International Economic Systems for International Public Goods: Incentive Mechanisms under Uncertainty

by

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June 2005

Running Title: International Economic Systems

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Abstract

We develop a model of international economic systems under uncertainty in a setup of international public goods. Based on the mechanism design theory, we examine international economic systems as coordinating entities for optimal supply of and demand for international public goods. Specifically, we characterize the environments where such IES could be constructed to internalizing economic uncertainty through transfers. By using our IES, the first-best allocation is implemented even though information is asymmetric.

Key Words: International Public Goods, International Economic Systems, Mechanism Design, Transfers

JEL numbers: H87, H41, F42, C72

1. Introduction

There are various fields in which many problems cross the border and affect international economy as well as each country's economy; for examples, international trade(goods, services and knowledge,..), disease(AIDS, bird flu,..), environment(climate change, yellow sand, greenhouse effect,..), and finance(financial crisis,..). National policy initiatives can not allow the problems in those fields to be dealt with efficiently. In order to solve the problems, we need globally coordinated efforts for the international public goods(IPGs), which include the rules that apply across borders, the institutions that supervise and enforce the rules, and the benefits that accrue without distinctions between countries. (1)

In the international economic society, there have already been various international public goods which have been undersupplied. Nowadays it also needs to get some new international public goods in various fields. Benefit and cost of providing and/or using the international public goods, which each country has got and/or expects to get, are different in size by country. Therefore, it is general for each country to get different position on providing and using the international public goods. The international economic society needs international coordination and cooperation in order to supply the international public goods efficiently and in appropriate size.

If there exists a supranational government, the adequate and efficient supply of and demand for the international public goods could be placed under it's control. As no such entity exists, the international economic society may substitute a certain institute or system for the supranational government. International economic system can play a critical convening function and a role as catalysts and supporters of coalition to provide the international public goods. (2) In particular, the well-designed international system or international economic system may be able to control the supply of and demand for the international public goods.

However, as Barrett (2002) points out, it is especially difficult for the international system to supply the international public goods in efficient quantities. The international

(1) Ferroni and Mody (2002), p1.
(2) ibid., p14.

economic system has not always been in the optimal situation to supply the international public goods in efficient quantities, because of the advent of free-rider countries, non-cooperative behaviors among countries, incomplete information and uncertainty, etc.

This paper investigates an incentive mechanism of the international economic system, which contains the decision rule about the supply of and demand for the international public goods under incomplete information. Based on the mechanism design literature, we analyze the possibility of the incentive mechanisms with transfers for the uncertainty of fundamental economic variables; the unit production costs and the parameter of utility functions. We use indirect utility functions to formalize the design problem and characterize a necessary and sufficient condition for the existence of the mechanism.

Our proposal of direct revelation mechanisms as an international economic system is novel in that the majority of international economic systems in the literature use the Nash Equilibrium concept. Ihori (1994) incorporated voluntary provision of public goods into a rigorous general equilibrium model of economic integration under uncertainty. And Ihori (1996) analyzed the impacts of productivity differentials in contributing to the international public good on national welfare in the context of non-cooperative voluntary provision of a pure public good. Laffont and Martimort (2005) recently analyze the design of incentive mechanisms for the provision of transnational public goods under asymmetric information among two countries.

The paper is organized as follows. In section 2, we draw on preliminary concerns based on the benchmark case with complete information and motivate our study by observing that there is a possibility of the first-best allocation under incomplete information through the direct revelation mechanisms with monetary transfers. Section 3 deals with the model and characterization of the direct revelation mechanisms. Section 4 is the application of the main result to concrete cases of uncertainty; the unit production costs and the parameter of utility functions. The conclusion is in section 5.

2. Preliminary - Benchmark

We assume that there are n countries that are interested in coordinating the production and consumption of international public good G. There are two kinds of economic goods in the model; the international public good G and a private good c. The preferences of a country i over her consumption amount c_i of the private good and G of the international public good can be represented as a (representative) utility function $u^i(c_i, G)$, which we assume to be continuously differentiable, monotonically increasing, and strictly concave. We restrict our attention into quasi-linear utility functions; $u^i(c_i, G) = c_i + b_i \ln(G)$ for i.

Each country must divide her given fixed income into the contribution to the international public good and the portion to the private good. The contributions of the countries sum up to the international public good; $G = g_1 + \cdots + g_n$ with g_i being the contribution of country i. Country i's budget constraint would be $c_i + p_i g_i = Y_i$, where p_i denotes country i's unit cost of the public good contribution, g_i the contribution of country i to the international public good, and Y_i country i's income.

When each country only considers her portions of the international public good and the private good with domestic perspective, country i's problem is to choose (c_i, g_i) maximizing the utility $u^i(c_i, g_1 + \cdots + g_n)$ subject to $c_i + p_i g_i = Y_i$, or to choose (c_i, G) maximizing the utility $u^i(c_i, G)$ subject to $c_i + p_i G = Y_i + p_i g_{-i}$ where $g_{-i} = G - g_i$. However, that problem does not consider two important issues over international public goods. First, one country's utility is influenced not only by her choice but also by the others' choices on international public goods due to non-rivalry and non-excludability. Second, there is asymmetry among nations over each nation's private information.

We introduce the framework of "games with incomplete information" into the situations and think of game forms of the rules of the games as international economic systems. We will consider the IES with complete information first, and then observe that the first-best allocations would be implemented. Under incomplete information due to the asymmetry of information, the first-best allocations would not be implemented even through Nash equilibrium behaviors. Therefore, we will consider direct revelation mechanisms as an IES under incomplete information.

(1) IES with complete information

First of all, consider an IES with complete information as a benchmark. When the

IES has complete information on the realized types of countries, the IES can solve the following maximization problem

$$\max_{\{g_i\}} \mathcal{L}(g_1, \cdots, g_n) \equiv \sum_i a_i u^i(Y_i - p_i g_i, \sum_i g_i) \tag{1}$$

with $\sum_i a_i = 1$. The Kuhn-Tucker first order necessary condition is then, for all $j \in I$

$$\begin{aligned} 0 \geq \frac{d\mathcal{L}}{dg_j} &\equiv -p_j a_j \frac{\partial u^j}{\partial c_j} + \sum_i a_i \frac{\partial u^i}{\partial G} \\ g_j &\geq 0 \\ \frac{d\mathcal{L}}{dg_j} \cdot g_j &= 0. \end{aligned} \tag{2}$$

Assuming that $g_j > 0$ for all j, we get that for all $j \in I$

$$p_j a_j \frac{\partial u^j}{\partial c_j} = \sum_i a_i \frac{\partial u^i}{\partial G}. \tag{3}$$

We observe then that the right-hand term of (3) is constant. Therefore, we obtain that for all i and j, $p_i a_i \frac{\partial u^i}{\partial c_i} = p_j a_j \frac{\partial u^j}{\partial c_j}$. Thus, the j's in the left-hand term of (3) can be replaced by i's. Then we can obtain $\sum_i \frac{MRS_i}{p_i} = 1$ where $MRS_i = (\frac{\partial u^i}{\partial G})/(\frac{\partial u^i}{\partial c^i})$ is the marginal rate of substitution. It means that the sum of the individual ratio of the marginal rate of substitution over the unit cost should be equal to 1. That ratio can be interpreted as a relative cost to producing and consuming the international public good from the private good. When p_i's are equal to p for all i, then the condition means that the sum of the MRS's is equal to the unit cost p. (3)

When the utility function is quasi-linear, $u^i(c_i, G) = c_i + b_i \ln(G)$ for all i, the amount of international public good is determined indifferently of the levels of the private good because we sequentially obtain $-p_j a_j + \sum_i a_i \frac{b_i}{G} = 0$, $p_i a_i = p_j a_j$, $\sum_i \frac{b_i}{p_i G} = 1$, thus $G = \sum_i \frac{b_i}{p_i}$. Furthermore, if we are considering the equally-weighted problem with $a_i = \frac{1}{n}$ for all i, we will have $p_i = p_j$ for all i and j since $p_i a_i = p_j a_j$ for all i and j. Thus, a necessary condition for the existence of interior solution is the unique cost. We can implement this allocation through the guidance of the IES. The IES knows the most

(3) It is called the Bowen-Lindahl-Samuelson condition.

efficient country to producing the international public good. Thus, the IES assign the production of the international public good to her. The IES also orders the other countries to pay the cost of production to the producing country.

(2) IES under incomplete information

Under incomplete information, we firstly consider non-cooperative behaviors of the countries by using Nash equilibrium. Nash equilibrium could be thought of as the result of strategic interaction of the countries. Each country considers the others' fundamental variables and actions as given and decides the allocation of the income into the public good and private good. Thus, there would be the typical underproduction of international public good G. Specifically, we assume that country i decides the choices for given p_i, Y_i, g_{-i}. Thus we examine the outcome of non-cooperative behaviors based on Nash equilibrium. Then by using the dual approach, we get the conditional consumption bundles and the expenditure function, $c_i(p_i, u^i)$, $g_i(p_i, u^i)$, $E^i(p_i, u^i)$ or $G(p_i, u^i)$ by solving the minimization problem $\min_{c_i, G} c_i + p_i G$ subject to $u^i(c_i, G) = \overline{u}$. Finally, by using the equilibrium identities $E^i(p_i, u^i) \equiv Y_i + p_i g_{-i}$ and $G^i(p_i, u^i) \equiv G$, we obtain the indirect utility function $U^i(p_i, Y_i, b_i)$. [4]

When there is uncertainty for each fundamental variable, p_i or b_i respectively, we can suppose that each country's concern is about the indirect utility function $U^i(p_i, Y_i, b_i)$. We introduce an explicite IES of direct revelation mechanisms. That is, the countries install the IES like a central agency that collects the reports on private information and decides the allocations and transfers.

Consider a case of two-country. We assume that $p_1 < p_2$. Each country's problem without considering the other's policy is solving $\max b_i \ln(g_i + g_{-i}) + c_i$ subject to $c_i + p_i g_i = Y_i$ given g_{-i}. Then the outcome pair of allocation is $c_i = Y_i - b_i$ and $g_i = b_i/p_i$. The total amount of the international public good would be $G = g_1 + g_2$. The utility of the country i is

$$U^i(p_i, Y_i, b_i) = b_i \ln(\frac{b_1}{p_1} + \frac{b_2}{p_2}) + Y_i - b_i \tag{4}$$

(4) Ihori (1996).

Considering the interactions of the two countries, we will think the above Cournot-Nash approach. [5] Then, $G = g_1 = b_1/p_1$. Each country's utility from Nash equilibrium is

$$U_N^1(p_i, Y_i, b_i) = b_1 \ln(b_1/p_1) + Y_1 - b_1, \tag{5}$$

$$U_N^2(p_i, Y_i, b_i) = b_2 \ln(b_1/p_1) + Y_2, \tag{6}$$

respectively. Here country 2 is the free-rider.

The mechanism approach introduces explicitely an international economic system [6] Since $p_1 < p_2$, the IES decides that the production of the international public good is done by the country 1 and that there would be a payment from 2 to 1. Country 1 additionally produces $g_2 = \frac{b_2}{p_2}$ instead of country 2. Country 2 pays the total costs $p_1 g_2 = (\frac{p_1}{p_2})b_2$ to country 1. Each country's utility from the direct mechanism of an IES is

$$U_M^1(p_i, Y_i, b_i) = b_1 \ln(\frac{b_1}{p_1} + \frac{b_2}{p_2}) + Y_1 - p_1(\frac{b_1}{p_1} + \frac{b_2}{p_2}) + (\frac{p_1}{p_2})b_2 \tag{7}$$

and

$$U_M^2(p_i, Y_i, b_i) = b_2 \ln(\frac{b_1}{p_1} + \frac{b_2}{p_2}) + Y_2 - (\frac{p_1}{p_2})b_2 \tag{8}$$

We observe that for some parameters the utility of country 2 from the direct mechanism in (8) is greater than that from the free-riding at Nash equilibrium in (6). [7] That is, we can find the environments where the individual rationality or voluntary participation condition is satisfied, for example with $p_2 = 2p_1$ and $b_1 = 4b_2$.

The Pareto efficiency leads us to the fact that the country with the lowest cost should produce the whole quantity of the international public goods. Because information about the production cost is asymmetric and private, there may be strategic relationship over the production cost. We assume that the countries want to build an association of the international public good, an international economic system to achieve the Pareto efficiency.

3. Model and Characterization Theorem

(5) ibid.
(6) For example, Makowski and Mezzetti (1994).
(7) Note that we set country 1's payoff remained.

Let $I = \{1, 2, \cdots, n\}$ be a finite set of countries, $\Theta_i = [\underline{\theta}_i, \overline{\theta}_i]$ a set of types for a country $i \in I$, and A a set of all feasible (alternative) outcomes. That is, $(c_1, \cdots, c_n, g_1, \cdots, g_n) \in A$ should satisfy each country's budget constraint. Country i has type-contingent valuation functions over A; $v_i(\cdot, \theta_i)$ for θ_i. We assume that $v_i(\cdot, \theta_i)$ is continuous for all i and all θ_i, and that Θ_i is *convex* [(8)] for all i in the sense that for any type $\theta'_i \in \Theta_i$, any type $\theta''_i \in \Theta_i$, and any number $\lambda \in [0, 1]$, there is a type $\theta_i(\lambda) \in \Theta_i$ such that for all $a \in A$,

$$v_i(a, \theta_i(\lambda)) = (1 - \lambda)v_i(a, \theta'_i) + \lambda v_i(a, \theta''_i). \tag{9}$$

We will below introduce uncertainty of some fundamental variables. Uncertainty of information is summarized by a set of states $\Theta \equiv \prod_{i=1}^{n} \Theta_i$ with a typical element $\theta = (\theta_1, \cdots, \theta_n)$. Thus, we use a private-values model. The common prior belief over the states is given by a probability measure F on the Borel subsets of Θ.

Timing of the whole scheme is as follow. There are three stages; ex ante, interim, and ex post. At ex ante stage, the countries set up an IES. The IES is equipped with rules and guidelines for any state. At interim stage, the true state is realized. Each country knows her own type but has a distribution of the others' types. Each country reports her type to the IES. Then, the IES decides which countries produce the international public good and allocates the amount of contributions to the international public good. At ex post stage, the countries follow the decision of the IES.

We assume that the IES wants to maximize the equally-weighted sum of countries' valuations at each state. Define the objective of the IES at each state θ as

$$g(\theta) \equiv \max_{a \in A}\{\sum_{i=1}^{n} v_i(a, \theta_i)\}. \tag{10}$$

Then the goal of the IES is to get $g(\theta)$ at each θ. We call $g(\cdot)$ a global gain function. We may think the objective function is realized when the state is complete information. However, because the IES does not have complete information on countries' types, it should induce the true information from the countries through a mechanism with incentive transfers.

A mechanism consisting of a message space and an outcome function should be devised to implement the global gain function $g(\cdot)$. Once a mechanism is installed and after each

[(8)] Holmström (1979).

country knows her type, the countries face a game with incomplete information. We use dominant strategy equilibria as our equilibrium concept in incomplete information games.

Since we are interested in the construction of mechanisms which satisfy dominant strategy incentive compatibility, we have no loss of generality in restricting our attention to direct mechanisms due to the Revelation Principle. [9]

A direct mechanism is denoted by $(\Theta, < s, t >)$ where Θ is the message space of type reports and $< s, t >$ is an outcome function which consists of a decision rule $s : \Theta \to A$ and a transfer scheme $t = (t_1, \cdots, t_n)$ with $t_i : \Theta \to \Re$. We will use the notation $< s, t >$ for a direct mechanism. Given $< s, t >$, country i's payoff with type θ_i from a report θ' is $v_i(s(\theta'), \theta_i) + t_i(\theta')$.

As a direct mechanism is installed and a state is realized, countries face a direct revelation game. Dominant-strategy incentive compatibility means that in any direct revelation game, the truth-telling is a dominant-strategy equilibrium for all countries. Formally, a mechanism $< s, t >$ is *dominant-strategy incentive compatible* (DSIC) if every country has an incentive to report her own type honestly regardless of the others' report schemes at any state, i.e., for all i, for all θ_{-i}, for all θ_i, and for all θ'_i,

$$v_i(s(\theta_{-i}, \theta_i), \theta_i) + t_i(\theta_{-i}, \theta_i) \geq v_i(s(\theta_{-i}, \theta'_i), \theta_i) + t_i(\theta_{-i}, \theta'_i). \tag{11}$$

A decision rule s is *outcome-efficient* if $\sum_{i=1}^{n} v_i(s(\theta), \theta_i) = g(\theta)$ for all θ, that is, if it always realizes the global gain in (10). A mechanism $< s, t >$ is a *first-best dominant-strategy* mechanism if it is outcome-efficient and dominant-strategy incentive compatible.

Since our setup satisfies the convexity condition in Holmström (1979), we can use the result that a mechanism is a first-best dominant-strategy if and only if it is a Groves mechanism. To describe the Groves mechanisms in our setup, we will introduce a concept between the individual country's concern and the global stake of interest.

Given any outcome-efficient mechanism $< s, t >$, define the *participation charge* [10] on country i at state θ as the difference of i's payoff from the global gain; $h_i(\theta) \equiv g(\theta) - v_i(s(\theta), \theta_i) - t_i(\theta)$ for all i and for all θ. A mechanism $< s, t >$ is a *Groves mechanism* if it

(9) Dasgupta, Hammond and Maskin (1979).
(10) Makowski and Mezzetti (1994).

is outcome-efficient and its participation charges on country i are independent of i's type for each i, i.e., for all i, for all θ_{-i}, for all θ'_i, and for all θ''_i, $h_i(\theta_{-i}, \theta'_i) = h_i(\theta_{-i}, \theta''_i)$. Then country i's payoff from the participation in a Groves mechanism at state θ is

$$v_i(s(\theta), \theta_i) + t_i(\theta) = g(\theta) - h_i(\theta_{-i}). \tag{12}$$

Since each country's participation charges are non-distortionary lump-sum in Groves mechanisms, there is no incentive for any country to tell a lie in a direct revelation game. One simple Groves mechanism is a mechanism with zero participation charges; $h_i(\theta) = 0$ for all i and for all θ. Then each country's payoff would be equal to the global gain $g(\theta)$ in (10) at each θ, and the zero-charge Groves mechanism incurs a deficit $g(\theta) - v_i(s(\theta), \theta_i)$ for agent i at state θ. The (ex ante) expected budget deficit for country i in the zero-charge Groves mechanism is

$$B_i \equiv E[g(\theta) - v_i(s(\theta), \theta_i)]. \tag{13}$$

where E is the expectation operator with respect to F. Set $B = \sum B_i$.

The main idea of Groves mechanisms is that the IES uses monetary transfers to give an incentive for the truthful reporting to each country. Generally speaking, the use of transfers happens ex post after the realization of the types of countries. However, the countries expect the possibility of incentive transfers and thus expect to have the common interest from the cooperation and coordination. Then ex ante at the time period of mechanism installation the countries can calculate the expectation value of net transfers. The following two conditions are concerned with the expectation of net-transfers between the IES and each country. A mechanism $< s, t >$ is *ex ante budget balanced* (EABB) if

$$E\Big[\sum_{i=1}^{n} t_i(\theta)\Big] = 0. \tag{14}$$

A mechanism $< s, t >$ is *zero expected net-transferred* (ZENT) if for all i,

$$E[t_i(\theta)] = 0. \tag{15}$$

The condition of ZENT might be interpreted as the condition of Individual Ex Ante Budget Balanced. In brief, the countries need to install an IES with incentive transfer scheme to induce the true information from themselves. Furthermore, the expected budget should be even in average for efficiency. The condition of ZENT requires the mechanism to satisfy both incentive and efficiency motivation beside other criteria for the mechanism.

Voluntary participation without any kind of non-economic enforcement should respect outside option payoffs after countries receive the terms of a mechanism at each state. Let $u_i^0(\theta)$ be the outside option payoff of country i at state θ; the payoff that country i would obtain if she decides not to participate in the mechanism. Formally, a mechanism $< s, t >$ is *ex post individually rational* (EPIR) if no country has an incentive to drop out from the mechanism after learning the state, i.e., for all i and for all θ,

$$u_i(s(\theta), \theta_i) + t_i(\theta) \geq u_i^0(\theta). \tag{16}$$

At this moment, we assume that $u_i^0(\theta) = 0$. Importantly and naturally, we might assume the outside option payoff is that from Nash equilibrium. For easy exposition, we assume the zero outside option payoff.

The ex post individual rationality (EPIR) condition of (16) in a first-best dominant-strategy mechanism is $g(\theta) - h_i(\theta_{-i}) \geq u_i^0(\theta)$ for all θ_i and for all θ_{-i}, or $g(\theta) - u_i^0(\theta) \geq h_i(\theta_{-i})$ for all i and for all θ_{-i}. Since the IES does not observe country i's type, the maximal amount the IES as a center can charge on country i without violating country i's EPIR condition is $c_i(\theta_{-i}) = \min_{\theta_\text{i}}\{g(\theta) - u_i^0(\theta)\}$ for all θ_{-i}. Then, the (ex ante) expected lump-sum charge without violating country i's EPIR condition is

$$E[c_i(\theta_{-i})] = E[\min_{\theta_\text{i}}\{g(\theta) - u_i^0(\theta)\}]. \tag{17}$$

The following theorem characterize the range of environments where we can construct the first-best weakly-robust mechanisms with ex ante zero-expected net-transfers. Since our characterization theorem has a necessary and sufficient condition, we not only construct the mechanism from the condition but also propose that the mechanism is unique.

Theorem: Let $s(\cdot)$ be an outcome-efficient decision rule. Then
(i) there exists a first-best dominant-strategy mechanism with ex post individual rationality (EPIR) and ex ante budget balanced (EABB) iff $\sum_i E[c_i(\theta_{-i})] \geq \sum_i E[g(\theta) - v_i(s(\theta), \theta_i)]$, and

(ii) there exists a first-best dominant-strategy mechanism with ex post individual rationality (EPIR) and zero-expected net -transfers (ZENT) iff $E[c_i(\theta_{-i})] \geq E[g(\theta) - v_i(s(\theta), \theta_i)]$ for all i.

Proof: (i) is done by Makowski and Mezzetti (1994). We will prove (ii).

(If) Define a transfer scheme t by $t_i(\theta) = g(\theta) - v_i(s(\theta), \theta_i) - c_i(\theta_{-i}) + K_i$, for all i and for all θ, where $K_i = E[c_i(\theta_{-i})] - E[g(\theta) - v_i(s(\theta), \theta_i)] \geq 0$. Then the resultant participation charges are lump-sum since $h_i(\theta) = c_i(\theta_{-i}) - K_i$ is θ_i-indifferent for all i and for all θ. Thus, $< s, t >$ is a Groves mechanism , i.e., a first-best dominant-strategy mechanism. [11] Since $E[t_i(\theta)] = 0$ for all i, $< s, t >$ is ZENT. Since $t_i(\theta) + v_i(s(\theta), \theta_i) - u_i^0(\theta) = g(\theta) - c_i(\theta_{-i}) + K_i - u_i^0(\theta) \geq g(\theta) - u_i^0(\theta) - c_i(\theta_{-i}) \geq 0$ for all i and for all θ, $< s, t >$ is EPIR.

(Only if) Since $< s, t >$ must be a Groves mechanism, $t_i(\theta) = g(\theta) - v_i(s(\theta), \theta_i) - h_i(\theta_{-i})$ for all i and for all θ. Since $< s, t >$ is EPIR in (9), $g(\theta) - u_i^0(\theta) \geq h_i(\theta_{-i})$ for all i and for all θ. Thus, $c_i(\theta_{-i}) \geq h_i(\theta_{-i})$ for all i and for all θ_{-i}. By the operator E, $E[c_i(\theta_{-i})] \geq E[h_i(\theta_{-i})]$ for all i. It suffices to show that $E[h_i(\theta_{-i})] = E[g(\theta) - v_i(s(\theta), \theta_i)]$ for all i because then $E[c_i(\theta_{-i})] \geq E[g(\theta) - v_i(s(\theta), \theta_i)]$ for all i. Since $< s, t >$ is ZENT in (8), $E[t_i(\theta)] = E[g(\theta) - v_i(s(\theta), \theta_i)] - E[h_i(\theta_{-i})] = 0$ for all i. ∎

We observe that the mechanism in (ii) is more restrictive than that in (i). Thus, if we can construct the mechanism in (ii), then we can also construct the mechanism in (i). The mechanism's net-transfers are worthy of noting. The net-transfer t_i of country i could be decomposed into three parts. The first $g(\theta) - v_i(s(\theta), \theta_i)$ is related to production and consumption of the international public good. It is the side of real economy over the international economic system. The second $c_i(\theta_{-i})$ is a kind of incentive scheme. Note that it is not dependent upon country i's type. Thus, it may be interpreted as the (negative) evaluation based on peer-group reporting. The third K_i is constant. It may be interpreted as the subsidy to country i from the IES.

4. Applications

[11] See Makowski and Mezzetti (1994) for that equivalence.

Applying the contents of the previous section, we will concretely analyze the cases of uncertainty about the fundamental economic variables; the unit production costs p_i's and the preferences parameters b_i's.

(1) Cost Uncertainty

We assume that b_i's and Y_i's are public information and given fixed. We assume that p_i's are variable and private information. Each country i knows her cost parameter p_i, but only knows the distribution of the others' cost parameters. It seems then that the most efficient country must produce the international public goods. Since the costs are private information, the important role of the IES would be how to induce the true information about p_i's for producer-selection from the countries.

We assume that the utility functions are quasi-linear, $u^i(c_i, G) = c_i + b_i \ln(G)$ for all i. From the results in section 2, we suppose that the Pareto allocations could be equivalent to the following allocation when the individual country's budget constraints hold.

$$g_i(p_1, \cdots, p_n; Y_1, \cdots, Y_n, b_1, \cdots, b_n) = \frac{b_i}{p^*}$$

$$c_i(p_1, \cdots, p_n; Y_1, \cdots, Y_n, b_1, \cdots, b_2) = Y_i - b_i$$

with $p^* = \min_j p_j$. When there are differences in the costs in producing the international public good, the Pareto allocation would be such that the most efficient country produces the world amount of the international public good. The other countries pay the costs of the contributions to the producer country. We will incorporate the Pareto allocation and design an international economic system for the international public good in order to achieve the Pareto allocation.

We will set $\theta_i = p_i$ for all i. Let $\Theta_i = [c, d]$ be the set of types for country $i \in I$. We assume that c the lower bound of the type is not zero. Thus, there is substantial cost burden to the international public goods. We assume also that each country's income is not so small in the sense that for all i

$$Y_i \geq \ln[(\frac{d}{c})^{\sum_{j \neq i} b_j} (\frac{d}{\sum_j b_j})^{b_i}] + b_i \qquad (18)$$

where n is the number of countries. Then the Pareto rule would be like the following: The IES receives the reports on the types from the countries. Based on the reports, the IES then decides the individual production amount of the international public good and (thus) the consumption levels of private goods of each country.

Denoting the reports as $\hat{\theta} = (\hat{\theta}_1, \cdots, \hat{\theta}_n)$ and with $\theta^* \equiv \min_j \hat{\theta}_j$, we can express the Pareto allocations, $(g_1(\hat{\theta}), \cdots, g_n(\hat{\theta}), c_1(\hat{\theta}), \cdots, c_n(\hat{\theta}))$ for any $\hat{\theta}$. Specifically, $g_i(\hat{\theta}) = \frac{b_i}{\theta^*}$ and $G(\hat{\theta}) = \sum_i (\frac{b_i}{\theta^*})$. [12] Only the most efficient country produces the international public goods and the other countries pay the costs. Then the consumption level of the private goods are $c_i(\hat{\theta}) = Y_i - b_i$ for the non-producing countries and $c_i(\hat{\theta}) = Y_i - \theta_i \sum_i (\frac{b_i}{\theta^*}) + \sum_{j \neq i} b_j$ for the producing country.

For easy understanding of the analysis, let us decompose the state set Θ into the subsets $\Theta_i^* = \{\theta \in \Theta | \theta_i = \min_j \theta_j\}$ for all i. The set Θ_i^* consists of the cases where country i has the lowest production cost.

Each type θ_i of country i has a valuation function $v_i(\cdot, \theta_i)$ over A. The payoff of country i with type θ_i from the reports $\hat{\theta}$ is

$$v_i((c(\hat{\theta}), g(\hat{\theta})), \theta_i) = b_i \ln(\sum_i (\frac{b_i}{\theta^*})) + Y_i + \mathbf{1}_{\Theta_i^*}(\hat{\theta})[-\theta_i \sum_i (\frac{b_i}{\theta^*}) + \sum_i b_i] - b_i \qquad (19)$$

where the index function $\mathbf{1}_X(x)$ has the value 1 if $x \in X$ or 0 otherwise.

It is easily proven that Θ_i is *convex* for all i since for all $a \in A$,

$$v_i(a, \theta_i(\lambda)) = (1-\lambda) v_i(a, \theta_i') + \lambda v_i(a, \theta_i'')$$

by taking $\theta_i(\lambda) = (1-\lambda)\theta_i' + \lambda \theta_i''$ for any number $\lambda \in [0, 1]$.

The set of states is denoted as $\Theta \equiv \prod_{i=1}^n \Theta_i$ with a typical element $\theta = (\theta_1, \cdots, \theta_n)$. The common prior belief on the states Θ is given by a probability measure F on the Borel subsets of Θ. We assume that each country knows her own type and has a common prior on the states Θ.

The global gain function from the Pareto allocation is

$$g(\theta) \equiv [\sum_i b_i \ln(\sum_i (\frac{b_i}{\tilde{\theta}})) + \sum_i Y_i - \sum_i b_i]$$

[12] Note that this amount is only dependent upon the sum of the income.

where $\tilde{\theta} = \min_i \theta_i$.

A direct mechanism is denoted by $(\Theta, < s, t >)$ where Θ is the message space of type reports and $< s, t >$ is an outcome function which consists of a decision rule $s : \Theta \to A$ and a transfer scheme $t = (t_1, \cdots, t_n)$ with $t_i : \Theta \to \Re$. Given $< s, t >$, country i's payoff with type θ_i from a report θ' is $v_i(s(\theta'), \theta_i) + t_i(\theta')$. We will use the notation $< s, t >$ for a direct mechanism.

The (ex ante) expected budget deficit for country i in the zero-charge Groves mechanism is

$$B_i \equiv E[g(\theta) - v_i(s(\theta), \theta_i)] = E[\sum_{j \neq i} b_j \ln(\sum_j \frac{b_j}{\tilde{\theta}}) + \sum_{j \neq i}(Y_j - b_j)], \tag{20}$$

and

$$B \equiv \sum_i B_i = E[(n-1)g(\theta)] = (n-1)E[\sum_i b_i \ln(\sum_i (\frac{b_i}{\tilde{\theta}})) + \sum_i Y_i - \sum_i b_i]. \tag{21}$$

The outside option payoff of country i at state θ is assumed to be $u_i^0(\theta) = 0$. Since the IES does not observe country i's type, the maximal amount the IES can charge on country i without violating country i's EPIR condition is $c_i(\theta_{-i}) = \min_{\theta_i}\{g(\theta) - u_i^0(\theta)\}$ for all θ_{-i}. Then, the (ex ante) expected lump-sum charge without violating country i's EPIR condition is

$$E[c_i(\theta_{-i})] = E[\min_{\theta_i}\{g(\theta) - u_i^0(\theta)\}].$$

Specifically,

$$E[c_i(\theta_{-i})] = E[\sum_j b_j \ln(\sum_j \frac{b_j}{\tilde{\theta}_{-i}}) + \sum_j Y_j - \sum_j b_j] \tag{22}$$

where $\tilde{\theta}_{-i} = \min_{j \neq i} \theta_j$.

It is found that we can construct the first-best weakly-robust mechanisms with ex ante budget-balanced transfers. We briefly show the case of two-country with $\Theta_i = [c, d] = [\frac{1}{2}, 1]$. Then

$$B = E[g(\theta)] = E[(b_1 + b_2) \ln(\frac{b_1}{\tilde{\theta}} + \frac{b_2}{\tilde{\theta}}) + (Y_1 + Y_2) - (b_1 + b_2)]$$

$$E[\sum_i c_i(\theta_{-i})] = E[(b_1 + b_2) \ln(\frac{b_1}{\theta_2} + \frac{b_2}{\theta_2}) + (b_1 + b_2) \ln(\frac{b_1}{\theta_1} + \frac{b_2}{\theta_1}) + 2(Y_1 + Y_2) - 2(b_1 + b_2)]$$

For each θ, we get the term of B at θ is greater than the term of $E[\sum_i c_i(\theta_{-i})]$ at θ by assuming that $Y_1 + Y_2$ is very large. Since the inequality holds for each state θ, so does the condition of the expectations in Theorem.

Furthermore, we would construct the first-best weakly-robust mechanisms with ex ante zero-expected net-transfers. For $i \neq j$,

$$B_i = E[b_j \ln(\frac{b_i}{\theta_j} + \frac{b_j}{\theta_j}) + Y_j - b_j]$$

$$E[c_i(\theta_j)] = E[(b_i + b_j)\ln(\frac{b_i}{\theta_j} + \frac{b_j}{\theta_j}) + (Y_i + Y_j) - (b_i + b_j)]$$

For each θ, we get the term of B_i at θ is greater than the term of $E[c_i(\theta_{-i})]$ at θ by assuming that Y_i is very large. Since the inequality holds for each state θ, so does the condition.

Proposition 1: For some large Y_i's with $c > 0$, there exists an IES which is first-best dominant-strategy incentive compatible, ex post individual rational (EPIR), and zero-expected net-transferred (ZENT).

Proof: By Theorem, we just only needs to show that for all θ, $c_i(\theta_j) - [g(\theta) - v_i(s(\theta), \theta_i)] \geq 0$ since then the condition of the expectations in Theorem holds. That is

$$Y_i - b_i \geq \sum_{j \neq i} b_j \ln[\sum_j \frac{b_j}{\tilde{\theta}}] - \sum_j b_j \ln[\sum_j \frac{b_j}{\tilde{\theta}_{-i}}]$$

which is equivalent to

$$Y_i \geq \ln[(\frac{\tilde{\theta}_{-i}}{\tilde{\theta}})^{\sum_{j \neq i} b_j} (\frac{\tilde{\theta}_{-i}}{\sum_j b_j})^{b_i}] + b_i.$$

By using (18), it holds. ∎

Since ZENT means EABB, the existence of Proposition 1 means that there exists an IES which is first-best dominant-strategy incentive compatible, ex post individual rational (EPIR), and ex ante budget balanced (EABB).

(2) Preferences Uncertainty

We assume that p_i's and Y_i's are public information and that b_i's are private information. Each country knows her preference parameter, but only knows the distribution of the others' parameters.

We assume that the utility functions are quasi-linear, $u^i(c_i, G) = c_i + b_i \ln(G)$ for all i. We assume that $p_i = p$ for all i. Then we can suppose that each country produces the optimal level of her contribution to the international public good, and that she consumes private good within the budget constraints.

From the results in section 2, we get that the Pareto allocations could be equivalent to the following allocation when the individual country's budget constraints hold.

$$g_i(p, Y_1, \cdots, Y_n, b_1, \cdots, b_n) = \frac{b_i}{p}$$

$$c_i(p, Y_1, \cdots, Y_n, b_1, \cdots, b_2) = Y_i - b_i$$

Then, we also obtain that

$$G(p, Y_1, \cdots, Y_n, b_1, \cdots, b_n) = \sum_i (\frac{b_i}{p})$$

and the indirect utility of country i

$$U_i(p, Y_1, \cdots, Y_n, b_1, \cdots, b_n) = b_i \ln(\sum_i (\frac{b_i}{p})) + Y_i - b_i$$

from the Pareto efficient allocation.

We will set $\theta_i = b_i$ for all i. Let $\Theta_i = [c, d]$ be the set of types for country $i \in I$. We assume that c the lower bound of the type is not zero. Thus, there is substantial cost burden to the international public goods. We assume also that each country's income is not so small in the sense that for all i

$$Y_i \geq \ln[(\frac{nd}{(n-1)d + c})^{(n-1)d} (\frac{p}{(n-1)d + c})^d] + c \tag{23}$$

where n is the number of countries. Then the Pareto rule would be like the following: The IES receives the reports on the types from the countries. Based on the reports, the IES then decides the individual contribution to the international public good and (thus) the consumption level of private goods for each country. Denoting the reports $\hat{\theta} = (\hat{\theta}_1, \cdots, \hat{\theta}_n)$, we can express the Pareto allocations, $(g_1(\hat{\theta}), \cdots, g_n(\hat{\theta}), c_1(\hat{\theta}), \cdots, c_n(\hat{\theta}))$ for any $\hat{\theta}$. Specifically, $g_i(\hat{\theta}) = \sum_i (\frac{\hat{\theta}_i}{p_i})$, $c_i(\hat{\theta}) = Y_i - \hat{\theta}_i$, and $G(\hat{\theta}) = \sum_i g_i(\hat{\theta})$. Let A be the set of all possible alternative outcomes. Then $A = \{(c_1, \cdots, c_n, g_1, \cdots, g_n)\}$ should satisfy the budget constraints for each country.

Each type θ_i of country i has a valuation function $v_i(\cdot, \theta_i)$ over A. The payoff of country i with type θ_i from the reports $\hat{\theta}$ is

$$v_i((c_i(\hat{\theta}), G(\hat{\theta})), \theta_i) = \theta_i \ln(G(\hat{\theta})) + c_i(\hat{\theta}) = \theta_i \ln(\sum_i (\frac{\hat{\theta}_i}{p})) + Y_i - \hat{\theta}_i \tag{24}$$

It is easily proven that Θ_i is *convex* for all i since for all $a \in A$,

$$v_i(a, \theta_i(\lambda)) = (1-\lambda) v_i(a, \theta_i') + \lambda v_i(a, \theta_i'')$$

by taking $\theta_i(\lambda) = (1-\lambda)\theta_i' + \lambda \theta_i''$ for any number $\lambda \in [0, 1]$.

The set of states is denoted as $\Theta \equiv \prod_{i=1}^n \Theta_i$ with a typical element $\theta = (\theta_1, \cdots, \theta_n)$. The common prior belief on the states is given by a probability measure F on the Borel subsets of Θ. We assume that each country knows her own type and has a common prior on the states.

The global gain function from the Pareto allocation is

$$g(\theta) \equiv [\sum_i \theta_i \ln(\sum_i (\frac{\theta_i}{p})) + \sum_i Y_i - \sum_i \theta_i]$$

A direct mechanism is denoted by $(\Theta, < s, t >)$ where Θ is the message space of type reports and $< s, t >$ is an outcome function which consists of a decision rule $s : \Theta \to A$ and a transfer scheme $t = (t_1, \cdots, t_n)$ with $t_i : \Theta \to \Re$. Given $< s, t >$, country i's payoff with type θ_i from a report θ' is $v_i(s(\theta'), \theta_i) + t_i(\theta')$. We will use the notation $< s, t >$ for a direct mechanism.

The (ex ante) expected budget deficit for country i in the zero-charge Groves mechanism is

$$B_i \equiv E[g(\theta) - v_i(s(\theta), \theta_i)] = E[\sum_{j \neq i} \theta_j \ln(\sum_j (\frac{\theta_j}{p})) + \sum_{j \neq i} Y_j \sum_{j \neq i} \theta_j. \tag{25}$$

Then,

$$B \equiv \sum_i B_i = E[(n-1)g(\theta)] = (n-1)E[\sum_i \theta_i \ln(\sum_i (\frac{\theta_i}{p})) + \sum_i Y_i - \sum_i \theta_i] \tag{26}$$

The outside option payoff of country i at state θ is assumed to be $u_i^0(\theta) = 0$. Since the IES does not observe country i's type, the maximal amount the IES can charge on country i without violating country i's EPIR condition is $c_i(\theta_{-i}) = \min_{\theta_i}\{g(\theta) - u_i^0(\theta)\}$ for all θ_{-i}. Then, the (ex ante) expected lump-sum charge without violating country i's EPIR condition is

$$E[c_i(\theta_{-i})] = E[\min_{\theta_i}\{g(\theta) - u_i^0(\theta)\}].$$

Specifically,

$$E[c_i(\theta_{-i})] = E[\sum_j \theta_j \ln(\sum_{j\neq i}(\frac{\theta_j}{p} + \frac{c}{p})) + \sum_j Y_j - \sum_{j\neq i}\theta_j - c] \tag{27}$$

For some problems with a regular condition in ( ), we can construct the first-best weakly-robust mechanisms with ex ante zero-expected net-transfers.

Proposition 2: For some large Y_i's with $c > 0$, there exists an IES which is first-best dominant-strategy incentive compatible, ex post individual rational (EPIR), and zero-expected net-transferred (ZENT).

Proof: By Theorem, we just only needs to show that for all θ $c_i(\theta_j) - [g(\theta) - v_i(s(\theta), \theta_i)] \geq$, since then the condition of the expectations in Theorem holds. That is

$$Y_i - c \geq \sum_{j\neq i}\theta_j \ln[\sum_j \frac{\theta_j}{p}] - \sum_j \theta_j \ln[\frac{\sum_{j\neq i}\theta_j + c}{p}]$$

which is equivalent to

$$Y_i \geq \ln[(\frac{\sum_j \theta_j}{\sum_{j\neq i}\theta_j + c})^{\sum_{j\neq i}\theta_j}(\frac{p}{\sum_{j\neq i}\theta_j + c})^{\theta_i}] + c.$$

By using (23), it holds. ∎

Since ZENT means EABB, the existence of Proposition 1 means that there exists an IES which is first-best dominant-strategy incentive compatible, ex post individual rational (EPIR), and ex ante budget balanced (EABB).

To be sure, we can find a range of problems where we can construct the first-best weakly-robust mechanisms with ex ante budget-balanced transfers, but we cannot construct the first-best weakly-robust mechanisms with ex ante zero-expected net-transfers.

We first suggest the range of parameters for which our argument holds and then prove it. We use the case of two-country with $\Theta_i = [c, d]$ and $p_i = p = 1$ for all i. We assume that

$$E[\ln[(\frac{(\theta_1+\theta_2)^2}{\theta_1 \times \theta_2})^{\theta_1+\theta_2}] + 2c] > Y_1 + Y_2 \geq E[\ln[(\frac{\theta_1+\theta_2}{\theta_1 \times \theta_2})^{\theta_1+\theta_2}] + 2c] \tag{28}$$

It is found that we can construct the first-best weakly-robust mechanisms with ex ante budget-balanced transfers. We only need to show that $E[\sum_i c_i(\theta_{-i})] \geq B$ holds. We obtain that

$$B = E[g(\theta)] = E[(\theta_1+\theta_2)\ln(\theta_1+\theta_2) + (Y_1+Y_2) - (\theta_1+\theta_2)]$$

$$E[\sum_i c_i(\theta_{-i})] = E[(\theta_1+\theta_2)\ln(\theta_2+c) + (\theta_1+\theta_2)\ln(\theta_1+c) + 2(Y_1+Y_2) - (\theta_1+\theta_2) - 2c].$$

It is easy to find that $E[\sum_i c_i(\theta_{-i})] \geq B$ is equivalent to the right (weak) inequality of (28).

However, we cannot construct the first-best weakly-robust mechanisms with ex ante zero-expected net-transfers. We only need to show that $E[c_i(\theta_j)] \geq B_i$ dose not hold for some i. Let's assume that it holds for all i. By calculation, we get that for $i \neq j$,

$$B_i = E[\theta_j \ln(\theta_i+\theta_j) + Y_j - \theta_j]$$

$$E[c_i(\theta_j)] = E[(\theta_i+\theta_j)\ln(\theta_j+c) + Y_i + Y_j - \theta_j - c].$$

It is easy to obtain that for all i, $E[c_i(\theta_j)] - B_i$ is equivalent to $Y_i \geq E[\ln[(\frac{\theta_1+\theta_2}{\theta_j+c})^{\theta_1+\theta_2}] + c$. Summing up over i results in the contradiction to the left inequality of (28).

5. Conclusion

We try to find out an optimal mechanism of the international economic system, which contains the decision rule about the supply of and demand for the international public goods under incomplete information. In particular, we analyze the cases of uncertainty in production costs and preferences on the international public goods. For this, we introduce the framework of the games with incomplete information and think of the international economic system as a rule of the game and implement the first best allocation with incomplete information through direct revelation mechanism.

Based on the mechanism theory for optimal supply of and demand for international public goods, we observe that there exists a first-best dominant-strategy mechanism with ex post individual rationality (EPIR), and ex ante budget balancedness (EABB) or zero expected net-transferred (ZENT). Conceptually, the first-best dominant-strategy mechanism with ZENT is more restrictive that the mechanism with EABB. We specifically determined the range of the parameters where the former is not possible while the latter is possible.

The results of the application to each case of uncertainty are interesting and meaningful. We can concretely find out that there exists a first-best dominant-strategy mechanism with ex post individual rationality and zero-expected net-transferred. Even though there is uncertainty of IPG's production cost over countries, the IES can find out the mechanism to satisfy both incentive and efficiency motivation through the transfers.

Our characterization theorem suggest the structure of the incentive transfers for the first-best dominant-strategy mechanism with ex post individual rationality and zero-expected net-transferred. The net-transfer could be decomposed into three parts. The first is related to production and consumption of the international public good. The second is the (negative) evaluation based on peer-group reporting. The third is the subsidy from the IES.

The first part for the case of cost uncertainty contains the payment of consuming-only countries to the producing country with the lowest cost. Thus, for the case of cost uncertainty the net-transfer has three different monetary transfers; the payment for the surrogate production of the international public good, the subsidy from the IES for the cooperative interest, and the evaluation charge to the IES based on the peer-group evaluation.

Our result depends on the assumptions of quasi-linear utility functions, utilitarian social welfare functions, linear cost functions for the international public good, etc. Since our characterization theorem has a condition not only necessary but also sufficient for the possibility of the mechanism, we can extend the domain of utility functions by sacrificing the 'necessary' part. Thus, we suggest our mechanism as a mechanism to the IES. We do not lose the generality of our analysis with other social welfare functions, but the construction of the IES for other social welfare functions needs other works. We assume that outside option payoff is zero in the sense that any country's exit from the IES would be autarky. However, there are many different scenarios. One open interesting question would be what might be the mechanism when the outside option is the Nash equilibrium.

There are so many kinds of international public goods for the international economic society. Each of them has different characteristics in its measure of uncertainty of production cost and preferences among countries. Therefore the international economic society should develop the very unique form of international economic entity for each good. This paper's results would be one framework for that development.

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