Nonlinear Phillips curve, NAIRU and monetary policy rules

Hyeon-seung Huh
Department of Economics
Yonsei University
134 Sinchon-dong Seodaemun-Gu
Seoul 120-749
South Korea
Tel: 82-2-2123-5499 Fax:82-2-393-1158
Email: hshuh@yonsei.ac.kr

Abstract

The U.S. Phillips curve is modeled with an LSTAR specification, which is flexible to allow various nonlinear shapes. Using this model, we present a method to derive model-consistent estimates of the NAIRU. An additional feature is that the NAIRU is defined as a leading indicator of inflation changes over the policy horizon. We then investigate the implications of this nonlinear Phillips curve for the derivation of optimal monetary policy rules. The optimal policy rule is proven to be nonlinear too. Empirical results show that this nonlinear rule, together with the model-consistent NAIRU measure, may offer some assistance in understanding the conduct of U.S. monetary policy.

Key words: Phillips curve; Nonlinearity; LSTAR; NAIRU; Monetary policy rule
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1. Introduction

Growing evidence suggests nonlinearities in the U.S. Phillips curve. This has important implications for the NAIRU (Non-Accelerating Inflation Rate of Unemployment), which is typically estimated in the context of the linear Phillips curve. For example, Debelle and Laxton (1997) and Laxton *et al*. (1999) show that the conventional NAIRU estimates can be seriously biased and are not appropriate for use with the nonlinear Phillips curve. They estimate the NAIRU under the alternative assumption that the Phillips curve has a hyperbolic shape, to impose convexity. However, there is no consensus in the literature about the precise nonlinear form of the Phillips curve. Eisner (1997) and Stiglitz (1997) present counter-evidence by concluding that the U.S. data are consistent with a concave Phillips curve. See Dupasquier and Ricketts (1998) for a survey of the microfoundations underlying the nonlinear Phillips curve.

In this paper, we construct a nonlinear Phillips curve and present a method to derive model-consistent estimates of the NAIRU. The nonlinearity is accommodated through a logistic smooth transition autoregression (LSTAR) specification. The LSTAR model is flexible to allow various nonlinear Phillips curve shapes and does not require assuming one specific form or another *a priori*. It also allows shapes that are convex in one region and concave in another region (i.e. kinked curve). This feature receives support from the suggestion of Dupasquier and Ricketts (1998) and Filardo (1998) that a model nesting more than one type of nonlinearity may be needed to fit the Phillips curve better. Further, the transition between the regimes is carried out smoothly, which is coherent with the fact that the slow adjustments and inertia in inflation and consumers' expectations are the main reasons for the tradeoff between inflation and the unemployment rate.[1]

[1] Inflation tends to move slowly over time generating a great deal of persistence and inertia. Consumers' expectations may also adjust slowly over time, perhaps being based on some sort of adaptive mechanism. Because decisions about wages and prices depend on expectations of future changes, slow adaptation is self-fulfilling, creating inertia.

Equally importantly, the NAIRU here is defined as the level of unemployment rate that would correspond to a *forecast* of no inflation change over the policy horizon. The same notion was used previously by Estrella and Mishkin (1999) in a linear setting. They differentiate this measure of the NAIRU from conventional ones like the natural rate of unemployment. In particular, they highlight its enhanced predictability for future changes in inflation. This property should be valuable as monetary policy actions must, necessarily, be preemptive, due to its transmission lags on the economy. Accurate inflation forecasts will help policymakers achieve targets without the need to generate large cyclical fluctuations down the track. As will be discussed, however, our actual implementation differs from the one by Estrella and Mishkin that is based on a multi-horizon prediction regression with the future change in inflation being the dependent variable.[2] The benefit is that, unlike their procedure, the resulting NAIRU estimates are readily available for policymakers faced with real-time policy decisions.

Nonlinearities in the Phillips curve can also lead to different implications for the conduct of monetary policy. To shed some light, we then derive an optimal monetary policy rule when the nonlinearity is characterized by an LSTAR specification. Optimal policy rules are linear under the typical assumptions of a quadratic objective function and a linear Phillips curve (i.e. Taylor, 1993; Svensson, 1997). However, Nobay and Peel (2000) and Corrado and Holly (2003) demonstrate that if the Phillips curve exhibit nonlinearities this will impart a bias to inflation when a linear rule is used. Several studies conclude that, in such cases, the optimal policy rule becomes also nonlinear: Shaling (1999), Dolado *et al*. (2003), Surico (2003), and Dolado *et al*. (2004), *inter alia*. We show, in particular, that the elasticities of the interest rate with respect to the inflation and NAIRU gaps are state-contingent, depending on the phase of the economy. This implies that inferences based on the linear rule may provide

[2] Their empirical application, for example, requires inflation data two years ahead in order to construct the current NAIRU.

misleading signals about the appropriate policy stance.

The remainder of this paper is organized as follows. Section 2 presents a method to estimate the NAIRU when the Phillips curve is augmented with the LSTAR specification. Its empirical application to the U.S. data is given in Section 3. Section 4 studies implications of this nonlinear Phillips curve for optimal monetary policy rules. Some empirical evidence is offered to demonstrate the practical relevancy with data. Section 5 concludes the paper. The appendix contains detailed derivations of the nonlinear monetary policy rule.

2. Nonlinear Phillips curve and NAIRU

Consider a standard Phillips curve augmented with an LSTAR component such as:

$$\Delta\pi_t = \sum_{i=1}^{p} \theta_i \Delta\pi_{t-i} + \sum_{j=1}^{q} \eta_i \Delta u_{t-j} + [\sum_{i=1}^{p} \theta_i^* \Delta\pi_{t-i} + \sum_{j=1}^{q} \eta_j^* \Delta u_{t-j}]\, F(z_{t-k}) + \varepsilon_t \qquad (1)$$

where Δ is the first difference operator; $\pi_t = 400\ln(CPI_t \,/\, CPI_{t-1})$ is quarterly inflation at an annual rate; u_t is the unemployment rate; and ε_t is a disturbance term. The logistic function $F(z_{t-k})$ is assumed to have the following form:

$$F(z_{t-k}) = (1 + \exp\{-\lambda(z_{t-k} - c)/\sigma_z\})^{-1} \qquad (2)$$

where $F(z_{t-k})$ lies in the range between 0 and 1, and λ>0. The variable z_{t-k} is a switching indicator that represents the state of the economy, and the parameter *c* represents the threshold around which the dynamics of the model change. The parameter λ is the smoothness parameter measuring how rapidly the transition between the regimes is processed.[3] The parameter σ_z is the standard deviation of the switching variable z_{t-k}. The smoothness parameter λ is not scale-free as its value depends on the magnitude of the switching variable z_{t-k}. Dividing by σ_z normalizes the deviations of z_{t-k} from the

[3] If λ approaches zero, $F(z_{t-k})$ converges to a constant, and the model becomes linear. If λ approaches infinity, the model becomes a threshold autoregression model.

threshold value and facilitates interpretation of the smoothness parameter. This also makes it easier to find sensible starting values in initiating the optimization process for estimation. See Granger and Teräsvirta (1993) for a detailed exposition of the LSTAR model.

The linear version of Eq. (1) is frequently adopted in the previous studies, for example, King and Watson (1994), King *et al*. (1995), and Staiger *et al*. (1997). Its single-equation specification assumes implicitly that Δu is exogenous to the process determining $\Delta\pi$. King and Watson and King *et al*. justify this assumption with empirical evidence that the U.S. unemployment rate is determined largely independent of the movement in inflation at all horizons.[4] However, there is an additional requirement when the Phillips curve is nonlinear with the LSTAR component. That is, $\Delta\pi$ should not act as the switching variable. Empirical results in the subsequent section show that, as the switching variable, Δu_{t-2} is most preferred while the evidence for any lagged values of $\Delta\pi$ is weak at best. In this context, we assume that the exogeneity of Δu remains appropriate for the present application. An implication is that Eq. (1) may be treated as 'structural', summarizing the behavioral interactions between Δu and $\Delta\pi$.

When Eq. (1) is structural, one will be able to conduct an experiment to analyze the effects on $\Delta\pi$ of policy interventions that alter Δu. Previously, Evan (1989) suggested a way of implementing a policy experiment to estimate U.S. potential GDP from a linear Okun's equation. We apply a similar strategy to the nonlinear Phillips curve. Through this policy simulation, the NAIRU is defined as the level of unemployment rate that would correspond to a forecast of no inflation change over the policy horizon. To see how, suppose that the sequence of unemployment changes over the future can be set precisely to chosen levels through macroeconomic policy. Eq. (1) can then be inverted to generate, at any time T

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[4] The exogeneity of unemployment also has some theoretical grounds. For example, it is embodied in the typical Keynesian model which assumes that the unemployment rate is essentially an indicator of demand. Real business cycle models, at the other extreme, also agree on this condition, but with the different interpretation that movements in real variables such as the unemployment rate are unaffected by nominal variables such as

with current and past $\Delta\pi$ and Δu given, the sequence of changes in the unemployment rate $\Delta u_t = GU_{T,t}$ for $t \geq T+1$ which, if chosen by policymakers, would generate $\Delta\pi_t$=0 for $t \geq T+2$ in the absence of random disturbances.

For ease of illustration, assume that p=q=1 and that the policy horizon for inflation is from 1 to (*S* +1) quarters ahead. Note that we replace z_{t-k} in Eq. (2) with Δu_{t-2} following the discussion above. At a given time T, the optimal point forecast of $\Delta\pi_{T+s}$ for s=1, $\cdots$, *S*+1 can be expressed as:

$$\begin{array}{lcl}
\Delta\hat{\pi}_{T+1} & = & \hat{\theta}_1\Delta\pi_T + \hat{\eta}_1\Delta u_T + [\hat{\theta}_1^*\Delta\pi_T + \hat{\eta}_1^*\Delta u_T]\,\hat{F}(\Delta u_{T-1}) \\
\Delta\hat{\pi}_{T+2} & = & \hat{\theta}_1\Delta\pi_{T+1} + \hat{\eta}_1\Delta u_{T+1} + [\hat{\theta}_1^*\Delta\pi_{T+1} + \hat{\eta}_1^*\Delta u_{T+1}]\,\hat{F}(\Delta u_T) \\
\Delta\hat{\pi}_{T+3} & = & \hat{\theta}_1\Delta\pi_{T+2} + \hat{\eta}_1\Delta u_{T+2} + [\hat{\theta}_1^*\Delta\pi_{T+2} + \hat{\eta}_1^*\Delta u_{T+2}]\,\hat{F}(\Delta u_{T+1}) \\
\vdots & \vdots & \qquad\qquad\qquad\vdots \\
\Delta\hat{\pi}_{T+S+1} & = & \hat{\theta}_1\Delta\pi_{T+S} + \hat{\eta}_1\Delta u_{T+S} + [\hat{\theta}_1^*\Delta\pi_{T+S} + \hat{\eta}_1^*\Delta u_{T+S}]\,\hat{F}(\Delta u_{T+S-1})
\end{array} \quad (3)$$

The optimal one-step ahead forecast of $\Delta\pi_{T+1}$, $\Delta\hat{\pi}_{T+1}$ is easily obtained given the information available at time T. Turn to the two-step forecast $\Delta\hat{\pi}_{T+2}$. $GU_{T,T+1}$ can be calculated by solving for Δu_{T+1} (that is, $GU_{T,T+1}=\Delta u_{T+1}$) after setting $\Delta\hat{\pi}_{T+2}$=0 and substituting $\Delta\pi_{T+1}$ with $\Delta\hat{\pi}_{T+1}$. Next, $GU_{T,T+2}$ is obtained from the three-step forecast $\Delta\hat{\pi}_{T+3}$ by solving for Δu_{T+2} with $\Delta\hat{\pi}_{T+3}$=0, $\Delta\pi_{T+2}$=0, and $\Delta u_{T+1}=GU_{T,T+1}$. In the same manner, $GU_{T,t}$ can be generated for t=T+3, T+4, $\cdots$, T+*S*.

A natural definition of the NAIRU gap in time T is given by

$$\tilde{u}_T = \sum_{t=T+1}^{S} [E_0(\Delta u_t) - GU_{T,t}] \quad (4)$$

where $E_0(\Delta u_t)$ is the steady-state value of Δu_t at $\Delta\pi$=0.[5] The values in $\tilde{u}_T$ are simply excess unemployment changes above a policy-engineered target of $\Delta\pi_t$=0 for

inflation.

[5] Clarida and Taylor (2003) applied a similar variant of the Beveridge and Nelson decomposition to other nonlinear models.

$T+2 \le t \le S+1$. The NAIRU at time T is then $u_T^N = u_T - \tilde{u}_T$, as this level results in neither accelerations nor decelerations of inflation over the policy horizon. When $\tilde{u}_T < 0$, Eq. (4) indicates that the unemployment rate needs to go higher to achieve a target of $\Delta\pi=0$ over the policy horizon. This implies that its current level is lower than the NAIRU with inflationary pressures mounting. When $\tilde{u}_T > 0$, a lower level of unemployment can be tolerated in pursuing a target of $\Delta\pi=0$ over the policy horizon. That is, the rate is currently higher than the NAIRU pointing to no signs of inflation.

3. Estimates of the U.S. NAIRU

The analysis outlined above is applied to quarterly U.S. data for the sample period 1954:Q1-2001:Q4. Definitions of the data series are as follows. The rate of inflation (π_t) is measured as the quarterly percentage change in the CPI all-items price index for all urban consumers at an annual rate. The civilian unemployment rate for 16 years and older is chosen for the variable (u_t). Both series were taken from FRED at the Federal Reserve Bank of St. Louis web site. They were provided monthly and averaged to quarterly values. Results of the augmented Dickey-Fuller test indicated that π_t and u_t are characterized as *I*(1) processes, consistent with the previous findings. These variables enter the regression in their first differences accordingly.

For testing the presence of LSTAR components, we apply the LM-type linearity test as described in Granger and Teräsvirta (1993). The procedure involves testing $H_0: \omega_1 = \omega_2 = \omega_3 = 0$ in an auxiliary regression of the form $\hat{\varepsilon}_t = \omega_0 x_t + \omega_1 x_t z_{t-k} + \omega_2 x_t z_{t-k}^2 + \omega_3 x_t z_{t-k}^3 + \upsilon_t$ where $\hat{\varepsilon}_t$ is the OLS residual from the linear regression, ω_i is a (1xm) vector of parameters, x_t is a (mx1) vector of regressors used in the linear regression, and z_{t-k} is the switching variable. Note that for the linear Phillips curve, Eq. (1) excluding the LSTAR component was estimated with the lag length of p=5 and q=4 on the basis of the AIC criterion. Under the null hypothesis of linearity, the LM test statistic is distributed as $\chi^2(3m)$. To perform these tests, the switching variable z_{t-k} must be assumed *a priori*. We experiment with one to five lagged values of changes in the rate of inflation ($\Delta\pi$) and in the unemployment rate (Δu) as possible switching variables.

The results of these linearity tests are reported in Table 1 in terms of their marginal significance levels (*p*-values). When the change in the rate of inflation is used as the switching variable, the evidence against linearity is weak, with a possible exception of $\Delta\pi_{t-2}$. By contrast, linearity is rejected comfortably when the change in the unemployment rate is used. The evidence is particularly strong with Δu_{t-2}. This variable is employed as the switching variable in the LSTAR model following Granger and Teräsvirta (1993), who suggest selecting the variable associated with the smallest *p* value if linearity is rejected for more than one switching variable.

Once the switching variable is chosen, Eq. (1) can be estimated by applying nonlinear-least squares, which is equivalent to maximum likelihood estimation in the case of normal errors. For the final model, we sequentially removed the variables with poor explanatory power using the t-values as a guide so long as this did not damage the overall fitness of the model. The parsimonious equation is given in Table 2. The parameters most easily interpreted are *c* and λ. The estimate $\hat{c} = 0.081$ marks the halfway point between two regimes of F=0 and F=1, as $\hat{F} = 1/2$ at $\Delta u_{t-2} - \hat{c} = 0$. To see the effects of $\hat{\lambda} = 6.232 / \sigma(\Delta u)$, Figure 1 depicts the estimated transition function $\hat{F}$ both against Δu_{t-2} and over time. The transition between two regimes is seen to be smooth, and extreme values of $\hat{F}$ (either 0 or 1) are not very frequent. The shaded areas shown in the right-hand figure are periods between the peaks and troughs of US business cycles dated by the NBER. In all cases, switches into the regime of F=1 occurred, which are associated with high unemployment rises.

Table 2 also reports the results of several diagnostic tests to check the statistical adequacy of the estimated model. Overall, they point to no evidence of any serious model inadequacy. The residual variance of the equation is about 85 percent of that from its linear counterpart, consolidating empirical support for the LSTAR specification. Before advancing to the estimation of Eq. (4), the policy horizon *S* must be determined. While there is no universally accepted rule as to how to choose it, we follow Estrella and Mishkin (1999) and assume a policy horizon of two years (that is, *S*=8).[6] This selection is reasonable as the studies find that the peak inflation responses exhibit a monetary transmission lag of one to two years (Christiano *et al*. 1996). The two-year target horizon is also regarded as a common estimate embodied in the forecasting and decision-making of many inflation-targeting central banks (Bernanke *et al*. 1999, p. 320).

Figure 2 depicts the estimated NAIRU together with the actual unemployment rate. Also shown are 95 percent confidence bounds generated using 300 bootstrap replications following the procedure in Li and Maddala (1999).[7] The figure demonstrates the high variability of NAIRU in contrast with conventional measures designed to estimate a natural rate. For example, Staiger *et al*. (1997) estimate a constant NAIRU of 6.2 percent for the period 1955:Q1 to 1994:Q4. Laubach (2001) applies a time-varying NAIRU model in which all the estimates remain in the range of 5 to 7 percent over the period 1970:Q1 to 1998:Q4. Nevertheless, our NAIRU measures are estimated more precisely. Over the sample period, the standard errors range from 0.38 to 0.93 with a mean of 0.50. Staiger *et al*. report a standard error of 0.75 whereas the average standard errors in Laubach are in the range from 0.54 to 1.7, depending on the assumptions for the process of the NAIRU.

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[6] The steady-state value of $E_0(\Delta u_t)$ in Eq. (4) is set at 0.008 calculated from Table (1). By comparison, the unconditional mean of Δu is 0.014.

[7] The bootstrapping was undertaken with $\hat{c}$ and $\hat{\lambda}$ fixed at 0.081 and 6.232 respectively, as in Table 2. This set-up saves computation time considerably but at the cost of sacrificing accuracy in estimation.

Figure 2 also shows the resulting NAIRU gap estimates ($u_t - u_t^N$). A negative (positive) value indicates mounting inflationary (disinflationary) pressure over the policy horizon. The NAIRU gap offers a signal for the appropriate stance of monetary policy. From 1993:Q1 to 1995:Q3, for example, the NAIRU gap remained negative, suggesting a need for monetary tightening. This is consistent with the sharp rises in the federal funds rate throughout 1994 and into early 1995. The NAIRU gap also indicated some pressure to tighten from mid-1996 through 1998. While actual inflation almost halved during this period, the federal funds rate remained virtually unchanged. From the end of 2000, the NAIRU gap suggested that monetary policy should be loosened. Coupled with declining inflation, the federal funds rate fell considerably. The usefulness of this NAIRU gap measure will be assessed more in the subsequent section.

4. Nonlinear monetary policy rules

This section investigates the implications of a nonlinear Phillips curve for the derivation of optimal monetary policy rules. In order to fix ideas without introducing unnecessary complications, we modify a minimalist forward-looking model of Svensson (1997) by allowing for an LSTAR specification in the Phillips curve. Although the system is a highly stylized description of the economy, it is representative of the type of models used in the literature on monetary policy rules.

Suppose that monetary policy is conducted by a central bank with an inflation target π^*. The central bank's objective in period t is to choose a sequence of current and future interest rates $\{r_\tau\}_{\tau=t}^{\infty}$ such that

$$\underset{\{r_t\}}{Min}\ E_t \sum_{\tau=t}^{\infty} \delta^{\tau-t} L(\pi_\tau) \tag{5}$$

where π_t is the annual inflation rate in period t, E_t denotes expectations conditional on the central bank's information set available in period t, discount factor δ fulfils $0 < \delta < 1$, and the loss function $L(\pi_t)$ is

$$L(\pi_\tau) = (\pi_\tau - \pi^*)^2 / 2 . \tag{6}$$

Thus, the central bank wishes to minimize the expected sum of discounted squared future deviations from the inflation target.

The evolution of the economy is represented by the following system:

$$\pi_{t+1} = \pi_t - \alpha \tilde{u}_t - \alpha^* \tilde{u}_t F(\tilde{u}_{t-1}) + \varepsilon^s_{t+1} \tag{7}$$

with

$$F(\tilde{u}_{t-1}) = (1 + \exp\{-\lambda \tilde{u}_{t-1} / \sigma_{\tilde{u}}\})^{-1} - 1/2$$

and

$$\tilde{u}_{t+1} = \beta \tilde{u}_t + \phi(r_t - \pi_t) + \varepsilon^d_{t+1} \tag{8}$$

where $\tilde{u}_t$ is the NAIRU gap ($u_t - u^N_t$), r_t is the monetary policy instrument (federal funds rate, say), ε^s_t and ε^d_t are zero-mean normally distributed shocks.[8] Eq. (7) represents an accelerationist Phillips curve (or AS schedule). Its nonlinearity is captured by the LSTAR component in which $\tilde{u}_{t-1}$ is the switching indicator with the threshold parameter of zero. Hence, the model's dynamics change according to the state of the economy, i.e. $\tilde{u} > 0$ (contractions) and $\tilde{u} < 0$ (expansions). This feature is embodied in theoretical models supporting the nonlinear feature of the Phillips curve. In the capacity constraints model, for example, the sensitivity of inflation increases as the economy strengthens (convex), while it reduces (concave) in the monopolistic competitive model.[9] Eq. (8) is the aggregate demand schedule in which the NAIRU gap is serially correlated, decreasing in the lagged real interest rate ($r_t - \pi_t$).

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[8] The logistic function $F(\tilde{u}_{t-1})$ is scaled so that its values are in the range $-1/2$ to $1/2$.

[9] See Dupasquier and Ricketts (1998) for a survey.

Since the interest rate affects inflation with a two-period lag, without any effects in periods t and t+1, the central bank can find the optimal interest rate at time t as the solution to the simpler period-by-period problem (see Svensson, 1997):

$$\underset{\{r_t\}}{\text{Min}} \ E_t \delta^2 [(\pi_{t+2} - \pi^*)^2 / 2]. \tag{9}$$

The Appendix shows that the first-order conditions for minimizing (9) subject to constraints (7) and (8) leads to the following policy reaction function:

$$r_t = \pi_t + \frac{1}{\phi}\left[\frac{(\pi_t - \pi^*)}{\alpha + \alpha^* F(\tilde{u}_t)}\right] - \frac{1}{\phi}\left[\beta + \frac{\alpha + \alpha^* F(\tilde{u}_{t-1})}{\alpha + \alpha^* F(\tilde{u}_t)}\right]\tilde{u}_t. \tag{10}$$

It shows that the optimal response of interest rates depends nonlinearly on the inflation and NAIRU gaps. Specifically, both elasticities of the interest rate are state-contingent, depending on the phase of the economy as measured by $\tilde{u}$. This implication is in line with Bec *et al.* (2002), who find that changes in the federal funds rate are influenced by the state of the business cycle (measured by output gap). When $\alpha^* = 0$, Eq. (10) reduces to the conventional linear rule as in Svensson (with the obvious exception that the NAIRU gap now replaces output gap). In this case, marginal changes in π_t and $\tilde{u}_t$ lead the central bank to change the nominal interest rate by $\partial r_t / \partial \pi_t = 1 + (1/\phi\alpha)$ and $\partial r_t / \partial \tilde{u}_t = -(\beta + 1)/\phi$, respectively. The responses are hence independent of inflation and NAIRU gaps.

Under the nonlinear rule of Eq. (10), the change in r_t with respect to inflation is $\partial r_t / \partial \pi_t = 1 + (1/\phi)[\alpha + \alpha^* \ F(\tilde{u}_t)]^{-1}$. As $\tilde{u}_t$ rises or, equivalently, the economy gets weaker, the value of $F(\tilde{u}_t)$ rises. Then, $\partial r_t / \partial \pi_t$ becomes smaller, implying that a lesser interest rate rise is needed to attain the inflation target. As an illustration, assume that $\phi = 0.5$, $\alpha = 2$, $\alpha^* = 1.5$, $\beta = 0.7$ and $\lambda = 6$.[10] When $\tilde{u}_t = 0$ (i.e. balanced economy), $\Delta\pi_t = 1$ induces $\Delta r_t = 2$. Note that in this case, the linear and nonlinear policy rules imply the same level of

[10] Under these values, $\partial r_t / \partial \pi_t = 2$ and $\partial r_t / \partial \tilde{u}_t = -3.4$ if the Phillips curve is linear. They correspond to the Svensson rule which responds to inflation and output gaps with elasticities of 2 and 1.7 (with Okun's law).

interest rates. However, $\Delta\pi_t$=1 induces a larger rise of the interest rate at Δr_t=2.52 when $\tilde{u}_t = -0.5$ (i.e. expansions) and a smaller one at Δr_t=1.75 when $\tilde{u}_t = 0.5$ (i.e. contractions). The elasticity $\partial r_t / \partial \pi_t$ becomes still smaller when the economy worsens. For example, $\Delta\pi_t$ induces Δr_t=1.71 at $\tilde{u}_t = 1.0$.

Similar asymmetries arise when considering the interest rate response to a change in the NAIRU gap. The change in r_t with respect to the NAIRU gap is given by:

$$\frac{\partial r_t}{\partial \tilde{u}_t} = -\frac{1}{\phi}\left[\beta + \frac{[\alpha + \alpha^* F(\tilde{u}_{t-1})][\alpha + \alpha^* F(\tilde{u}_t)] + [\pi_t - \pi^* - \alpha\tilde{u}_t - \alpha^* \tilde{u}_t F(\tilde{u}_{t-1})]\ \alpha^* F'(\tilde{u}_t)}{[\alpha + \alpha^* F(\tilde{u}_t)]^2}\right]$$

where $F'(\tilde{u}_t) = \partial F / \partial \tilde{u}_t = (\lambda / \sigma_{\tilde{u}})\exp(-\lambda \tilde{u}_t / \sigma_{\tilde{u}})(F(\tilde{u}_t) + 1/2)^2$. For an illustration, assume that $\pi_t = \pi^*$, $\tilde{u}_{t-1} = 0$, and $\sigma_{\tilde{u}} = 0.4$ together with the same parameter values as the ones listed above. When $\tilde{u}_t = 0$, $\Delta\tilde{u}_t$=1 induces $\Delta r_t = -3.4$. Again, this is the same level implied in the linear policy rule. However, the fall of the interest rate becomes larger at $\Delta r_t = -3.60$ when $\tilde{u}_t = 0.5$ but smaller at $\Delta r_t = -3.27$ when $\tilde{u}_t = -0.5$. The elasticity $\partial r_t / \partial \tilde{u}_t$ gets still smaller at -3.15 when the economy is expanded to $\tilde{u}_t = -1.0$.

In order to examine the empirical relevance of the nonlinear monetary policy rule, we estimate Eqs (7) and (8) with quarterly U.S. data over the period 1954-2001. This exercise is not intended to be a definite empirical contribution but instead is intended to explore what the consequences might be if in fact nonlinearities were an important aspect of the relationship between inflation and the NAIRU gap.[11] The series for r_t is the quarterly average of federal funds rates, and estimates of the NAIRU gap in Section 3 are used for $\tilde{u}_t$. Table 3 shows the estimation results. The parameters α^* and λ are estimated to be 0.61 and 8.01, respectively,

respectively (see Schaling, 1999).

[11] Because the model by Svensson is originally designed for annual data, its application to quarterly data can be somewhat arbitrary. Obviously, we need to construct a full optimization model in the context of the quarterly equations. In such a model, however, complicated lag structure renders the analytic derivation of optimal policy rules difficult. The effect is particularly severe when the Phillips curve is nonlinear. To get around this hardship, the studies assessing nonlinear policy rules employ the Svensson model with quarterly or even monthly data, for example, Corrado and Holly (2003), Dolado *et al*. (2003), Surico (2003), and Dolado *et al*. (2004).

and are statistically significant from zero at standard levels. Again, the LSTAR specification receives empirical support. The implied optimal policy rule in Eq. (10) can be constructed using the estimates in the Table. The inflation target (π^*) is assumed to be at 2 percent per year, as in most studies. Dolado *et al*. (2003) observe that this level is consistent with the Fed target rates reported by the U.S. Council of Economic Advisors.

It is interesting to see how well this implied policy rule fares against the well-known Taylor rule (1993). The Taylor rule has tracked reasonably accurately the actual federal funds rate since the Chairmanship of Greenspan (1987:Q3-present). It also has normative uses providing a recommendation of a good way to conduct monetary policy (Rudebusch and Svensson, 1999). In fact, the linear optimal rule derived from Eq. (7) and (8) is essentially a forward-looking version of the Taylor rule. The policy rule proposed by Taylor takes the form of $r_t = i^* + \pi_t + 0.5(\pi_t - \pi^*) + 0.5\tilde{y}_t$, with the assumptions of the equilibrium real federal funds rate i^* and the inflation target π^* both at 2 percent. The output gap $\tilde{y}_t$ is constructed as the percentage deviation of the level of real GDP from the level of potential real GDP, which is measured by a log linear trend of real GDP. Figure 3 shows the nonlinear optimal policy rule, Taylor rule and actual federal funds rates during the tenure of Greenspan.

The three series are shown to move quite closely together until the end of 1992. Since then, however, there are substantial differences detected between the two policy rules. Through 1995, for example, the actual rate of inflation remained stable in the range of 2.4 to 3.1 percent. The Taylor rule suggested a neutral stance of monetary policy, whereas the nonlinear policy rule signaled a need for monetary tightening as the NAIRU gap indicated mounting inflation pressures (see Figure 2). Actual federal funds rates rose considerably to 5.8 percent from 3.0 during this period. From mid-1996 through the end of 1998, the actual inflation rate almost halved to 1.5 percent. The Taylor rule suggested that monetary policy should be loosened. The actual rate remained at high levels instead. The nonlinear policy rule supported this as the NAIRU gap suggests that inflation pressures remained in the pipeline. In fact, Figure 3 shows that the actual behavior of the federal funds rate is replicated better by the nonlinear policy rule than the Taylor rule. As an indication, the absolute deviations of the nonlinear policy rule from the actual rate is at an average of 0.54 while the corresponding figure for the Taylor rule is twice as high at 1.16.

5. Conclusion

There is a strand of theoretical and empirical evidence that suggests nonlinearities in the Phillips curve. Yet there is no consensus in the literature about the precise nonlinear form of the Phillips curve. In this paper, we model the U.S. Phillips curve with an LSTAR specification, which is flexible to allow various nonlinear shapes. The bottom line is that there is no need to assume one specific form of nonlinearity or another *a priori*. Using this model, the paper proposes a method to derive model-consistent estimates of the NAIRU. An additional feature is that the NAIRU is defined to enhance its predictability for future changes in inflation. This paper then investigates the implications of this nonlinear Phillips curve for the derivation of optimal monetary policy rules. The optimal policy rule is shown to be nonlinear too. In particular, the elasticities of the interest rate with respect to the inflation and NAIRU gaps become state-contingent, depending on the phase of the economy. Empirical results also show that the implied policy rule has tracked reasonably well the actual federal funds rate since the Chairmanship of Greenspan. Taking all results together, this paper suggests that the policy rule and the NAIRU measure built on the LSTAR model may offer some assistance in understanding the conduct of U.S. monetary policy.

Appendix

This appendix derives the optimal monetary policy rule when the Phillips curve is nonlinear represented by an LSTAR specification. The dynamic problem of the central bank can be decomposed into a sequence of period-by-period problems:

$$\underset{\{r_t\}}{\text{Min}} \ E_t\delta^2[(\pi_{t+2}-\pi^*)^2/2] \tag{A1}$$

subject to

$$\pi_{t+1}=\pi_t-\alpha\tilde{u}_t-\alpha^*\tilde{u}_tF(\tilde{u}_{t-1})+\varepsilon^s_{t+1} \tag{A2}$$

$$\tilde{u}_{t+1}=\beta\tilde{u}_t+\phi(r_t-\pi_t)+\varepsilon^d_{t+1} \tag{A3}$$

where $F(\tilde{u}_{t-1})=(1+\exp\{-\lambda\tilde{u}_{t-1}/\sigma_{\tilde{u}}\})^{-1}-1/2$, with all notations as defined in the text.

The first-order condition for Eq. (A1) with respect to r_t is:

$$\frac{\partial E_t\delta^2[(\pi_{t+2}-\pi^*)^2/2]}{\partial r_t}=\delta^2[E_t(\pi_{t+2}-\pi^*)(-\phi)(\alpha+\alpha^*F(\tilde{u}_t))]=0 \tag{A4}$$

using the relation that

$$E_t[\pi_{t+2}]=\pi_t-\alpha\tilde{u}_t-\alpha^*\tilde{u}_tF(\tilde{u}_{t-1})-E_t[\alpha\tilde{u}_{t+1}+\alpha^*\tilde{u}_{t+1}F(\tilde{u}_t)] \tag{A5}$$

and the chain rule that

$$\partial E_t\pi_{t+2}/\partial r_t=E_t[(\partial\pi_{t+2}/\partial\tilde{u}_{t+1})(\partial\tilde{u}_{t+1}/\partial r_t)]=-\phi(\alpha+\alpha^*F(\tilde{u}_t)).$$

Then, the first-order condition in (A4) can be written as:

$$E_t\pi_{t+2}=\pi^*$$

because $(\alpha+\alpha^*F(\tilde{u}_t))$ and ϕ cannot be zero. Substituting Eq. (A3) into Eq. (A5), letting $E_t\pi_{t+2}=\pi^*$ and solving for r_t yields:

$$r_t=\pi_t+\frac{1}{\phi}\left[\frac{(\pi_t-\pi^*)}{\alpha+\alpha^*F(\tilde{u}_t)}\right]-\frac{1}{\phi}\left[\beta+\frac{\alpha+\alpha^*F(\tilde{u}_{t-1})}{\alpha+\alpha^*F(\tilde{u}_t)}\right]\tilde{u}_t.$$

which corresponds to Eq. (10) in the text.

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Table 1: The *p* values of the LM-type linearity test against LSTAR

Switching variables	LM tests	Switching variables	LM tests
$\Delta\pi_{t-1}$	0.14	Δu_{t-1}	0.03
$\Delta\pi_{t-2}$	0.08	Δu_{t-2}	0.00
$\Delta\pi_{t-3}$	0.15	Δu_{t-3}	0.08
$\Delta\pi_{t-4}$	0.22	Δu_{t-4}	0.13
$\Delta\pi_{t-5}$	0.31	Δu_{t-5}	0.19

Note:
Figures reported are the marginal significance levels (*p* values).

Table 2: Estimation results

$$\Delta\pi_t = \underset{(0.101)}{0.021} - \underset{(0.079)}{0.296}\Delta\pi_{t-1} - \underset{(0.084)}{0.223}\Delta\pi_{t-2} - \underset{(0.352)}{1.557}\Delta u_{t-1} + \underset{(0.457)}{0.437}\Delta u_{t-2} - \underset{(0.373)}{0.987}\Delta u_{t-4} +$$

$$(-\underset{(0.126)}{0.197}\Delta\pi_{t-1} - \underset{(0.141)}{0.483}\Delta\pi_{t-2} - \underset{(0.624)}{1.307}\Delta u_{t-1} + \underset{(0.651)}{0.609}\Delta u_{t-2} - \underset{(0.583)}{0.833}\Delta u_{t-3})\times$$

$$[1+\exp\{\underset{(2.012)}{-6.232}\times(\Delta u_{t-2} - \underset{(0.033)}{0.081})/\sigma_{\Delta u}\}]^{-1} + \hat{\varepsilon}_t$$

R^2=0.55	$\hat{\sigma}^2$ = 1.02	$\hat{\sigma}^2/\hat{\sigma}_L^2$ =0.85	F_{LSTAR} = 6.23 [0.00]
AUTO (4)=2.34 [0.67]	ARCH (4)=4.13 [0.39]	JB=4.11 [0.12]	LSTAR=9.81 [0.45]

Notes:
The sample standard deviation of Δu ($\sigma_{\Delta u}$) is 0.381. Figures in parentheses below the coefficient estimates are standard errors. R^2 is the coefficient of determination, $\hat{\sigma}^2$ is the residual variance, and $\hat{\sigma}_L^2$ is the residual variance of the corresponding linear model. Figures in squared brackets are *p*-values of the diagnostic tests. F_{LSTAR} is the F test for the null hypothesis of linearity that the coefficients on the $\hat{F}(\Delta u_{t-2})$ terms are jointly equal to zero. Auto (4) and ARCH (4) refer to the F versions of the LM test for fourth-order serial correlation and the LM test for fourth-order ARCH effects, respectively. JB reports the result of the Jarque-Bera test for normality. LSTAR checks for remaining nonlinearity in residuals using the LSTAR test described previously.

Table 3: Estimates of the stylized economy

Equation (7)		Equation (8)	
Coefficient	Estimate	Coefficient	Estimate
Intercept	–0.02 (0.03)	Intercept	–0.03 (0.04)
α	0.92 (0.16)	β	0.66 (0.06)
α^*	0.61 (0.28)	ϕ	0.42 (0.08)
λ	8.01 (4.03)	φ	–0.25 (0.07)
Adj-R^2	0.39	Adj-R^2	0.45

Notes:

Eq. (8) includes an additional regressor of $(r_{t-1} - \pi_{t-1})$ with the coefficient of φ to take account of serial correlation in the residuals. The sample standard deviation of $\tilde{u}$ $(\sigma_{\tilde{u}})$ is 0.39. Figures in parentheses beside the coefficient estimates are standard errors.

Figure 1: Estimated transition function

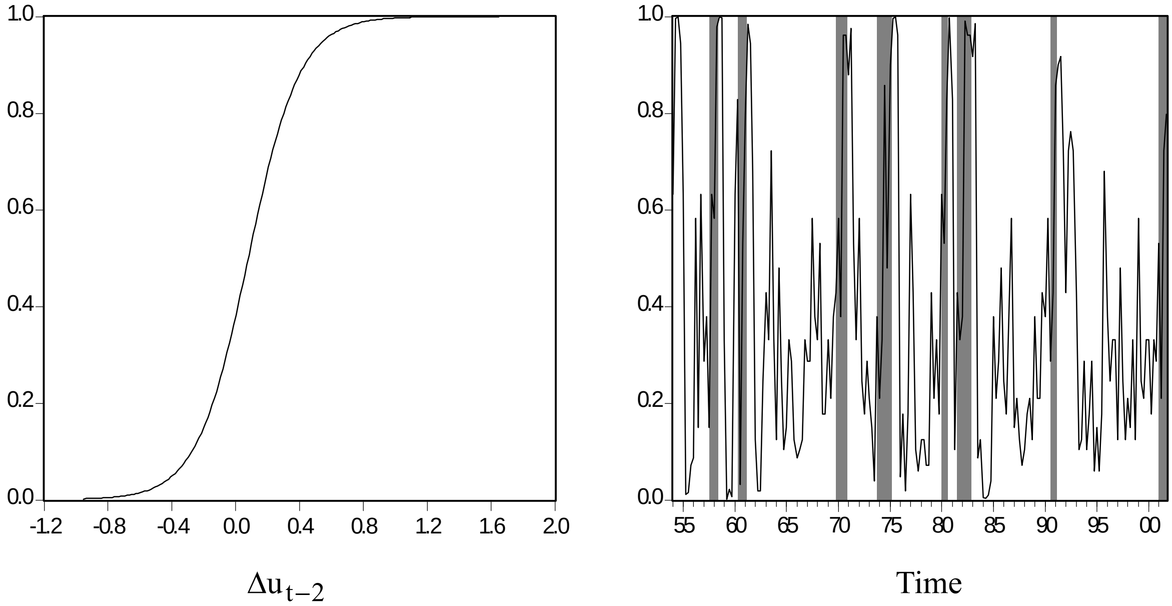


Figure 2: Estimates of the NAIRU and NAIRU gap

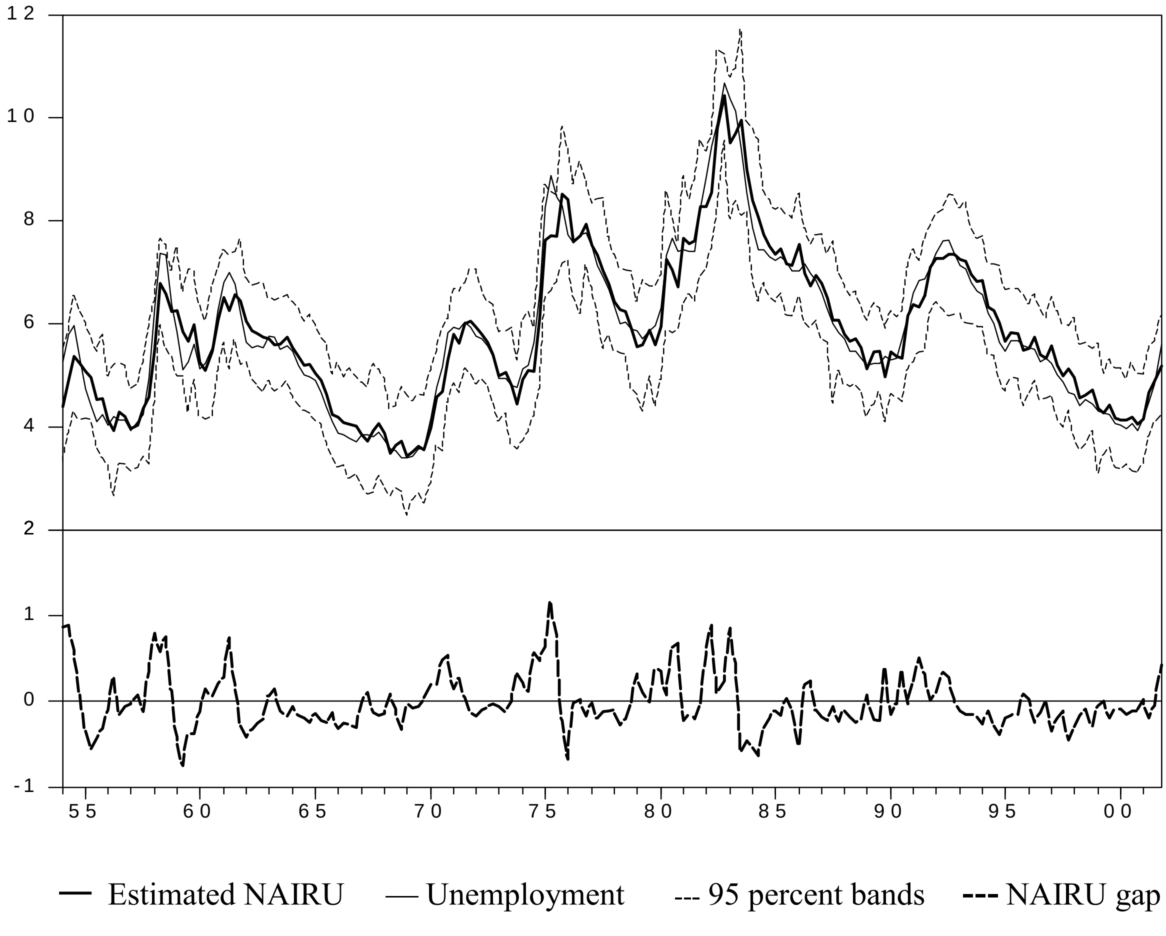
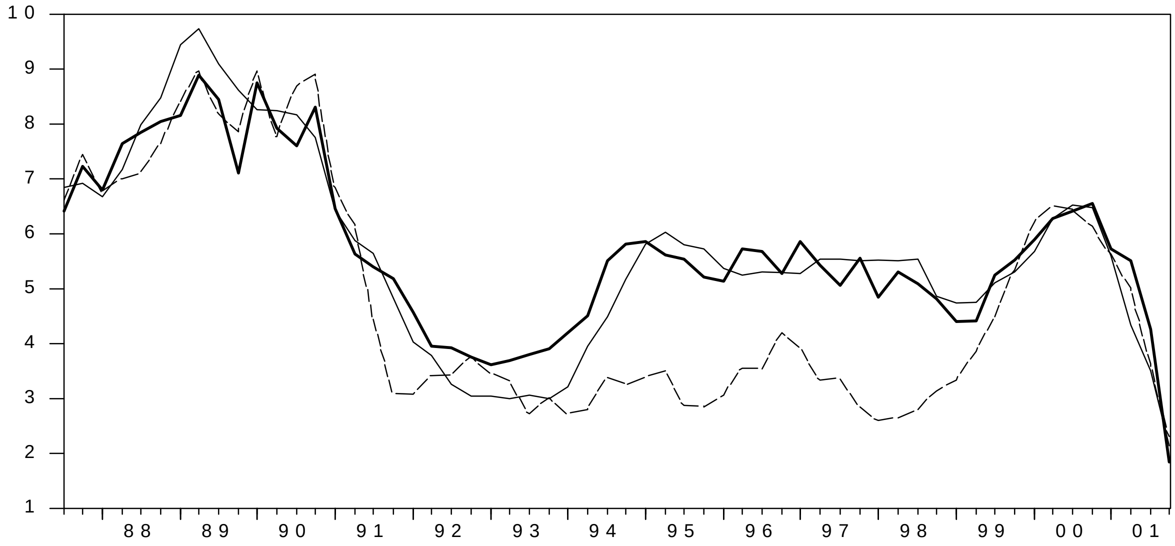


Figure 3: Federal funds rates: Target versus Actual



— Nonlinear policy rule — Federal funds rates ---- Taylor rule