega-1} a_t^x N_t^x}{\sum_{x=T_w}^{T_r-1} h_x N_t^x}, \forall t \in \mathcal{T}, \tag{27}$$

5. Next, determine the marginal product of capital resulting from (27), that is
$$r_t^n + \delta = \alpha \kappa_t^{\alpha-1}, \forall t \in \mathcal{T}. \tag{28}$$

6. If $||\mathbf{r}^i - \mathbf{r}^n|| < \epsilon$ then STOP.

7. Else, compute a new vector of interest rates and salaries
$$\begin{aligned} r_t^{i+1} &= (1-\xi) r_t^i + \xi r_t^n, & (29) \\ w_t^{i+1} &= (1-\alpha) \left(\frac{r_t^{i+1} + \delta}{\alpha} \right)^{\frac{\alpha}{\alpha-1}}, \forall t \in \mathcal{T}. & (30) \end{aligned}$$

8. Calculate the unexpected bequest
$$h_t^x = (1+r_t^{i+1}) \sum_{s=T_w}^{x} \frac{N_{t-x}^s \hat{f}_{t-x}^s f_{fab}}{N_{t-x}^0} \frac{l_t^{s+x} d_t^{s+x}}{o_t^{s+x}} a_t^{s+x} + (1+r_t^{i+1}) \frac{q_t^x}{p_t^x} a_t^x I_{x<2T_w}, \tag{31}$$
and old-age support
$$\phi_t^x = (1-\pi) b^o \sum_{s=T_w}^{\Omega-1-x} \frac{N_{t-x}^s \hat{f}_{t-x}^s}{N_t^0} \frac{l_t^{s+x}}{o_t^{s+x}} I_{s+x \geq T_r}, \tag{32}$$
$$\tau_t = \pi b^{ss} \frac{\sum_{x=T_r}^{\Omega-1} N_t^x}{\sum_{x=T_w}^{T_r-1} N_t^x}, \tag{33}$$
for all $x \in \mathcal{X} = \{T_w, \ldots, \Omega\}$ and $t \in \mathcal{T} = \{t_0, t_0+1, \ldots, T\}$.
Then go to step 4.

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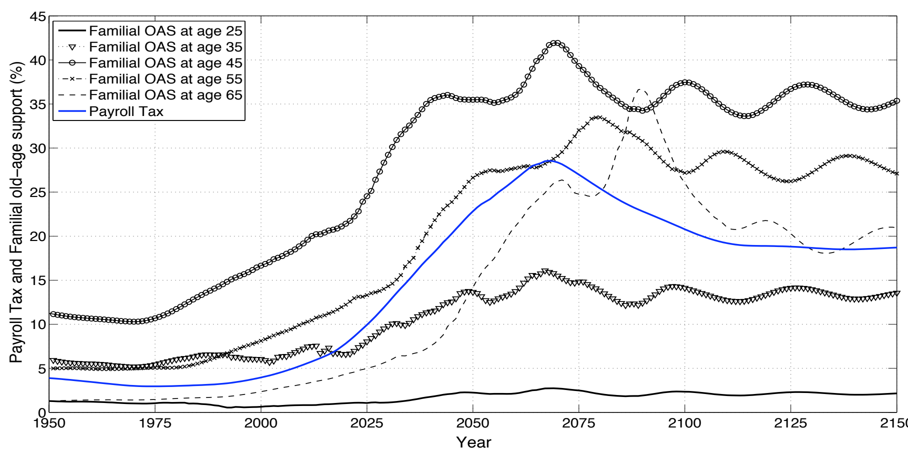

Figure 1: Pseudo-Taiwan. Estimated Payroll Taxes and Familial Support Ratios by Age from 1950 to 2150.

NOTES: Payroll taxes estimated based on a replacement rate of 40% in both systems.

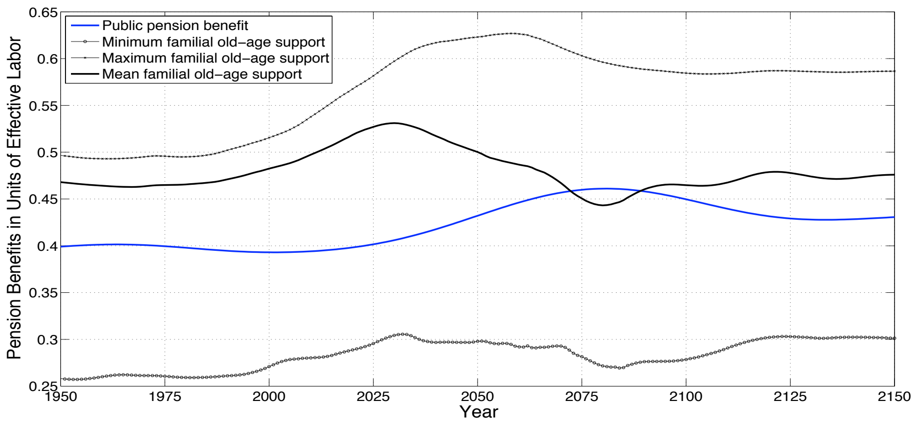

Figure 2: Pseudo-Taiwan. Estimated pension benefits from 1950 to 2150.

NOTES: Pension benefits are estimated based on a replacement rate of 40% in both systems. In the familial old-age support the amount of money received from offspring depends on their average labor income and age. Maximum and minimum familial old-age supports are respectively the highest and the lowest pension benefit paid in that year.

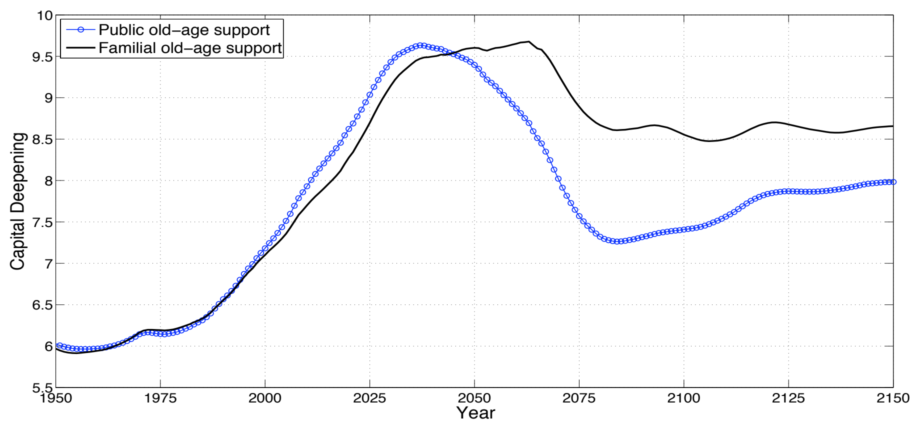

Figure 3: Pseudo-Taiwan. Estimated capital-to-labor ratio from 1950 to 2150.

NOTES: Capital-to-labor ratio estimated based on a replacement rate of 40% in both systems.