Common Trends, Common Cycles and Forecasting

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Abstract

Given that the presence of cointegration in economic time series has a number of implications in time series analysis, the recent literature has devoted considerable attention to estimation of and testing for cointegrating relationships. As for the usage of cointegrated systems, many researchers have presented evidence that imposing long-run cointegrating relations improves forecasts. In this paper, we show that the forecasting performance can further be enhanced by taking into account of short-run common cycles. That is, we can improve our forecasts by using more information on both the short-run and the long-run co-movements.

In doing so, we first discuss the testing procedure for the existence of common cycles among cointegrated variables developed in Vahid and Engle (1993). The method is then used to examine whether and to what extent Korean GDP, consumption, and investment share common cycles, and evidence is presented for the existence of short-run common features.

Based on simulation experiments, forecasting performances of the alternative econometric models are compared. The basic finding is that the restricted error correction models, which consider common-cycle restrictions as well as cointegrating relations, outperform the usual error correction models, which take only cointegrating restrictions into account.

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I. Introduction

There are many economic theories that imply comovement of economic time series. It is also a well-known stylized fact that numerous sets of economic variables move together. As comovement among time series data has an important implication in modeling the dynamic behavior of economic variables, an increasing attention has been devoted to testing for the existence of such common components and estimating their patterns.

A substantial research in this area has stemmed from the seminal work on cointegration developed by Granger (1983) and Engle and Granger (1987). The idea of cointegration is concerned with long-run comovement among nonstationary time series, which indicates the existence of common stochastic trends. Another topic in this area is common serial correlation feature discussed in Engle and Kozicki (1993), which suggests that many stationary series may share common cycles. These two types of comoving features has recently been playing an important role in charactering the dynamics of many economic time series that can be characterized as long-run equilibrium relations.

Given the impact of cointegration analysis on both empirical and theoretical econometrics, numerous papers have been published on testing for the existence of long-run comovements and estimation techniques of cointegrating parameters. However, relatively little attention has been given to investigation into short-run comovement and its implications. It is very recently that the literature on common features has gained some momentum, which has brought a special issue on the theme [see Anderson *et al.* (2006) and papers therein].

It is now well known that the use of cointegrating restrictions, if correctly imposed, may lead to efficiency gain in estimating the dynamic models of many economic time series. It is also expected that efficiency gain can further be enhanced by using common-factor restrictions on short-run components (common cycles). That is, the joint use of common-trend and common-cycle restrictions would lead to more precise estimates of the dynamic models than those based on common-trend restrictions alone. In fact, Issler and Vahid (2001) and Giannone *et al.* (2006) provided evidence that common-factor restrictions on short-run components (common cycles) are particularly important for accurate estimation of vector error correction models, and hence for impulse-response calculations.

Econometric models are often used to generate forecasts for future values of time series under consideration as well as to identify the relationship among the variables, and the presence of cointegrating relations has another important implication in forecasting economic time series, in particular, at long horizons, as discussed earlier in Engle and Yoo (1987) [see also Clements and Hendry (1995), Lin and Tsay (1996) and Löf and Franses (2001)]. It is naturally expected that the forecasting performance of econometric models can further be enhanced by incorporating common-cycle restrictions as well as common-trend restrictions. This paper presents evidence that we can improve our forecasts by using more information on both the short-run and the long-run co-movements.

In doing so, we first discuss the testing procedure for the existence of common cycles among cointegrated variables developed in Vahid and Engle (1993). The method is then used to examine whether and to what extent Korean GDP, consumption, and investment share common cycles. Evidence for the existence of short-run common features is presented, which confirms earlier results in this area.

Based on simulation experiments as well as real data sets, forecasting performances of alternative econometric models are then compared. Our simulation results show that the restricted error correction models, which consider common-cycle restrictions as well as cointegrating relations, outperform the usual error correction models, which take only long-run relations into account.

This paper is organized as follows. Section II discusses a simple theoretical framework for the existence of both common trends and common cycles. Section III explains estimation and testing procedures for cointegration and common cycles, and presents empirical results. Section IV discusses forecasting performance of alternative econometric models, using simulation experiments. Concluding remarks are in section V, which summarizes the significance of our empirical results with a brief discussion about its potential implications on building econometric models for generating forecasts.

II. Common trends and common cycles

There are many empirical and theoretical models which suggest the existence of common features such as common trends and common cycles among economic variables [see, *inter alia*, Vahid and Engle (1993) and Anderson *et al.* (2006)]. A simple theoretical framework for the existence of both common trends and common cycles can be illustrated by the standard real business cycle model of King *et al.* (1988).

As discussed in Issler and Vahid (2001), the dynamic stochastic general equilibrium model developed by King *et al.* (1988) suggests that output, consumption and investment have a common trend and a common cycle. Under the assumption that the labor augmenting technological progress (X_t) follows a random walk with drift, [1] i.e., $\log(X_t) = \mu + \log(X_{t-1}) + \varepsilon_t$, the closed-form solutions for the logarithms of these variables can be represented as:

$$\log(Y_t) = \log(X_t) + \log(y) + \hat{y}_t \tag{2.1}$$

$$\log(C_t) = \log(X_t) + \log(c) + \hat{c}_t, \tag{2.2}$$

[1] The solutions can also be obtained under additional specific assumptions on preferences, production function and depreciation rates as presented by King *et al.* (1988).

$$\log(I_t) = \log(X_t) + \log(i) + \hat{i}_t, \tag{2.3}$$

where y, c and i are the steady-state values of $\frac{Y_t}{X_t}$, $\frac{C_t}{X_t}$ and $\frac{I_t}{X_t}$, respectively, and the hats (ˆ) denote percent deviations from steady state values.

If the economy under consideration is stationary, then the growth rates of the transformed capital stock, $\hat{k}_t \equiv \widehat{\left(\frac{K_t}{X_t}\right)}$ will be expected to have a stationary process, that is, $\hat{k}_t = \mu_1 \hat{k}_{t-1} - \varepsilon_t$ where $\mu_1 < 1$. Furthermore, since there exist no transitory parts of technology under our assumption, we can easily conjecture that $\hat{y}_t$, $\hat{c}_t$ and $\hat{i}_t$ respond only to $\hat{k}_t$ as followings:

$$\hat{y}_t = \pi_{yk}\hat{k}_t,\ \hat{c}_t = \pi_{ck}\hat{k}_t,\ \hat{i}_t = \pi_{ik}\hat{k}_t. \tag{2.4}$$

Therefore, the closed-form solutions for the logarithms of output, consumption and investment can be described as linear combinations of a random walk part, $\log(X_t)$, which is called the 'stochastic trend' , plus a stationary component, $\hat{k}_t$, which is called the 'cycle'.

Note first that the following linear combinations have no stochastic trend:

$$\log(Y_t) - \log(C_t) = \log(y) - \log(c) + (\pi_{yk} - \pi_{ck})\hat{k}_t, \tag{2.5}$$

$$\log(Y_t) - \log(I_t) = \log(y) - \log(i) + (\pi_{yk} - \pi_{ik})\hat{k}_t. \tag{2.6}$$

Thus, it is straightforward to see that "great ratios" are stationary, so that $\log(Y_t)$, $\log(C_t)$ and $\log(I_t)$ are cointegrated, as discussed in Engle and Granger (1987). The existence of two cointegrating relationships indicates that the variables share only one common stochastic trend.

On the other hand, consider the following linear combinations:

$$\pi_{ck}\log(Y_t) - \pi_{yk}\log(C_t) = (\pi_{ck} - \pi_{yk})\log(X_t) + \pi_{ck}\log(y) - \pi_{yk}\log(c) \tag{2.7}$$

$$\pi_{ik}\log(Y_t)-\pi_{yk}\log(I_t)=(\pi_{ik}-\pi_{yk})\log(X_t)+\pi_{ik}\log(y)-\pi_{yk}\log(i). \qquad (2.8)$$

These linear combinations having no cyclical component implies that $\log(Y_t)$, $\log(C_t)$ and $\log(I_t)$ have a common cycle, as pointed out by Vahid and Engle (1993). Consider also the first differences of the logarithms of output, consumption and investment defined as:

$$\Delta\log(Y_t)=\varepsilon_t+\pi_{yk}\Delta\hat{k}_t,\ \ \Delta\log(C_t)=\varepsilon_t+\pi_{ck}\Delta\hat{k}_t,\ \ \Delta\log(I_t)=\varepsilon_t+\pi_{ik}\Delta\hat{k}_t. \qquad (2.9)$$

Given that ε_t is white noise, it follows that the growth rates of these macroeconomic aggregates are serially correlated due to a single common factor, $\Delta\hat{k}_t$, and hence that the cycles in the growth rates of these variables are synchronized.

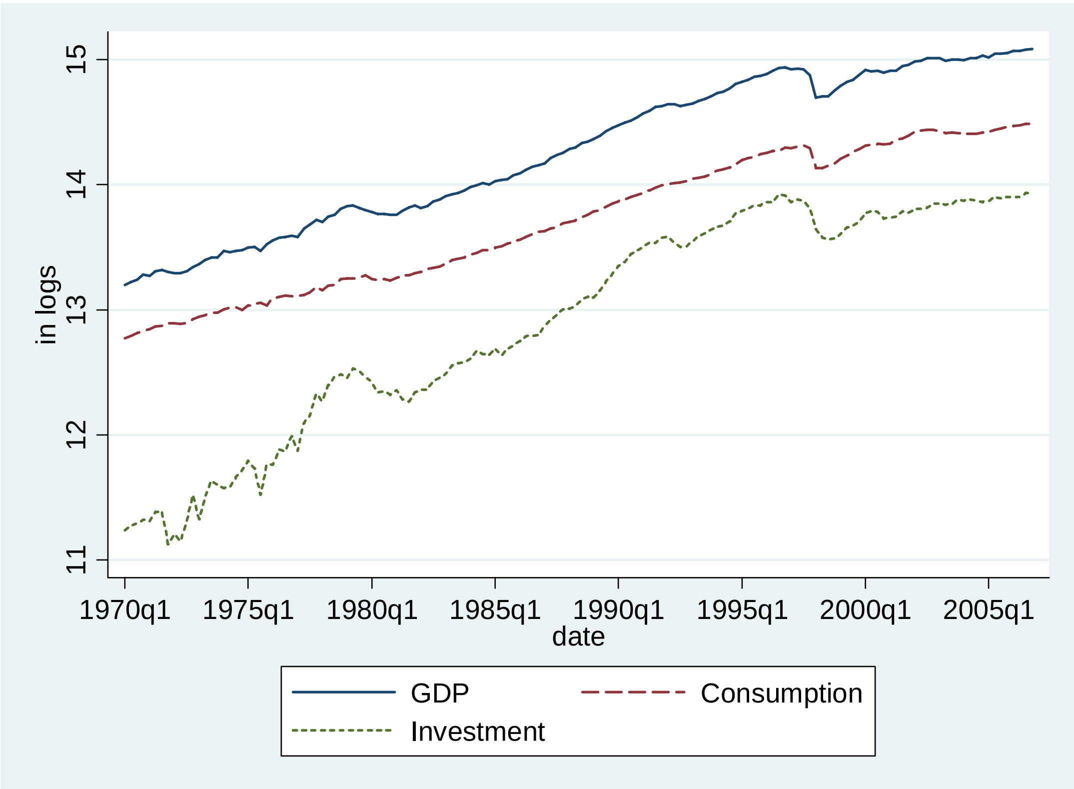
III. Empirical results

As developed by King *et al.* (1988) and further discussed by Issler and Vahid (2001) in the context of common features, we examine in this section whether per-capita income, consumption and investment share common cycles using Korean data. Quarterly data on these variables for the period of 1970.Q1-2006.Q4 are available from The Bank of Korea. The data used here consist of c = log of per-capita personal consumption, y = log of per-capita private output, and i = log of per-capita fixed investment (i.e., gross fixed capital formation).

Many real business cycle models, which are the theoretical background of this paper, assume a closed economy. However, Korea should generally be regarded as a small-open economy. Thus, we use an alternative notion of real output, obtained by subtracting net export from gross domestic output (GDP), to filter out foreign factors from the data.

The plots of (logged) per-capital real output, consumption and investment are presented in Figure 1. As pointed out in Isser and Vahid (2001), the data are extremely smooth and appear to be trending in the long-run, which are typical of most $I(1)$ variables. Notice also that the data display similar short-run behavior.

Figure 1. Logged per-capital real output, consumption and investment



Note: As mentioned above, GDP in the figure is somewhat different from the usual measure of GDP.

In order to check whether the three series y, c and i are non-stationary, we test for unit roots in these variables. As shown in Table 1, the real flow variables y, c and i appear to be characterized as $I(1)$ processes.

Table 1. Augmented Dickey-Fuller unit root tests

	t-Statistic	Prob.[a]
y	-1.377611	0.5918
c	-1.311880	0.6232
i	-1.727930	0.4151

a. MacKinnon (1996) one-sided p-values.

b. The critical values at the 1%, 5% and 10% are -3.4755, -2.8813, -2.5774, respectively.

3.1 Test for common trends

Tests for cointegration can easily be carried out by using Johansen's maximum-likelihood approach, which is available on many statistical package programs such as EViews and STATA. In order to adequately capture the dynamics of the series under consideration, we first need to determine p, the required number of lags in the VECM. In doing so, we start by estimating VARs of different lengths in levels and select one with the smallest SC (Schwarz criterion). A VAR of order 2 turns out to minimize the SC, which suggests a VECM of order 1 if the series are cointegrated.

The outcome of the test results is presented in Table 2, which shows the trace statistics and, the 1% and 5% critical values. We can first see that the data support the existence of two cointegrating relationships (and hence only one common stochastic trend) among the three variables y, c and i.

Table 2. Johansen Cointegration tests

Hypothesized no. of CE(s)	Eigenvalue	Trace Statistic	5% critical values	1% critical values
None	0.20427	60.5627	34.91	41.07
At most 1	0.14631	27.2019	19.96	24.60
At most 2	0.02774	4.1067	9.24	12.97

A normalized version of point estimates for the cointegration space is obtained as:

$$\hat{\alpha}' = \begin{pmatrix} -0.95 & 1 & 0 \\ -1.04 & 0 & 1 \end{pmatrix},$$

which indicates that there are two cointegrating vectors close to $(-1,1,0)'$ and $(-1,0,1)'$, respectively. Such result implies that 'great ratios' are $I(0)$ processes. In fact, the joint null hypothesis that $(-1,1,0)'$ and $(-1,0,1)'$ are a basis for the cointegrating space is not rejected.[2] This finding is consistent with earlier results in King *et al.* (1991), and Issler and Vahid (2003) observed on U.S. data.

3.2 Test for common cycles

Given the finding that output, consumption and investment have one stochastic trend, the next step is to test whether there exist common cycles among these variables. In order to test for common cycles, the canonical-correlation analysis developed in Vahid and Engle (1993) is applied to the VECM with the cointegration restrictions, $(-1,1,0)'$ and $(-1,0,1)'$. The testing procedure for common cycles can be summarized as follows:

1. Compute the sample squared canonical correlations between {$\Delta\log(y_t)$, $\Delta\log(c_t)$, $\Delta\log(i_t)$} and {$\log(c_{t-1}/y_{t-1})$, $\log(i_{t-1}/y_{t-1})$, $\Delta\log(y_{t-1})$, $\Delta\log(c_{t-1})$, $\Delta\log(i_{t-1})$}, labeled λ_j, $j = 1, 2, 3$.
2. Test whether the first smallest s canonical correlations are zero by

[2] The test statistic for restrictions on the cointegrating space $H_0 : \hat{\alpha}' = \begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$ is obtained as $\chi^2(2) = 1.929983$, with $p-\text{value} = 0.380986$.

computing the statistics:

$$-T\sum_{i=1}^{s}\log\left(1-\lambda_i\right),$$

which has a limiting χ^2-distribution with $s\left(np+r\right)-s\left(n-s\right)$ degrees of freedom[3] under the null, where r is the number of cointegrating relationships (in this case, $r=2$).

The outcome of the test results is presented in Table 3. At the 5% significance level, the hypothesis that the smallest canonical correlation is zero cannot be rejected, which implies that the cofeature rank s equals 1. Therefore, we can conclude that there exists only one common cycle among these variables. The cofeature vector, obtained as the canonical covariate corresponding to the smallest canonical correlation after normalization, is presented in Table 4.

Table 3. Vahid and Engle common-cycle tests[a]

Squared Canonical correlations[a] $\left(\lambda_i\right)$	Test Statistics (degrees of freedom)	5 percent Critical value	Null Hypotheses
0.6432	52.19 (15)	24.99	Current and all smaller $\left(\lambda_i\right)$ are zero
0.4154	17.87 (8)	15.51	Current and all smaller $\left(\lambda_i\right)$ are zero
0.2911	5.69 (3)	7.81	Current and all smaller $\left(\lambda_i\right)$ are zero

[a] The canonical correlations can be calculated using CANON in STATA 9.

Table 4. Cointegrating and cofeature vectors

	Cointegrating and cofeature vectors		
	y	c	i
Cointegration 1	−1.000	1.000	0.000
Cointegration 2	−1.000	0.000	1.000
Cofeature	1.000	−0.983	1.084

[3] Since $p=1$ in this VECM, the degrees of freedom equals $s\left(r+s\right)$.

Since $n = r + s$ holds in this case, we can derive a special trend-cycle decomposition of the data, as discussed in Issler and Vahid (2003). Given the observation that there exists one common stochastic trend in this system, the trend component of output, consumption and investment is the same, which is generated by the linear combination of the data that uses the cofeature vector. On the other hand, these variables will have cycles that combine two distinct $I(0)$ serially correlated components, which are in turn generated by the linear combinations of the data using the cointegrating vectors.

IV. Forecasting performances

An attempt is made in this section to assess the implication of common-cycle restrictions on forecasting performance, based on small simulation experiments. That is, the forecasting performance of the restricted vector error correction model (RVECM), which considers common-cycle restrictions as well as cointegrating relations, is compared with three alternative models which take neither short-run restrictions nor cointegrating restrictions into account. Specifically, the unrestricted vector error correction model (UVECM) which considers only cointegrating relations is compared with the RVECM to examine whether it is worth incorporating the information on short-run dynamics in addition to the knowledge of cointegrating relationship. The usual VAR model in differences (DVAR) with no restrictions is considered. Finally, the naïve model (Naive), which generates forecasts by simply providing the current values for all forecasting horizons, is also considered to assess the improvement in forecasts using any *a priori* information. In doing so, the UVECM is used here as the basic forecasting framework, and three alternative models (RVECM, DVAR, Naïve) are compared to examine the relative forecasting performances.

In order to evaluate the forecasting performances, we divide the whole sample into estimating and forecasting subsamples as follows. Initial estimates of the alternative forecasting models are first obtained for the period of 1970.Q1-1999.Q2 to start the out-of-sample forecasts for the first forecasting subsample, consisting of 30 data points for the periods of 1999.Q3 and onwards, which results in one-step to 30-step ahead forecasts. The models are then estimated for the period of 1970.Q1-1999.Q3 to generate forecasts for the periods of 1999.Q4 and afterwards, which results in one-step to 29-step ahead forecasts. The parameter estimates of the forecasting models are recursively updated in this fashion as new observations become available from the previously forecasting subsamples, until we estimate the forecasting models for the period 1970.Q1-2006.Q3, which is used to generate just a single one-step-ahead forecast. It is assumed that there are two cointegrating relations among the three variables in both the UVECM and RVECM throughout the estimating periods.

For each forecasting horizon, we use the square root of the average of the trace of error-covariance matrix, which is calculated from the forecast errors as follows:

$$RMSE(h) = \sqrt{trace\left[\sum_{t=1}^{31-h} \mathbf{e}_t(h)\mathbf{e}_t'(h) \Big/ (31-h)\right]}, \quad h = 1, 2, \cdots, 20 \qquad (4.1)$$

where $\mathbf{e}_t(h)$ denotes the vector of h-step-ahead forecast error, which reduces to the usual *RMSE* (root mean squared error) measure in univariate forecasting.

As we end up with 30 one-step-ahead forecasts, 29 two-step-ahead forecasts, and so on, the RMSE-type measures are calculated using 30 forecast errors for one-step-ahead forecasts, 29 forecast errors for two-step-ahead forecasts, until we have twenty-step-ahead forecasts, for which we use 11 forecast errors. The choices of using the last 30 observations in forecasting evaluation and of the longest forecasting horizon are made to keep the estimating subsamples sufficiently large and also to keep the forecasting subsamples relatively large so that we can obtain reliable forecasting evaluation.

The results of forecasting comparison are presented in Table 5. The first column gives the forecasting horizons, and next to which the RMSE-estimates of the UVECM are provided. Notice first that the RMSE-statistics increase with forecasting horizon, which is expected when forecasts for unit-root nonstationary variables are generated. The remaining columns show the ratios of the RMSE for alternative forecasting models to that of the UVECM. The ratios are in percentage so that values less than 100 indicate that the corresponding RMSE-measures are smaller than those of the UVECM.

As expected, the RVECM appears to outperform alternative models including the UVECM for all forecasting horizons. The relative forecasting performance of the RVECM compared to the UVECM turns out to be better in short-term forecasts than in long-term forecasts, which is what we expect given that the information on the common feature is concerned with short-term dynamics. Notice also that the RVECM compares favorably with the UVECM even in long-term forecasts, although the improvement seems rather small. Such result is not usually expected, since the advantage of the RVECM over the UVECM stems from the short-run common-cycle restrictions. However, as longer-horizon forecasts are generated sequentially based on shorter-term forecasts, the initial improvement in short-term forecasts is expected to carry over to long-term forecasts.

Table 5. Forecasting comparison for y, c and i

Forecasting Horizon	UVECM	RVECM	DVAR	Naïve
1	0.109	93.80	103.17	99.28
2	0.109	95.25	102.93	98.94
3	0.212	93.56	103.50	101.05
4	0.222	93.93	104.78	101.76
5	0.312	92.47	105.01	99.99
6	0.423	94.40	106.98	102.18
7	0.413	96.03	106.32	102.24
8	0.430	98.56	107.64	101.52
9	0.531	97.93	108.29	101.52
10	0.560	99.38	109.00	103.72
11	0.631	99.57	108.42	104.15
12	0.642	99.38	108.70	102.90
13	0.632	99.31	109.73	104.14
14	0.675	99.43	110.54	105.23
15	0.742	99.15	112.30	106.43
16	0.823	99.06	113.44	107.62
17	0.842	97.04	114.92	108.12
18	0.923	97.94	115.87	109.54
19	0.944	99.80	116.95	108.73
20	1.144	98.25	117.01	109.96

The simple VAR model with differences (DVAR) fares poorly, in particular, for long-term forecasts. In fact, the differences in the RMSE-statistics increase with forecasting horizon. This observation is consistent with that of Lin and Tsay (1996) in that correct specification of the number of unit roots provides better forecasts. That is, it pays to take the information on the long-run relationship into account in generating forecasts.

Finally, naive forecasts, which are simply given by the current value for all forecasting horizons, turn out to perform better than the UVECM for short-term forecasts. They also provide decent forecasts in medium and long horizons. This finding contrasts with the observation in Hall, Anderson and Granger

(1992), where the error correction model leads to a reduction in RMSE over the naive model. However, the RVECM compares favorably with the naive model for all forecasting horizons. Anyway, the failure of obtaining significantly better forecasts via the econometric models is suggestive of possible model mis-specification.

The basic observation drawn from the RMSE measures in Table 5 is that it is worth incorporating the information on the short-run dynamics in addition to the knowledge of cointegrating relationship. As the result in Table 5 is based on a single data set, however, we cannot draw too strong a conclusion. In order to assess the improvement in forecasts using the information on common feature, further simulation experiments based on simulated data are also carried out. The models used in the simulation are trivariate VAR(1) models generated by two different specifications: ① the UVECM (unrestricted error correction model on which only long-run restriction imposed) and ② the RVECM (restricted error correction model with both long-run and short-run restrictions). The innovation series are sequences of three-dimensional i.i.d standard normal vectors with zero mean and identity covariance matrix generated by the GAUSS random number generator. The estimates presented in Table 4 are chosen as the coefficients of each model used to generate data. Specifically, the models are generated by :

$$\Delta \mathbf{y}_t = \boldsymbol{\beta} z_{t-1} + \mathbf{A}_1 \Delta \mathbf{y}_{t-1} + \boldsymbol{\varepsilon}_t \tag{4.2}$$

$$\Delta \mathbf{y}_t = \mathbf{B}^{-1} \begin{bmatrix} \boldsymbol{\beta} & \mathbf{A}_1 \end{bmatrix} \begin{bmatrix} z_{t-1} \\ \Delta \mathbf{y}_{t-1} \end{bmatrix} + \boldsymbol{\varepsilon}_t \tag{4.3}$$

where $z_{t-1} = \boldsymbol{\alpha}' \mathbf{y}_{t-1}$, $\boldsymbol{\alpha}' = \begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$, $\mathbf{A}_1 = \begin{pmatrix} 0.892 & -0.060 & -0.327 \\ 0.488 & -0.329 & -0.004 \\ 0.237 & 0.007 & -0.014 \end{pmatrix}$,

$$\boldsymbol{\beta} = \begin{pmatrix} -0.007 & 0.001 \\ -0.003 & -0.009 \\ -0.025 & 0.037 \end{pmatrix}, \ \mathbf{B} = \begin{pmatrix} 1 & -0.983 & 1.804 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ and } \boldsymbol{\varepsilon}_t \sim N\left[\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right].$$

The whole sample consists of 150 data points, and the smallest sample size used for estimation is 120 in order to get reasonable parameter estimates. One-step to 30-step ahead forecasts are generated at different reference points, starting at 121 and onwards, until we reach the last (150*th*) observation in the simulated data. Thus, we have 30 one-step-ahead forecasts, 29 two-step-ahead forecasts, and so on until we have 11 twenty-step-ahead forecasts. Similarly to (4.1), we compute the square root of the average of the trace of error-covariance matrix for each forecasting horizon as follows:

$$RMSE(h) = \sqrt{\frac{\sum_{j=1}^{100} trace_j \left[\sum_{t=1}^{31-h} \mathbf{e}_t(h)\mathbf{e}_t'(h) \Big/ (31-h) \right]}{100}}, \quad h = 1, 2, \cdots, 20 \qquad (4.4)$$

where $j = 1, 2, \cdots, 100$ denotes the total number of data sets used in the simulation.

The results of forecasting comparison of four alternative forecasting models for the two different data generating processes are respectively given in Tables 6 and 7. As in Table 5, the first column gives the forecasting horizons, and the second column the RMSE-statistics of the UVECM. The remaining columns show the ratios of the RMSE for alternative forecasting models to that of the UVECM. The ratios are in percentage so that values less than 100 indicate that the corresponding RMSE-measures are smaller than those of the UVECM.

Table 6 shows the RMSE-statistics when the data are generated by the UVECM in (4.2). In this case, the UVECM compares favorably with other forecasting models including the RVECM, which is somewhat expected given that the true DGP follows an unrestricted UVECM. In particular, long-term forecasts of the UVECM outperform those of the DVAR and Naive models which do not use cointegration restrictions. As long as the knowledge of cointegrating relationship is incorporated as in the RVECM, however, the cost of misspecification in the short-run dynamics seems rather small. As expected, the RVECM outperforms DVAR and Naive models, which use no comovement restrictions at all, in long-term forecasts.

Table 6. Forecasting comparison of alternative models for the UVECM specification

Forecasting Horizon	UVECM	RVECM	DVAR	Naïve
1	3.432	103.43	103.22	103.19
2	3.544	102.91	103.50	103.24
3	3.565	102.50	104.09	103.96
4	3.589	103.22	104.87	104.16
5	3.589	103.01	105.22	104.10
6	3.590	102.22	105.50	104.20
7	3.590	102.30	106.09	104.31
8	3.600	102.49	107.87	104.43
9	3.611	103.22	107.81	104.52
10	3.615	102.00	109.20	104.60
11	3.628	103.33	110.37	104.62
12	3.636	102.70	111.07	104.27
13	3.654	102.87	112.31	104.88
14	3.661	102.65	113.28	105.21
15	3.679	102.30	114.95	105.56
16	3.732	101.44	115.96	105.28
17	3.744	102.92	116.30	106.29
18	3.750	102.90	117.73	106.82
19	3.805	102.95	117.95	107.11
20	3.808	102.01	118.76	107.60

On the other hand, the DVAR model with no error correction term performs worst, for all forecasting horizons, indicating that it certainly pays to consider the long-run relationship in generating forecasts. Notice that the DVAR compares poorly with the Naive model, in particular, for long-term forecasts.

Table 7 presents the results of the forecasting comparison when the data are generated by the RVECM in (4.3). While the differences are not so critical, the RVECM produces better forecasts than the UVECM, which is again expected as it is the correct specification of error correction models. This result indicates that it is worth incorporating the information on the short-run dynamics in addition to the long-run relationship.

Table 7. Forecasting comparison of alternative models for the RVECM specification

Forecasting Horizon	UVECM	RVECM	DVAR	Naïve
1	4.252	99.06	102.29	102.11
2	4.782	99.97	102.98	102.23
3	5.290	99.30	103.88	103.85
4	5.558	98.21	104.45	103.97
5	5.607	99.94	105.62	104.22
6	5.860	99.98	106.99	104.89
7	5.908	99.21	107.58	105.20
8	6.004	99.20	107.88	105.41
9	6.119	99.22	108.96	105.90
10	6.425	99.15	109.00	106.11
11	6.608	99.18	110.21	106.80
12	6.832	99.20	112.08	107.43
13	6.996	99.22	113.24	107.48
14	7.098	99.22	114.75	107.80
15	7.205	99.21	115.66	107.91
16	7.231	99.05	116.82	108.10
17	7.423	98.90	117.86	108.92
18	7.509	98.93	118.90	109.28
19	7.643	99.01	120.39	109.60
20	7.754	99.05	121.84	109.90

In this case, the DVAR performs even worse compared to the case when the data are generated by UVECM, as neither the short-run nor long-run restrictions are incorporated in the VAR model. Notice also that the Naive model, which simply produces the current value for generating forecasts, turns out to compare fairly good with the DVAR for all forecasting horizons.

In summary, the forecasting comparison of the two alternative forecasting models (RVECM *vis-à-vis* UVECM) shows that it pays to account for the additional information on the common cycles in generating future values of the series under investigation. That is, use of more information concerning the short-run dynamics can certainly improve our forecasts.

V. Concluding remarks

The cointegration literature suggests that forecast errors may be reduced by incorporating the knowledge of cointegrating relationship into models used to generate forecasts. In this paper, we investigate whether the forecasting performance can further be enhanced by taking into account of short-run dynamics. Using estimation and testing procedures presented in section 2, we first examine whether and to what extent Korean GDP, consumption, and investment share common cycles, and evidence is presented the existence of short-run common features. The results indicate that output, consumption and investment share both one common trend and two common cycles, which is consistent with the implication of the standard real business model.

Based on the forecasting performance of two alternative models (restricted *vis-à-vis* unrestricted error correction models), it is also found that the restricted error correction models, which consider common-cycle restrictions as well as cointegrating relations, outperform the usual error correction models, which take only long-run relations into account.

While we cannot draw too strong a conclusion from the limited empirical examples, the observations presented in this paper suggest that some of the usual VECM specifications do not effectively use all the information available. This finding will enable us to improve our understanding of both short-run and long-run dynamics in economic time series, and to capture it in a statistical model so that the forecasting performance can be enhanced. That is, we can improve our forecasts by using more information on short-run common features. Thus, the result in this paper suggests that testing for common cycles should always precede econometric estimation whenever short-run co-movements are likely to be present.

While some interesting observations are presented in this paper by employing the common-cycle approach, much work still remains to be done. We first need to consider various forms of common features discussed in Anderson *et al.* (2006) and papers therein, and to develop estimation procedures for cointegrating vectors under common-feature restrictions. We also have some practical problems to investigate for future empirical applications, for instance, the importance of forecasting horizons in forecasting evaluation and the finite sample and asymptotic properties of the common-cycle tests adopted here.

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