Endogenized Monetary Policy and Exchange Rate Determination

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Abstract

This paper develops a monetary model of exchange rate determination by incorporating McCallum's (1994) monetary policy function into the traditional models. It shows that specific institutional arrangement or policy setting may provide yet another explanation for why the traditional monetary approach fails to explain and predict exchange rate movements. The traditional monetary model takes money supply as exogenous, among other assumptions, and focuses on the behavior of portfolio selection of the private sector. If the policy action of the monetary authority is incorporated, the modified model then contains features that are present both in the random walk model and the traditional model, and may be able to explain empirical exchange rate behavior better.

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This paper develops a monetary model of exchange rate determination by incorporating McCallum's monetary policy function [McCallum (1994)], and shows that specific institutional arrangement or policy setting may provide yet another explanation on why the traditional monetary approach fails to explain and predict exchange rate movements. The traditional monetary model takes money supply as exogenous, among other assumptions, and focuses on the behavior of portfolio selection of the private sector. If a certain type of monetary policy is incorporated into the traditional model, the modified model will contain features present both in the random walk model and the traditional model, which could help explain exchange rate behavior better.

This paper is organized as follows. Section 1 describes the monetary approach with a simple small open economy model and explains why this approach experiences difficulties in predicting exchange rate movements. Section 2 introduces McCallum's monetary policy function into a monetary model originally developed by Weymark (1995), as her model specifies certain aspects of money supply behaviors, thus allowing us to incorporate McCallum's type of monetary policy easily. Section 3 presents the rational expectations solution of the modified model and compares this reduced-form equation with the traditional model described in section 1. The final section concludes.

1. The monetary approach to exchange rate determination

The monetary approach to exchange determination in a small open economy can be described with the system of equations below

$$m_t^d = p_t + \beta_1 y_t - \beta_2 i_t \tag{1}$$

$$p_t = p_t^* + e_t \tag{2}$$

$$i_t - i_t^* = E_t\left(e_{t+1}\right) - e_t \tag{3}$$

$$m_t^s = m_t \tag{4}$$

Equation (1) is the standard money demand function. Real money demand is determined by income (y_t) and domestic interest rates (i_t). (2) and (3) are the purchasing power parity and uncovered interest parity condition, respectively. p_t, p_t^*, e_t, i_t, and i_t^* are domestic prices, foreign prices, exchange rates, domestic interest rates, and foreign interest rates, respectively. In a small open economy, they also represent goods market and financial market equilibrium conditions. Equation (4) is money supply, which is exogenously given. Except for i_t and i_t^*, all other variables are represented in natural log form.

Substitute (2)~(4) into (1), we get

$$e_t = \left(\frac{1}{1+\beta_2}\right)m_t - \left(\frac{1}{1+\beta_2}\right)p_t^* - \left(\frac{\beta_1}{1+\beta_2}\right)y_t + \left(\frac{1}{1+\beta_2}\right)i_t^* + \left(\frac{\beta_2}{1+\beta_2}\right)E_t\left(e_{t+1}\right) \tag{5}$$

Let $K = (1,\ -1,\ -\beta_1,\ 1)$, $X_t' = \left(m_t,\ p_t^*,\ y_t,\ i_t^*\right)$. Here, the matrix X_t' can be regarded as economic fundamentals.

We can rewrite (5) as

$$e_t = \left(\frac{1}{1+\beta_2}\right)KX_t + \frac{\beta_2}{1+\beta_2}E_t\left(e_{t+1}\right) \tag{6}$$

Using the reiterative method, we get

$$e_t = \left(\frac{1}{1+\beta_2}\right)\sum_{i=0}^{\infty}\left(\frac{\beta_2}{1+\beta_2}\right)^i KE_t\left(X_{t+i}\right) \tag{7}$$

(7) is the well-known present-value model. According to (7), the sum of current and future expected values of fundamentals determines the current

exchange rate. This in turn means the source of exchange rate fluctuations is determined by changes in the current and future expected values of fundamentals

$$\Delta e_t = \left(\frac{1}{1+\beta_2}\right)\sum_{i=0}^{\infty}\left(\frac{\beta_2}{1+\beta_2}\right)^i K \Delta E_t\left(X_{t+i}\right) \tag{8}$$

Suppose the expectation formation mechanism follows a simple AR(1) process, (7) becomes

$$e_t = \left(\frac{1}{1+\beta_2}\right) K \left(I - \frac{\beta_2}{1+\beta_2}\rho\right)^{-1} X_t \tag{9}$$

From (9), the current exchange rate is determined by current fundamentals. Since fundamentals are not volatile as the exchange rate, the explaining power of the monetary approach collapses.

2. The Model

(1) McCallum's monetary policy function

$$i_t - i_t^* = a\left(e_t - e_{t-1}\right) + b\left(i_{t-1} - i_{t-1}^*\right) \tag{10}$$

The monetary authority sets the difference between domestic interest rate and foreign interest rates in the current period based on the leaning-against-the-wind strategy $\left(e_t - e_{t-1}\right)$ and the interest- rate smoothing policy $\left(i_{t-1} - i_{t-1}^*\right)$.

In the following model, we put the leaning-against-the-wind strategy $\left(e_t - e_{t-1}\right)$ in the monetary reaction function and the interest-rate smoothing policy $\left(i_{t-1} - i_{t-1}^*\right)$ in the foreign exchange intervention function. Although we separate McCallum's monetary policy function into two parts, in the end of this section we will show that his policy function can be derived from our modified model.

(2) The modified monetary model

$$m_t^d = p_t + \beta_1 y_t - \beta_2 i_t + \xi_t^{md} \tag{11}$$

The money demand function (11) is similar to (1), except that here we add an error term.

$$p_t = p_t^* + e_t \tag{12}$$

The purchasing power parity condition is the same as (2).

$$i_t - i_t^* = E_t\left(e_{t+1}\right) - e_t + \delta_t \tag{13}$$

Equation (13) is a modification of the uncovered interest parity condition. We add risk premium in this condition to indicate that domestic and foreign assets are imperfect substitutes. Therefore the monetary authority is able to conduct sterilization policy to isolate impacts from the foreign exchange market. Without this term, in equilibrium the private sector will be indifferent to holding domestic assets or foreign assets. For simplicity, we assume risk premium follows a random walk process.

$$m_t^s = m_{t-1}^s + \Delta d_t + \Delta r_t \tag{14}$$

$$\Delta d_t = \frac{\mu_t D_t - \mu_{t-1} D_{t-1}}{M_{t-1}} \tag{15}$$

$$\Delta r_t = \frac{\mu_t R_t - \mu_{t-1} R_{t-1}}{M_{t-1}} \tag{16}$$

(14) is the money supply equation. Money supply in t is equal to money supply in $t-1$ plus changes in the monetary authority's domestic credit and foreign assets scaled by nominal money supply in $t-1$. (15) and (16) give

the definitions of domestic credit (Δd_t) and (Δr_t) [1]. μ_t is the money multiplier.

$$\Delta d_t = -\theta_1 \Delta r_t^c + \theta_2 \left(i_{t-1} - i_{t-1}^*\right) + \xi_t^{dc} \tag{17}$$

Equation (17) specifies the monetary authority's reaction function. It contains two parts. The first one is sterilization policy. We assume it is proportional to the changes in foreign assets of the monetary authority. The second one is the interest-rate smoothing policy from McCallum's monetary policy function. ξ_t^{dc} is the innovation in monetary policy.

$$\Delta r_t = \Delta r_t^a + \Delta r_t^c \tag{18}$$

Changes in foreign assets of the monetary authority also contain two parts. The first one is interest received from holding foreign exchange reserves $\left(\Delta r_t^a\right)$. Since it has nothing to do with foreign exchange intervention, it should be excluded beforehand. The second one is the amount of intervention that the monetary authority carried out in the foreign exchange market, scaled by nominal money supply of the previous period $\left(\Delta r_t^c\right)$.

$$\Delta r_t^c = -\vartheta \Delta e_t + \xi_t^{rc} \tag{19}$$

Equation (19) specifies the foreign exchange intervention function. The monetary authority adopts the leaning-against-the-wind strategy to smooth movements of the exchange rate. It is similar to McCallum's leaning-against-the-wind strategy in his monetary policy function, except that in our setting it operates through changes in foreign asset first. We also add ξ_t^{rc} to indicate shocks in the foreign exchange intervention function.

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[1] (14) is obtained using the relationship $d \ln x = dx/x$. The sources of reserve money are the sum of domestic credit and foreign assets of the monetary authority.

(3) McCallum's policy function in the modified model

From the model, we can write changes in the money demand and supply as

$$\Delta m_t^d = \left(p_t^* - p_{t-1}^*\right) + \left(e_t - e_{t-1}\right) + \beta_1 \left(y_t - y_{t-1}\right) - \beta_2 \left(i_t - i_{t-1}\right) \tag{20}$$

$$\Delta m_t^s = -\left(1-\theta_1\right) \vartheta \left(e_t - e_{t-1}\right) + \theta_2 \left(i_{t-1} - i_{t-1}^*\right) \tag{21}$$

Therefore

$$\begin{aligned} &\left(p_t^* - p_{t-1}^*\right) + \left(1-\theta_1\right) \vartheta \left(e_t - e_{t-1}\right) - \theta_2 \left(i_{t-1} - i_{t-1}^*\right) \\ &= -\beta_1 \left(y_t - y_{t-1}\right) + \beta_2 \left(i_t - i_{t-1}\right) \\ &= -\beta_1 \left(y_t - y_{t-1}\right) + \beta_2 \left(i_t - i_t^*\right) - \beta_2 \left(i_{t-1} - i_{t-1}^*\right) + \beta_2 \left(i_t^* - i_{t-1}^*\right) \end{aligned} \tag{22}$$

Rearranging terms, we get

$$i_t - i_t^* = -\frac{1}{\beta_2}\left(p_t^* - p_{t-1}^*\right) + \left(y_t - y_{t-1}\right) - \left(i_t^* - i_{t-1}^*\right) + \frac{1+\left(1-\theta_1\right)\vartheta}{\beta_2}\left(e_t - e_{t-1}\right) + \frac{\beta_2 - \theta_2}{\beta_2}\left(i_{t-1} - i_{t-1}^*\right) \tag{23}$$

Equation (23) is similar to McCallum's monetary policy function. As is evident from this equation, his monetary policy function is a partial solution contained in our model.

3. The rational expectations solution of the modified monetary model

Changes in nominal money demand can be written as

$$\Delta m_t^d = \Delta p_t^* + \left(1+\beta_2\right) \Delta e_t + \beta_1 \Delta y_t - \beta_2 \Delta i_t^* - \beta_2 \Delta E_t \left(e_{t+1}\right) - \beta_2 \Delta \delta_t + \Delta \xi_t^{md} \tag{24}$$

And changes in nominal money supply as

$$\Delta m_t^s = -\left(1-\theta_1\right) \vartheta \Delta e_t + \theta_2 E_{t-1}\left(e_t\right) - \theta_2 e_{t-1} + \theta_2 \delta_{t-1} + \xi_t^{dc} - \left(1-\theta_1\right) \xi_t^{rc} \tag{25}$$

Since money demand equals money supply, we get

$$\begin{aligned}&\left[(1+\beta_2)+(1-\theta_1)\vartheta\right]e_t-\left[(1+\beta_2)+(1-\theta_1)\vartheta-\theta_2\right]e_{t-1}\\&=KX_t-KX_{t-1}+\beta_2E_t(e_{t+1})+(\theta_2-\beta_2)E_{t-1}(e_t)+v_t\end{aligned}\tag{26}$$

where

$$K=(-1,\ \beta_1,\ \beta_2)=(-1,\ 1,\ \beta_2),\quad X_t'=\left(p_t^*,\ y_t,\ i_t^*\right)$$

$$v_t=\beta_2\delta_t+(\theta_2-\beta_2)\delta_{t-1}-\xi_t^{md}+\xi_{t-1}^{md}+\xi_t^{dc}-(1-\theta_1)\xi_t^{rc}$$

Here for simplicity, in the following we assume that $\beta_1=1$. The matrix X_t' contains three variables: foreign prices, domestic income, and foreign interest rates. We refer them as fundamentals. Note that the meaning of fundamentals here is a little bit different from that in the traditional monetary model. In addition to the three variables, fundamentals in the traditional model also include monetary aggregates, which are exogenously given. In the modified model money supply behaviors are specified and therefore monetary aggregates do not appear in the fundamentals matrix.

Taking the rational expectations operator in both sides of (26), we get

$$\begin{aligned}&\left[(1+\beta_2)+(1-\theta_1)\vartheta\right]E_{t-1}(e_t)-\left[(1+\beta_2)+(1-\theta_1)\vartheta-\theta_2\right]E_{t-1}(e_{t-1})\\&=KE_{t-1}(X_t)-KE_{t-1}(X_{t-1})+\beta_2E_{t-1}(e_{t+1})+(\theta_2-\beta_2)E_{t-1}(e_t)+E_{t-1}(v_t)\end{aligned}\tag{27}$$

We define the forward operator as

$$F^ix_t=E_tx_{t+i}\tag{28}$$

And rewirte (27) as

$$\begin{aligned}&\left[F^2-\frac{(1+\beta_2)+(1-\theta_1)\vartheta-(\theta_2-\beta_2)}{\beta_2}F+\frac{(1+\beta_2)+(1-\theta_1)\vartheta-\theta_2}{\beta_2}\right]E_{t-1}(e_{t-1})\\&=-\frac{1}{\beta_2}KE_{t-1}(X_t)+\frac{1}{\beta_2}KE_{t-1}(X_{t-1})-\frac{1}{\beta_2}E_{t-1}(v_t)\end{aligned}\tag{29}$$

We also define the lag operator

$$E_{t-1}(e_{t-1}) = LE_{t-1}(e_t) \tag{30}$$

Equation (29) becomes

$$\begin{aligned}&\left[F^2 - \frac{(1+\beta_2)+(1-\theta_1)\vartheta-(\theta_2-\beta_2)}{\beta_2}F + \frac{(1+\beta_2)+(1-\theta_1)\vartheta-\theta_2}{\beta_2}\right]LE_{t-1}(e_t)\\ &= -\frac{1}{\beta_2}KE_{t-1}(X_t) + \frac{1}{\beta_2}KE_{t-1}(X_{t-1}) - \frac{1}{\beta_2}E_{t-1}(v_t)\end{aligned} \tag{31}$$

Therefore

$$(F-\lambda_1)(F-\lambda_2)LE_{t-1}(e_t) = -\frac{1}{\beta_2}KE_{t-1}(X_t) + \frac{1}{\beta_2}KE_{t-1}(X_{t-1}) - \frac{1}{\beta_2}E_{t-1}(v_t) \tag{32}$$

λ_1 and λ_2 are characteristic roots with the following relationships

$$\begin{aligned}\lambda_1 + \lambda_2 &= \frac{(1+\beta_2)+(1-\theta_1)\vartheta-(\theta_2-\beta_2)}{\beta_2}\\ \lambda_1\lambda_2 &= \frac{(1+\beta_2)+(1-\theta_1)\vartheta-\theta_2}{\beta_2}\end{aligned} \tag{33}$$

If the model exhibits saddle-point path property, then $|\lambda_1|<1$ and $|\lambda_2|>1$.

Moving $F-\lambda_2$ to the right-hand side, and note that

$$(F-\lambda_2)^{-1} = -\lambda_2^{-1}\left(1-\lambda_2^{-1}F\right)^{-1} = -\lambda_2^{-1}\sum_{i=0}^{\infty}\lambda_2^{-i}F^i$$

(33) becomes

$$(F-\lambda_1)LE_{t-1}(e_t) = \frac{1}{\beta_2\lambda_2}E_{t-1}\left[\sum_{i=0}^{\infty}\lambda_2^{-i}F^i\left(KX_t - KX_{t-1} + v_t\right)\right] \tag{34}$$

Or

$$(1-\lambda_1 L)E_{t-1}(e_t) = \frac{1}{\beta_2\lambda_2}E_{t-1}\left[\sum_{i=0}^{\infty}\lambda_2^{-i}F^i\left(KX_t - KX_{t-1} + v_t\right)\right] \tag{35}$$

Using (30), the left-hand side of (25) becomes

$$(1-\lambda_1 L) E_{t-1}(e_t) = E_{t-1}(e_t) - \lambda_1 e_{t-1} \tag{36}$$

and using (28), the right-hand side of (35) becomes

$$\frac{1}{\beta_2\lambda_2}\sum_{i=0}^{\infty}\lambda_2^{-i}E_{t-1}\left(KX_{t+i}-KX_{t+i-1}+v_{t+i}\right) \tag{37}$$

Substitute (36) and (37) into (35)

$$E_{t-1}(e_t)=\lambda_1 e_{t-1}+\frac{1}{\beta_2\lambda_2}\sum_{i=0}^{\infty}\lambda_2^{-i}E_{t-1}\left(KX_{t+i}-KX_{t+i-1}+v_{t+i}\right) \tag{38}$$

Therefore

$$E_t(e_{t+1})=\lambda_1 e_t+\frac{1}{\beta_2\lambda_2}\sum_{i=0}^{\infty}\lambda_2^{-i}E_t\left(KX_{t+i+1}-KX_{t+i}+v_{t+i+1}\right) \tag{39}$$

Substitute (38) and (39) back to (26), we have

$$\begin{aligned}
&\left[(1+\beta_2)-(1-\theta_1)\vartheta\right]e_t-\left[(1+\beta_2)-(1-\theta_1)\vartheta-\theta_2\right]e_{t-1}\\
&=\beta_2\left[\lambda_1 e_t+\frac{1}{\beta_2\lambda_2}\sum_{i=0}^{\infty}\lambda_2^{-i}E_t\left(KX_{t+i+1}-KX_{t+i}+v_{t+i+1}\right)\right]\\
&\quad+(\theta_2-\beta_2)\left[\lambda_1 e_{t-1}+\frac{1}{\beta_2\lambda_2}\sum_{i=0}^{\infty}\lambda_2^{-i}E_{t-1}\left(KX_{t+i}-KX_{t+i-1}+v_{t+i}\right)\right]\\
&\quad+KX_t-KX_{t-1}+v_t
\end{aligned} \tag{40}$$

From the characteristic equation we have $\beta_2\lambda_1=(1+\beta_2)+(1-\theta_1)\vartheta-\theta_2$, therefore

$$\begin{aligned}
&\left[(1+\beta_2)-(1-\theta_1)\vartheta\right]e_t-\left[(1+\beta_2)-(1-\theta_1)\vartheta-\theta_2\right]e_{t-1}\\
&-\beta_2\lambda_1-(\theta_2-\beta_2)\lambda_2 e_{t-1}=\theta_2\left(e_t-\lambda_1 e_{t-1}\right)
\end{aligned} \tag{31}$$

Substitute (31) into (40) and re-arrange terms

$$e_t = \lambda_1 e_{t-1} + \frac{1}{\theta_2}\sum_{i=0}^{\infty}\lambda_2^{-i}E_t\left(KX_{t+i} - KX_{t+i-1}\right) + \frac{\theta_2 - \beta_2}{\theta_2\beta_2\lambda_2}\sum_{i=0}^{\infty}\lambda_2^{-i}E_{t-1}\left(KX_{t+i} - KX_{t+i-1}\right)$$
$$+\frac{1}{\theta_2}\sum_{i=0}^{\infty}\lambda_2^{-i}E_t\left(v_{t+i}\right) + \frac{\theta_2 - \beta_2}{\theta_2\beta_2\lambda_2}\sum_{i=0}^{\infty}\lambda_2^{-i}E_{t-1}\left(v_{t+i-1}\right) \qquad (32)$$

In period $t-1$ and t, the decision units learn their $t-1$ and t expectation errors, respectively, while the expected future expectation error are zero, therefore (32) can be rewritten as

$$e_t = \lambda_1 e_{t-1} + \frac{1}{\theta_2}\sum_{i=0}^{\infty}\lambda_2^{-i}E_t\left(KX_{t+i} - KX_{t+i-1}\right) + \frac{\theta_2 - \beta_2}{\theta_2\beta_2}\sum_{i=0}^{\infty}\lambda_2^{-i-1}E_{t-1}\left(KX_{t+i} - KX_{t+i-1}\right) + w_t$$

(33)

where

$$w_t = \frac{1}{\theta_2}v_t + \frac{\theta_2 - \beta_2}{\theta_2\beta_2\lambda_2}\left[\left(\theta_2 - \beta_2\right)\delta_{t-1} - \xi_t^{md}\right]$$

(33) is the rational expectations solution of the exchange rate. Based on (33), the current exchange rate (e_t) is determined by the exchange rate in the previous period (e_{t-1}), revised expectation on the discount sum of expected fundamentals made in period t [$\sum_{i=0}^{\infty}\lambda_2^{-i}E_t(KX_{t+i} - KX_{t+i-1})$], and revised expectation on the discount sum of expected fundamentals made in period $t-1$ [$\sum_{i=0}^{\infty}\lambda_2^{-i}E_{t-1}(KX_{t+i} - KX_{t+i-1})$]. In other words, while the modified model is based on the monetary approach, it contains features both in the random walk model and the traditional monetary model. The interest-rate smoothing policy in the monetary authority's monetary policy function is the main cause for this hybrid result. As the monetary authority smoothes interest rates based on the difference between domestic and foreign interest rates of the previous period, under rational expectations decision units take this monetary action into consideration to form their views on current

exchange rates. Through the uncovered interest parity condition, this in turns means that the lagged exchange rate plays a role in explaining movements of exchange rates. The decision units also revise their expectations on fundamentals in period $t-1$ and t based on new information, which creates another source of exchange rate fluctuations.

4. Concluding Remark

The traditional monetary model explains movements of exchange rates through the behavior of portfolio selection of the private sector, taking money supply exogenously given. In a small open economy under rational expectations, this implies the current exchange rate is determined by the sum of current and future expected values of fundamentals. However, in reality expected fundamentals are not volatile as the exchange rate, so the explaining power of the monetary approach collapses. In the real world, monetary policy actions of the monetary authority may constantly change the public's expectations. If a certain type of monetary policy, namely McCallum's monetary policy function, is incorporated into the traditional model, the new model will contain features present both in the random walk model and the traditional model. Furthermore, the exchange rate is also determined by the revised expectation on the discount sum of expected fundamentals made in period t and $t-1$, which may create another source of exchange rate fluctuations. We believe models that consider specific institutional arrangement or policy setting help explain exchange rate behavior better than the traditional monetary model.

Reference

McCallum, B. T. (1994) "A Reconsideration of the Uncovered Interest Parity Relationship," *Journal of Monetary Economics* 33, 105-132.

Weymark, D. N. (1995) “Estimating Exchange Market Pressure and the Degree of Exchange Market Intervention for Canada,” *Journal of International Economics* 39, 273-295.