

The Role of Ownership on Control Right Allocation and Compensation Contract Design

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Abstract

We propose a corporate control model that deals with both control rights and managerial incentive problems. To square with observed facts, we pinpoint the distinction of ownership versus contract in inspiring managers' due diligence and safeguarding the interests of investors. This study draws upon a special feature of human nature, i.e., loss aversion affiliated with ownership, to buttress managers' working incentive. By incorporating this prospect-type utility function in entrepreneur's welfare, we find that it helps mitigate the inefficiency stemming from moral hazard. The entrepreneur's effort is enhanced and the social welfare is improved. Besides, less control right is transferred to investors when the entrepreneur has higher initial ownership. Nevertheless this beneficial effect might disappear once the entrepreneur emphasizes his prospect-type payoff so much as to generate the "over-effort" paradox. Under this circumstance it is very likely that the social welfare is reduced.

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1. Introduction

The issues of corporate governance arouse intense debates in recent years as daunting corporate scandals emerged, and the deficiency of governance structure were found to be blameful even in the advanced market economies. Corporate governance is usually viewed from the perspective of agency problem, often referred to as the separation of ownership and control. It deals with the mechanism design that can efficiently protect the interests of outside investors, especially when the corporate managers behave in a way quite different from what the outside investors entrust them to do.

The most common approaches to corporate governance include the managerial contract design and the transfer of some right or power to investors in exercising control on the firm (Tirole 2001, Shleifer and Vishny 1997). The contract design aims to encourage the managers to behave in the investors' interests. For example the performance-based compensation such as bonuses (cash or stocks) and stock options are widely used in corporations nowadays. Besides, offering some stock ownership to managers has positive effect on the resolution of conflicts of interests. Among the various designs for the latter approach, the legal protection from expropriation by managers, the monitoring of large investors (concentrated ownership) and the allocation of control rights to investors are well-analyzed and implemented in practices.

In this paper we synthesize these two approaches by exploring the relationship between control right allocation and the design of compensation contract; and examining how the interaction of corporate control and compensation design affects the firm's total welfare. In particular, we are probing the difference of stock ownership from contract relationship in stimulating the managerial incentive. Drawing upon Barberis, Huang and Santos (2001) we assume that the manager is irrational in that he engages in narrow framing: he gets utility from changes in the value of his original stock holdings (Barberis and Thaler 2002). The presence of ownership thereby creates a special feel or sense of gain or loss for the entrepreneur, which is characterized based on the implication of prospect theory (Kahneman and Tversky 1979). Contrary to other models of managerial irrationality based on belief bias such as "overconfidence" (Roll 1986, Heaton 2002), this specific aversion toward the loss of his stock ownership will thereby strengthen the entrepreneur's accountability and working incentive and generate a distinct impact on either control right allocation or compensation design in our model.

According to Tirole (2006, p.387), control right is defined to be "the right for a party (or groups of parties) to affect the course of action in certain circumstances once

the firm has gotten started.” The importance of control rights in corporate finance was first introduced by Aghion and Bolton (1992) and then developed by Hart (1995) and Hart and Moore (1998). In these studies allocating control rights to investors can substitute for limited cash flow rights and relax the financing constraint. Although this attribute is still sustained in our model, what we focus on is totally different. Specifically we consider an entrepreneur with a profitable project but with insufficient capital to start with. After financing the project by raising new equity, the entrepreneur and outside investors form the board of directors to determine the compensation contract and the allocation of control rights. The compensation contract is characterized by the ratio of stock bonus per unit of ownership; and the control right is the authority to make decisions on whether or not to take some interim action once the project is in progress.

Although taking the interim actions can increase the success probability of the project, it engenders high private cost on the entrepreneur who will not take the action once he retains the control. As a result, there exists conflict of interests in exercising the control right. The board must determine the allocation of control rights (to the entrepreneur or investors) to maximize firm’s total welfare. Furthermore in deciding the control right allocation, the board should also take into account its impact on the entrepreneur’s incentive and the investors’ financing constraint. Therefore we solve the board’s problem in two stages by adopting the concept of subgame perfectness. Starting with the second stage we derive the compensation contract and the entrepreneur’s resulting effort according to the entrepreneur’s incentive constraint and the investors’ financing constraint, conditional on the control right allocation. Then in the first stage the board determines the optimal control right allocation by taking these compensation contract and incentive-compatible effort into its consideration.

By retaining the control rights of undertaking some interim actions, which is virtually a substitute for cash flow rights to investors, the investors can hereby agree to offer the entrepreneur higher compensation without violating their individual rationality constraint. The entrepreneur’s effort will then be elevated accordingly so does the social welfare as long as his effort does not exceed the social efficient level. Since an increase in either the entrepreneur’s initial stock ownership or his subjective weight put on the prospect-type utility (gain and loss) can help strengthening the entrepreneur’s working incentives and improving his effort, the investors’ participation or individual rationality constraint will become less binding. The investors will no longer rely so much on the transfer of control right to secure their investment interest. As a result we find that more control rights can be retained in the hand of the entrepreneur when his initial share of stock ownership is bigger, or when

he puts greater weight on his utility from gain and loss.

Moreover we find that the entrepreneur's effort level may exceed the social efficient one when he weighs his prospect-type utility so much as that his recognized payoff from additional effort surpasses the project's gross return. Once this "too-much-effort" paradox is triggered, the effort-enhancing effect from higher ownership or larger weight on the utility from gain and loss will instead impair the social welfare. Although no control right should be allocated to outsiders under this circumstance, the entrepreneur who retains all the control right will possibly take some interim actions that are supposed not to undertake. This further jeopardizes social welfare.

This rest of the paper is organized as follows. Section 2 introduces the basic model. Section 3 analyzes the determination of the entrepreneur's compensation contract and the resulting effort conditional on the allocation of control rights. Some important attributes are also discussed. The optimal allocation of control rights between the entrepreneur and outsiders are examined in section 4. We present a numerical example to clarify the result. Section 5 provides some concluding remarks.

2. The Model

An entrepreneur has a project that requires a fixed investment of one dollar. This project yields verifiable return R if succeeds and yields no return if fails. The probability of success is denoted by $p(e)$, where e is the effort level exerted by the entrepreneur and is unobservable to outsiders. Thus this project is subject to moral hazard. The entrepreneur has an initial wealth of $\beta < 1$ which can either be invested in this project or in the financial market earning a risk-free return r per dollar. To implement the project the entrepreneur must finance $1 - \beta$ in the capital market. We assume that $1 - \beta$ is raised in the form of new equity. β can then be interpreted as the inside equity or the "ownership right" of the entrepreneur in this model.

Once $1 - \beta$ is financed from the outside stockholders, these stockholders and the entrepreneur form the board of directors to determine two issues: one is the compensation contract for the entrepreneur and the other is the allocation of "control rights". The compensation contract is characterized by (w, α) , where w is the salary determined exogenously in the model and α is the stock bonus per unit of ownership.¹ The entrepreneur can receive this compensation only when the project

¹ In fact α is the stock bonus in a relative sense. Denoting the absolute amount of stock bonus by x , the ownership ratio of the entrepreneur becomes $\frac{\beta + x}{1 + x}$ after the grant of stock bonus. We have in mind that $\frac{\beta + x}{1 + x}$ equals $(1 + \alpha)\beta$. This α can also be interpreted as a dilution factor. The ownership of

succeeds. As for the control rights, we follow Tirole (2006, p.389) by assuming that control right is represented by an authority to make some interim decision once the project is in progress. Specifically, there exists K dimensions of interim actions each of which enhances the probability of success by θ_k uniformly (i.e., θ_k is independent of the effort level e) and imposes a private cost γ_k on the entrepreneur if the interim action is undertaken in the dimension k ($k=1, \dots, K$). We define $Z_k=1$ if the interim action k is taken and $Z_k=0$ otherwise. Therefore the probability of success becomes $p(e) + \sum_{k=1}^K \theta_k Z_k$ and the private cost is $C(e) + \sum_{k=1}^K \gamma_k Z_k$ for the entrepreneur, where $C(e)$ is the private cost of making an effort level e .

The outsiders are assumed to be risk-neutral and will finance the project only if the following ‘‘individual rationality’’ constraint (abbreviated as ‘‘ IR ’’ hereafter) is satisfied,

$$(p(e) + \sum_{k=1}^K \theta_k Z_k) [1 - (1 + \alpha)\beta] (R - W) \geq (1 - \beta) r. \quad (1)$$

The entrepreneur is also risk-neutral and is intended to maximize the following utility function,

$$\begin{aligned} & (p(e) + \sum_{k=1}^K \theta_k Z_k) [W + (1 + \alpha)\beta(R - W)] \\ & + \delta [(p(e) + \sum_{k=1}^K \theta_k Z_k) \beta(R - W - r) - (1 - p(e) - \sum_{k=1}^K \theta_k Z_k) \lambda \beta r] - C(e) - \sum_{k=1}^K \gamma_k Z_k. \end{aligned} \quad (2)$$

The first term in eq.2 represents the expected final wealth of the entrepreneur. Given the stock bonus α , the ownership of the entrepreneur increases from β to $(1 + \alpha)\beta$. His final wealth includes monetary salary W and the value of his stock ownership $(1 + \alpha)\beta(R - W)$ when the project succeeds. In addition, we add a prospect-type utility (i.e., the second term in eq.2) to the entrepreneur’s objective function. We suppose that the ownership right imparts a sense of gain or loss on the entrepreneur. If the project succeeds, the entrepreneur perceives a gain of $\beta(R - W - r)$ from his original ownership β on which he expects an average return of βr . If the project fails, he suffers a sense of loss by an amount equal to $\lambda \beta r$ with the loss aversion factor $\lambda > 1$ representing a magnified sense of loss. $\delta < 1$ is the weight the entrepreneur puts on the prospect-type utility. Higher δ reflects that the entrepreneur emphasizes more heavily on the gain or loss associated with his ownership.

To pinpoint the conflict of interests between the entrepreneur and outsiders as a

outsiders is diluted from $1 - \beta$ to $1 - (1 + \alpha)\beta$. It is required that α be smaller than $\frac{1 - \beta}{\beta}$.

result of control right allocation, we impose assumption 1 as

Assumption 1: $\theta_k R < \gamma_k$ for all $k=1, \dots, K$.

This assumption means that the entrepreneur is reluctant to take interim actions once he is given the control right.² On the contrary, outsiders equipped with the control right will select the interim action because they partake of the benefit (i.e., $\theta_k (1-(1+\alpha)\beta)(R-W)$) but bear no cost. We can now characterize the governance or corporate control structure by the allocation $Z \equiv \{Z_1, \dots, Z_k\}$ of control rights, where $Z_k=1$ if outsiders obtain the control of decision k and $Z_k=0$ if the entrepreneur keeps the control. Besides we arrange k such that $\theta_K R / \gamma_K < \dots < \theta_2 R / \gamma_2 < \theta_1 R / \gamma_1 < 1$.

Finally, the board of directors maximizes social welfare (the aggregate expected payoff of the entrepreneur and outsiders) subject to the entrepreneur's "incentive compatibility" constraint (abbreviated as "IC" hereafter) and outsiders' IR constraint,

$$\begin{aligned}
 (P) \quad & \text{Max}_{(\alpha, Z_k)} \quad (p(e) + \sum_{k=1}^K \theta_k Z_k) R - 1 - C(e) - \sum_{k=1}^K \gamma_k Z_k \\
 \text{subject to} \quad & (p(e) + \sum_{k=1}^K \theta_k Z_k) [1 - (1 + \alpha)\beta] (R - W) \geq (1 - \beta) r, \quad (IR) \\
 & e \in \arg \max \quad (p(e) + \sum_{k=1}^K \theta_k Z_k) [W + (1 + \alpha)\beta(R - W)] \\
 & + \delta [(p(e) + \sum_{k=1}^K \theta_k Z_k) \beta(R - W - r) - (1 - p(e) - \sum_{k=1}^K \theta_k Z_k) \lambda \beta r] - C(e) - \sum_{k=1}^K \gamma_k Z_k. \quad (IC)
 \end{aligned}$$

By utilizing the first order approach we can rewrite the above IC constraint as

$$p'(e) [W + (1 + \alpha + \delta)\beta(R - W) + \delta(\lambda - 1)\beta r] = C'(e). \quad (3)$$

The board of directors solves the problem P as follows. Firstly, the compensation α and the effort level e are determined by eq.3 and the binding IR constraint contingent on the allocation of control rights (i.e., given $\sum_{k=1}^K \theta_k Z_k$). These results are then substituted into the objective function to resolve the allocation of control rights. To sum up, the board determines the allocation of control rights by backward induction, taking into account the impact of control right allocation on the compensation contract and the entrepreneur's effort level.

² By differentiating eq.2 with respect to Z_k , it is found that the entrepreneur will take the interim action k only if $\theta_k [W + (1 + \alpha + \delta)\beta(R - W) + \delta(\lambda - 1)\beta r] > \gamma_k$ is satisfied. The term in the bracket is smaller than R when $\delta=0$. Therefore assumption 1 implies that the entrepreneur with small δ will not take any interim action when he obtains the control right. However the entrepreneur might take some interim actions if his weight on the prospect-type utility (δ) is rather high such that the term in the bracket exceeds R . We discuss this situation later in section 4.

3. The determination of compensation contract and effort level given the allocation of control rights

In this section we take the allocation of control rights to be exogenously given and derive the compensation contract and the corresponding effort level for the entrepreneur that satisfy both “IC” and “IR” constraints. For simplicity we assume $p(e) = ae$, $C(e) = ce^2$, $a > 0$ and $c > 0$. Then the social efficient effort level e^E should be equal to $\frac{aR}{2c}$. By simultaneously solving eq.3 and the binding IR

constraint, the compensation contract α and the effort level e can be found as³

$$\alpha(\Sigma) = \frac{1-\beta}{\beta} \left[1 - \frac{4cr}{(R-W) [a^2D + 2c\Sigma + \sqrt{(2c\Sigma + a^2D)^2 - 8a^2c(1-\beta)r}]} \right] \quad (4)$$

$$e(\Sigma) = \frac{a^2D - 2c\Sigma + \sqrt{(2c\Sigma + a^2D)^2 - 8a^2c(1-\beta)r}}{4ca}, \quad (5)$$

where $\Sigma \equiv \sum_{k=1}^K \theta_k Z_k$ and $D \equiv R + \delta[\beta(R-W-r) + \lambda\beta r]$. Σ is the incremental increase in the probability of success as to be determined by the allocation of control rights. The more control rights are allocated to outsiders, the higher Σ is. D can be visualized as the maximal marginal benefit of effort for the entrepreneur (i.e., when $(1+\alpha)\beta = 1$ or when outsiders’ ownership is diluted to zero), which includes the project’s return R and the entrepreneur’s evaluation of his gain and loss $\delta[\beta(R-W-r) + \lambda\beta r]$. The derivations of eq.4 and 5 are provided in appendix 1.

At first we consider the benchmark case in which $\delta = 0$ and $\Sigma = 0$ (i.e., the entrepreneur derives utility only from his final wealth and no control right is allocated to outsiders). For this benchmark case, the board can find a reasonable design or solution of (α, e) that satisfies both “IC” and “IR” constraints only when the following two assumptions are met.

Assumption 2: $\beta \geq 1 - \frac{a^2R^2}{8cr}$.

Assumption 3: $cr \leq \max\left(\frac{Ra^2(R-W)}{4}, \frac{a^2(R-W)[\beta(R-W)+W]}{2}\right)$

Assumption 2 is the necessary condition to warrant that this solution for (α, e) is a real one. Assumption 3 is the necessary condition for this compensation or stock bonus α to be nonnegative. If assumption 2 or 3 is not satisfied, there is no nonnegative real α that can induce the entrepreneur to exert such an effort that

³ In fact there exists two solutions of (α, e) that satisfy both “IC” and “IR” constraints. However in appendix 1 we explain that we are merely interested in the (α, e) solution with a larger value. Thus in eq.4 and eq.5 we just show the solution with a larger value.

makes external equity financing feasible. These assumptions are required to prevent the moral hazard problem from hindering the financing even though the project itself is socially beneficial. Among the factors affecting the firm's ability to raise external equity we are particularly interested in the entrepreneur's ownership β . As can be seen from assumption 2 and 3 the higher β (the higher the internal capital or net worth), the more capacity the entrepreneur can attract external equity in the presence of moral hazard problem. If assumption 2 or 3 were not satisfied, the wealth-constrained entrepreneur could boost his financing ability either by allocating some control right to investors (i.e., increasing Σ) as proposed by Aghion and Bolton (1992) etc.; or by convincing investors that he is seriously concerned with any gain or loss associated with his ownership (i.e., having positive δ). This latter mechanism in relaxing financing constraint is a main thrust and contribution of our model. However, it is not necessarily beneficial to have a positive δ if the entrepreneur is not initially financially constrained. This situation will be discussed later. By now we summarize the outcome for the case of $\delta=0$ and $\Sigma=0$ in lemma 1.

Lemma 1: *Assume (i) the entrepreneur only derives utility from his final wealth and does not get any utility directly from his ownership gain and loss (i.e., $\delta = 0$), and (ii) none of the control rights is allocated to investors (i.e., $\Sigma = 0$). The board will select the compensation contract α and indirectly the entrepreneur's effort level e as follows.*

$$\alpha = \frac{1-\beta}{\beta} \left[1 - \frac{4cr}{a(R-W)(aR + \sqrt{a^2 R^2 - 8c(1-\beta)r})} \right] \quad \text{and} \quad e = \frac{aR + \sqrt{a^2 R^2 - 8c(1-\beta)r}}{4c}$$

This contract design and effort level is achievable only if $\beta \geq 1 - \frac{a^2 R^2}{8cr}$ and

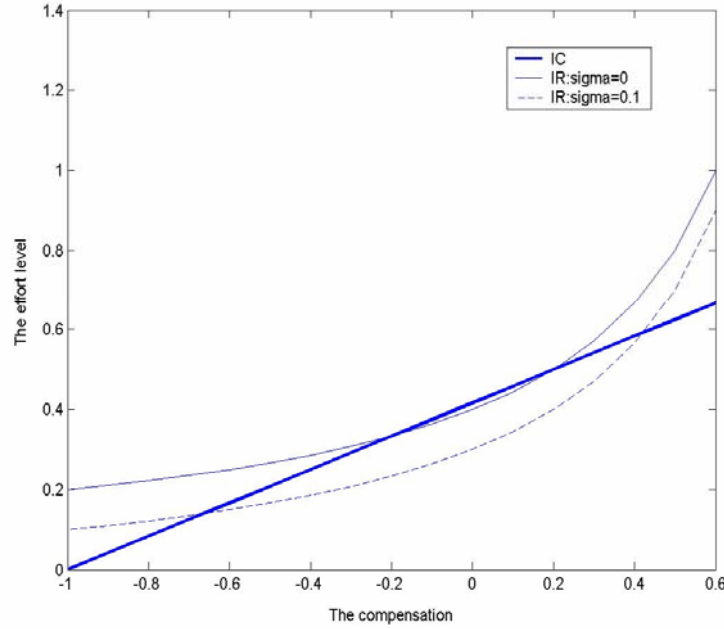
$$cr \leq \max \left(\frac{Ra^2(R-W)}{4}, \frac{a^2(R-W)[\beta(R-W)+W]}{2} \right) \text{ are both satisfied. Otherwise the entrepreneur}$$

is financially constrained. These financial constraints can be loosened by increasing Σ or δ .

Proof: appendix 1.

Next we proceed to investigate the interesting case $\delta > 0$ where the prospect-type utility plays a significant role in affecting the entrepreneur's decision. To simplify the analysis we still maintain that assumption 2 and 3 are satisfied. Otherwise the minimum value of δ above which the non-negative real solutions of (α, e) can be found must be proposed. Figure 1 depicts the solutions of (α, e) at the intersection of two lines. The straight line, denoted as "IC", illustrates all the pairs of (α, e) that satisfy "IC" constraint (eq.3); and the convex curve, denoted as "IR", is the set of points

(α, e) that satisfy “*IR* binding” constraint.⁴ Besides when Σ is enhanced (i.e., when more control rights are allocated to outsiders) the line “*IC*” remains intact while the “*IR*” curve moves downward. We thereby come to the conclusion that for the larger (α, e) solution that we are interested, transferring more control rights to investors will lead to bigger bonus and higher effort (i.e. $\partial \alpha / \partial \Sigma > 0$ and $\partial e / \partial \Sigma > 0$)⁵. Similarly when the parameters δ and λ (the two parameters that characterize the prospect-type utility) increase, the “*IC*” line moves upward and the “*IR*” curve remains intact. Again we can derive that $\partial \alpha / \partial \delta > 0$ and $\partial e / \partial \delta > 0$ for the larger (α, e) solution.



⁴ From eq.3 the slope of the straight line is $\frac{a}{2c} \beta (R - W)$. From eq.1 the slope of the “*IR* binding” curve is $\frac{1}{a} \times \frac{\beta(1-\beta)r}{(1-(1+\alpha)\beta)^2(R-W)}$. This curve is convex if and only if $\alpha < \frac{1-\beta}{\beta}$.

⁵ With some calculation $\frac{\partial \alpha}{\partial \Sigma} = \frac{1-\beta}{\beta} \left[\frac{8c^2 r (1 \pm \frac{(2c\Sigma+a^2D)}{\sqrt{(2c\Sigma+a^2D)^2-8a^2c(1-\beta)r}})}{[a^2D+2c\Sigma \pm \sqrt{(2c\Sigma+a^2D)^2-8a^2c(1-\beta)r}]^2} \right]$ and

$\frac{\partial \alpha}{\partial \delta} = \frac{1-\beta}{\beta} \left[\frac{a^2 P \times 4cr (1 \pm \frac{(2c\Sigma+a^2D)}{\sqrt{(2c\Sigma+a^2D)^2-8a^2c(1-\beta)r}})}{[a^2D+2c\Sigma \pm \sqrt{(2c\Sigma+a^2D)^2-8a^2c(1-\beta)r}]^2} \right]$. These effects are positive (negative)

for the larger (smaller) value solution.

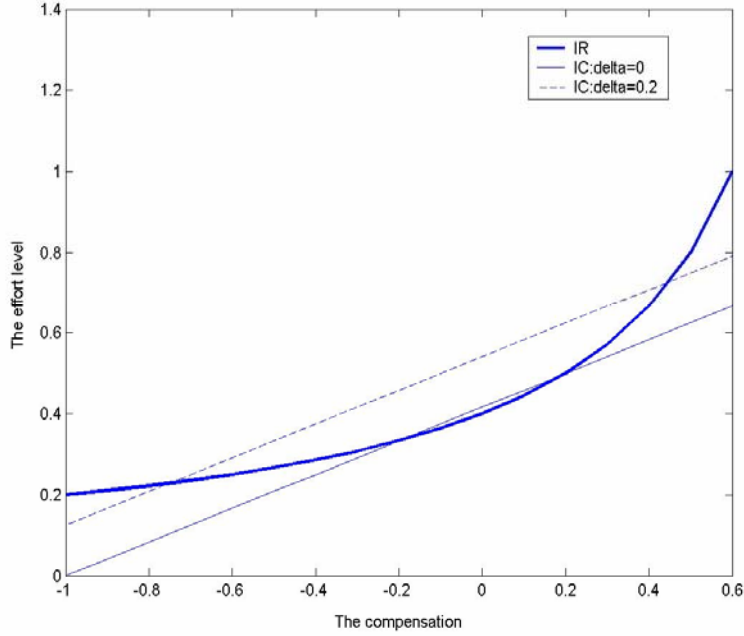


Figure 1: The determination of the compensation contract and the effort level

Proposition 1: Assume that in addition to the final wealth, the entrepreneur also derives utility directly from his ownership gain or loss (i.e., $\delta > 0$). Then given the allocation of control rights Σ , the board will choose the following compensation contract α and indirectly the entrepreneur's effort level e .

$$\alpha(\Sigma) = \frac{1-\beta}{\beta} \left[1 - \frac{4cr}{(R-W) [a^2 D + 2c\Sigma + \sqrt{(2c\Sigma + a^2 D)^2 - 8a^2 c(1-\beta)r}]} \right],$$

$$e(\Sigma) = \frac{a^2 D - 2c\Sigma + \sqrt{(2c\Sigma + a^2 D)^2 - 8a^2 c(1-\beta)r}}{4ca},$$

where $D \equiv R + \delta[\beta(R-W-r) + \lambda\beta r]$. The sufficient conditions for this compensation contract and the effort level to be attainable for all δ and Σ are imposed in assumption 2 and 3.

In addition to the allocation of control rights Σ , this compensation contract and the corresponding effort level rely on other exogenous factors. Among these factors the entrepreneur's ownership β and his evaluation of the prospect-type utility δ are particularly interesting. The next corollary shows their influences.

Corollary 1: For a given allocation of control rights, the compensation contract α and the effort level e that satisfy both investors' "IR" and the entrepreneur's "IC" constraints will be decided according to the share of the entrepreneur's ownership β and the weight δ he has on the prospect-type utility.

(i) The compensation contract α offered to the entrepreneur will decrease as the

entrepreneur attains a higher share of ownership except when the weight of prospect-type utility is small and the original ownership is rather low. By all means the effort level exerted by the entrepreneur will be boosted as his ownership is enhanced.

(ii) The compensation contract α offered to the entrepreneur will increase as the entrepreneur imposes a higher weight on his prospect-type utility. His effort level will also be elevated accordingly.

Furthermore, the compensation contract α and the effort level will increase unambiguously if more control right is allocated to investors.

Proof: The impact of δ can be obtained from figure 1. The entrepreneur will make higher effort for any given compensation contract α when he puts more emphasis on his prospect-type utility. Thus the “IC” line moves upward as δ is uplifted in figure 1. The contentment of his stock ownership in the company by way of the prospect-type utility will now render the entrepreneur’s effort more rewarding and induce his greater effort exertion. Investors’ “IR” constraint is hereby loosened due to this effect. As a result, the initial compensation α that satisfies original “IR” constraint can now be augmented due to the relaxation of “IR” constraint. The impact of Σ can be similarly deduced from figure 1. Investors’ “IR” constraint is directly loosened when they receive more control rights. This is because they can command the entrepreneur to take some interim actions that increase the success probability but without incurring any cost to them. Thus the resulting relaxation of “IR” constraint will now facilitate the investors to offer more generous compensation package to the entrepreneur so as to arouse his greater effort exertion.

Finally the impact of ownership β is a little complicated since it affects the “IC” and “IR” constraint in an opposite way. When the entrepreneur is concerned with the wealth change of his initial stock ownership, his strong sense loss aversion will drive him to work harder as his share of ownership rises. Thus the “IC” line move upward as β is enhanced, and the investor’s “IR” constraint is indirectly loosened. However the “IR” constraint is also directly influenced by β . Though higher β reduces the investors’ opportunity cost associated with their capital input (i.e., $(1-\beta)r$), it will reduce their pledgeable income $(1-(1+\alpha)\beta)(R-W)$ at the same time. We find that overall the latter effect overwhelms the former.⁶ The “IR” constraint becomes more stringent, and “IR” curve moves upward as β is uplifted. As for the ultimate impact on effort level, it will be unambiguously improved as the upward movement of the “IC” line (becoming loosened) and the “IR” curve (becoming stringent) will both

⁶ From “IR” constraint we can derive the equation: $e = \frac{1}{a} \left(\frac{(1-\beta)r}{(1-(1+\alpha)\beta)(R-W)} - \Sigma \right)$. For a given α value, $\frac{\partial e}{\partial \beta} = \frac{1}{a} \times \frac{\alpha r}{(1-(1+\alpha)\beta)^2 (R-W)} > 0$. The “IR” constraint becomes stricter as β is enhanced.

prompt the entrepreneur to work harder. However these two forces of movement will work in an opposite direction toward their impact on compensation. The force of loosened “*IC*” tends to makes higher compensation feasible, while that of stringent “*IR*” pushes the compensation lower. When the entrepreneur starts with a trivial stock ownership and puts a negligible weight on the prospect-type utility, the force of loosened “*IC*” will dominate that of stringent “*IR*”. The entrepreneur’s compensation will then be raised. Otherwise, the impact of stringent “*IR*” from increased β takes rein over loosened “*IC*”, resulting in a declining compensation. Figure 2 below depicts the evolvement of α with respect to β for three kinds of scenarios.

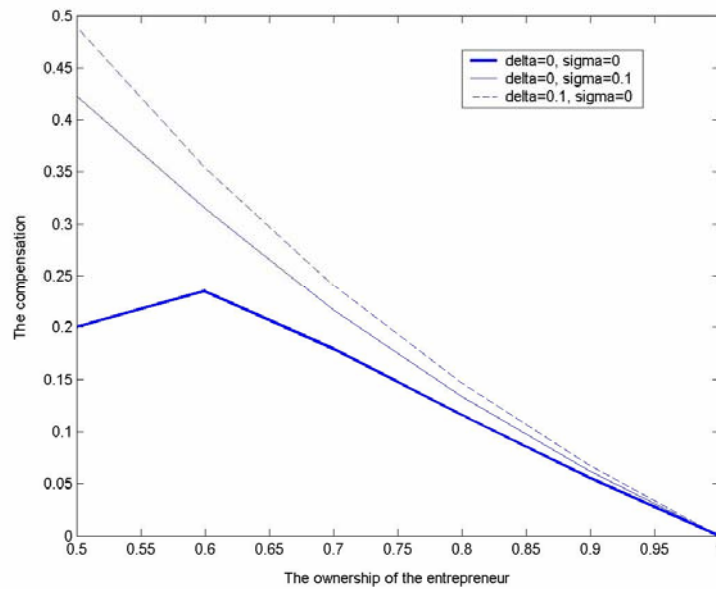


Figure 2: The changes of the compensation α with respect to the ownership

($a=1, c=1.5, R=2.5, r=1$ and $W=0$)

Q.E.D

Since the relative size of effort level $e(\Sigma)$ and $e^E(=aR/2c)$ is crucial for our analysis of control right allocation, we examine their relationship in proposition 2 below. As shown in Corollary 1, an increase in δ induces the entrepreneur to exert greater effort. However, the entrepreneur’s effort might exceed the social efficient level once δ becomes excessively high. We will discuss this important property more thoroughly in the next section. Proposition 2 points out two critical δ 's such that for δ larger than the high critical value, $e(\Sigma)$ is bigger than $aR/2c$ for all $\Sigma \geq 0$; and for δ smaller than the low critical value, $e(\Sigma)$ is smaller than $aR/2c$ for all $\Sigma \geq 0$. The derivation of these two critical values is provided in appendix 2.

Proposition 2: The relative size of the entrepreneur’s effort level $e(\Sigma)$ and the social

efficient level $\frac{aR}{2c}$ can be characterized below depending on the level of δ .

(i) For δ greater than $\frac{2c(1-\beta)r}{a^2R \times P} \equiv \delta^H$, $e(\Sigma)$ is bigger than $\frac{aR}{2c}$ for all $\Sigma \geq 0$. In this

situation the entrepreneur exerts too much effort as a result of his over-emphasis on the ownership gain or loss (even when $\Sigma=0$).

(ii) For δ smaller than $\frac{(1-\beta)r}{p} \equiv \delta^L$, $e(\Sigma)$ is smaller than $\frac{aR}{2c}$ for all $\Sigma \geq 0$. In this

situation the positive thrust from his ownership concern is not sufficient to counteract the negative impact of the inherent moral hazard problem. The effort level exerted by the entrepreneur will then be less than the social efficient one.

(iii) For δ between δ^H and δ^L , $e(\Sigma)$ could be greater or smaller than $\frac{aR}{2c}$. The

higher Σ , the more likely $e(\Sigma)$ will exceed $\frac{aR}{2c}$.

Proof: appendix 2.

Finally, some constraint is needed to ensure that the success probability $p(e)$ and $p(e) + \Sigma$ are no bigger than one. We describe it in appendix 2.

4. The determination of control right allocation

After examining the properties of (α, e) solution contingent on the allocation of control rights Σ , we continue to study the determination of control right allocation in this section. By substituting $e(\Sigma)$ in eq.5 (the one with larger value) into the board's objective function $(p(e) + \Sigma)R - 1 - C(e) - \sum_{k=1}^K \gamma_k Z_k$ and differentiating it with respect to Z_k , we derive the first order condition as follows

$$\left(a \frac{\partial e}{\partial \Sigma} + 1 \right) \theta_k R - 2c e(\Sigma) \frac{\partial e}{\partial \Sigma} \times \theta_k - \gamma_k \begin{cases} \geq 0 & \text{then } Z_k = 1 \\ < 0 & \text{then } Z_k = 0 \end{cases} \quad (8)$$

In the absence of moral hazard problem (or the entrepreneur's effort is observable) the first best solution for the board is to abandon all rights, or equivalently surrender all control rights to the entrepreneur who will not implement these rights as a result of $\theta_k R < \gamma_k$ for all $k = 1, \dots, K$ according to assumption 1. When entrepreneur's effort is unobservable, eq.8 introduces another channel, i.e., $\left(a \frac{\partial e}{\partial \Sigma} \right) \theta_k R - 2c e(\Sigma) \frac{\partial e}{\partial \Sigma} \times \theta_k$ that affects the determination of control right allocation through its influence on the effort level. When $Z_k = 1$ (allocating the control right k to outsiders) the effort level changes

by $(\frac{\partial e}{\partial \Sigma})\theta_k$ which affects the project's expected return by an amount of $(a\frac{\partial e}{\partial \Sigma})\theta_k R$.

Meanwhile the entrepreneur's effort cost changes by $2ce(\Sigma)\frac{\partial e}{\partial \Sigma}\times\theta_k$. If

$(a\frac{\partial e}{\partial \Sigma})\theta_k R - 2ce(\Sigma)\frac{\partial e}{\partial \Sigma}\times\theta_k$ is positive and greater than the loss $\theta_k R - \gamma_k$, then

allocating control right k to outsiders would become beneficial in the second-best sense. Otherwise allocating control right to outsiders is detrimental even in the second best sense.

We can rewrite eq.8 as follows,

$$\frac{\theta_k R}{\gamma_k} \geq \frac{1}{1 + \frac{\partial e}{\partial \Sigma}(a - \frac{2ce}{R})} \equiv \phi, \text{ then } Z_k = 1; \quad \text{otherwise } Z_k = 0 \quad (9)$$

Where $\frac{\partial e}{\partial \Sigma}$ is calculated as

$$\frac{\partial e}{\partial \Sigma} = \frac{1}{2a} \left[\frac{2c\Sigma + a^2 D}{\sqrt{(2c\Sigma + a^2 D)^2 - 8a^2 c(1-\beta)r}} - 1 \right] \quad (10)$$

Since $\theta_k R < \gamma_k$ for all $k = 1, \dots, K$ by assumption 1, the board could possibly take back some control right (i.e., $Z_k = 1$ for some k) only if the threshold ϕ is smaller than one or equivalently $\frac{\partial e}{\partial \Sigma}(a - \frac{2ce}{R}) > 0$. From eq.10 or corollary 1 $\frac{\partial e}{\partial \Sigma}$ is positive, so the

sign of $\frac{\partial e}{\partial \Sigma}(a - \frac{2ce}{R})$ hinges on the relative size of $e(\Sigma)$ and the efficient level $aR/2c$.

From proposition 2, $e(\Sigma)$ is bigger than $aR/2c$ for all $\Sigma \geq 0$ when δ is greater than δ^H . Therefore $\frac{\partial e}{\partial \Sigma}(a - \frac{2ce}{R})$ is negative and no control right will ever be

allocated to investors when the entrepreneur puts such high emphasis on the gain or loss of his ownership interest. Intuitively allocating control right to investors would be second best only if it improves the effort level *toward* the efficient level. If $e(\Sigma)$ is bigger than $aR/2c$, then allocating control right to investors would make $e(\Sigma)$ even further deviate from the efficient level and dampen the aggregate welfare.

For other δ 's that are less than δ^H , we need to solve eq.5, 9 and 10 simultaneously in order to decide the allocation of control rights. This is because ϕ is a function of Σ that is in turn determined by eq.9. In other words, in solving eq.9 the board should take into account the influence of Σ on the effort. The allocation $Z_k = 1$ or 0 should be resolved based on the concept of subgame perfectness. We will

not probe into the exact determination of Σ in this paper. Rather we are more interested in how the threshold ϕ is affected by the changes in other exogenous factors. For a given allocation of control right Σ , a lower ϕ implies that the board is more willing to transfer more control rights to investors. We examine in proposition 3 below the impact of exogenous elements such as the ownership β or the weight on the prospect-type utility δ on this threshold ϕ , and consequently on the allocation of control rights.

Proposition 3: *Regarding the allocation of control rights, the threshold level above which the board intends to transfer some control rights to investors is an increasing function of the entrepreneur's ownership β and the weight δ he puts on the prospect-type utility as long as δ is less than $\frac{2c(1-\beta)r}{a^2 R \times P}$ ($\equiv \delta^H$). Therefore the board tends to transfer less control rights to investors if the entrepreneur has larger ownership or puts greater weight on his prospect-type utility.*

Proof: From eq.9 the board transfers control right k to investors only if their relative benefit $\theta_k R / \gamma_k$ is no less than some threshold ϕ . This ϕ is composed of two terms:

$\frac{\partial e}{\partial \Sigma}$ and $(a - \frac{2ce}{R})$. The latter term decreases as β or δ increases according to corollary 1 (i.e., $\partial e / \partial \delta > 0, \partial e / \partial \beta > 0$). The former term, after some calculation, is also a decreasing function of β and δ . More specifically,

$$\frac{\partial(\frac{\partial e}{\partial \Sigma})}{\partial \delta} = \frac{a^2 \frac{\partial D}{\partial \delta} [\sqrt{(2c\Sigma + a^2 D)^2 - 8a^2 c(1-\beta)r} - \frac{(2c\Sigma + a^2 D)^2}{\sqrt{(2c\Sigma + a^2 D)^2 - 8a^2 c(1-\beta)r}}]}{(2c\Sigma + a^2 D)^2 - 8a^2 c(1-\beta)r} < 0$$

$$\frac{\partial(\frac{\partial e}{\partial \Sigma})}{\partial \beta} = \frac{a^2 \frac{\partial D}{\partial \beta} [\sqrt{(2c\Sigma + a^2 D)^2 - 8a^2 c(1-\beta)r} - \frac{(2c\Sigma + a^2 D)^2}{\sqrt{(2c\Sigma + a^2 D)^2 - 8a^2 c(1-\beta)r}}] - \frac{4a^2 cr(2c\Sigma + a^2 D)}{\sqrt{(2c\Sigma + a^2 D)^2 - 8a^2 c(1-\beta)r}}}{(2c\Sigma + a^2 D)^2 - 8a^2 c(1-\beta)r} < 0$$

Therefore ϕ is an increasing function of β and δ .

Q.E.D

We depict the results of proposition 3 in figure 3. In the figure we draw two lines

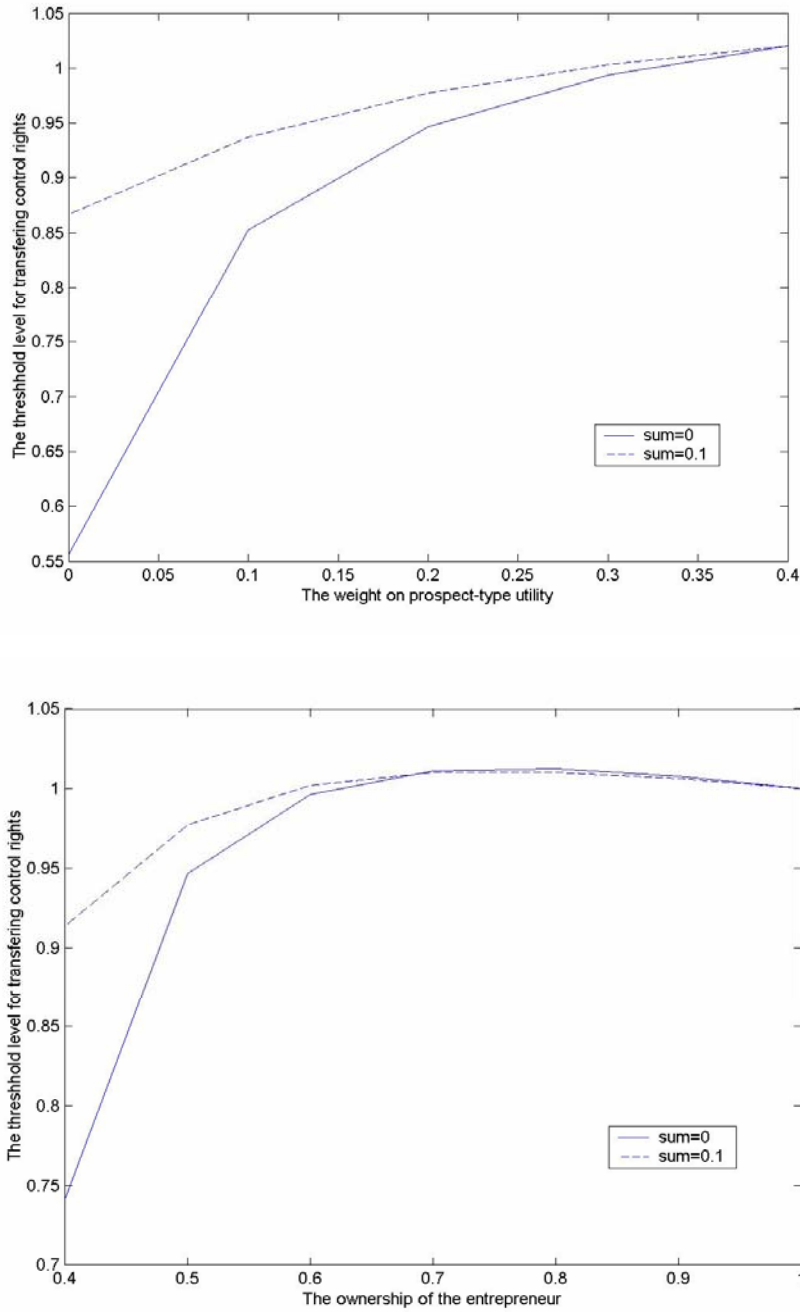


Figure 3 The threshold level for transferring control rights with respect to δ and β
 $(a=1, c=1.5, R=2.5, r=1, W=0, \beta=0.5(\text{left})$ and $\delta=0.2(\text{right})$)

representing the relationship between ϕ and δ (β). The lower line corresponds to $\Sigma=0$ and the higher line corresponds to $\Sigma=0.1$.⁷ Take the case of $\delta=0.1$ for

⁷ We can prove that $\frac{\partial \phi}{\partial \Sigma} (a - \frac{2ce}{R})$ also decreases as Σ is enhanced for any given δ and β . Thus the threshold ϕ increases as Σ is uplifted.

example. According to the left figure, the threshold level is about 0.85 when $\Sigma=0$. If there exists some control rights k such that their $\theta_k R / \gamma_k$ (although smaller than 1) is bigger than 0.85, then the board has motives to transfer these rights to investors. However Σ would be uplifted once these control rights were transferred. So the line would move upward and the original threshold is no longer valid. Based on this new line we can derive another new threshold level and the control rights that would be transferred. Since the new threshold is larger than the original one, less control rights would be allocated to investors. This reduces Σ from the one derive in the second step, and thus another new line lower than the one obtained in the second step is obtained. This procedure will be iterated until we find a consistent Σ level (a consistent line), at which the Σ determined by the amount of control right allocation is exactly the Σ level the line represents. We propose a numerical example to illustrate this procedure in table 1 and 2.

The parameters specified in this numerical example are as follows. The project's return in success $R=2.5$, the coefficient of success probability $a=1$ and the coefficient of the entrepreneur's effort cost function $c=1.5$. The social efficient effort level is 0.833. Other parameters are opportunity cost $r=1$ and the wage level $W=0$. As for the specification of control rights, we assume there are total twenty rights to be determined (i.e., $K=20$). To simplify the numerical analysis each control right k is assumed to have the same incremental success probability $\theta_k=0.005$ for all $k=1, \dots, K$. These control rights differ in the cost γ_k . We arrange k such that $0.0125 < \gamma_1 < \gamma_2 \dots < \gamma_K$. We let $\gamma_1=0.013$ and each of the following γ 's is then increased by 0.0005 subsequently.

Given the ownership β equal to 0.5, table 1 examines the impact of δ on the allocation of control right, the compensation contract and the resulting entrepreneur's effort. According to proposition 3 we find that the δ^H above which it is certain that no control right would be transferred to investors is equal to 0.32 when $\beta=0.5$. Thus in table 1 only the case of $\delta=0, 0.1$ and 0.2 are investigated.⁸ Table 2, on the other hand, examines the impact of β with the weight of prospect-type utility δ given to be 0.15. In these tables we assume $\Sigma=0$ in the first step. Contingent on $\Sigma_1=0$ we can find the correspondent threshold ϕ , the compensation contract α and effort level e . Besides according to eq.9 we can decide whether the control right k should be transferred to investors; and a new level of Σ is thereby generated. For example in table 1 when $\Sigma_1=0$ the relative benefit $\theta_k R / \gamma_k$ of all control rights are greater than the threshold level 0.56, so the twenty control rights are all allocated to investors and Σ is

⁸ We choose $\beta=0.5$ in table 1 based on assumption 2. Assumption 2 would be violated by lower β . Then no solution exists and the entrepreneur is financially constrained when $\delta=0$.

elevated to be 0.1. This $\Sigma=0.1$ becomes the initial Σ in the second step. This process is iterated until we find some Σ which is consistent with the resulting Σ after the calculation. However this iteration process may not converge because our assumption of the incremental change in K may not be fine enough (or the number of control rights is not large enough). In our numerical example we can't find the convergence only in the case of $\delta=0, \beta=0.5$ in table 1..

Table 1 The determination of control right allocation, compensation contract, effort level and social welfare with respect to different δ

$\delta=0 (\beta=0.5)$	ϕ	Σ	α	e
$\Sigma_1=0$	0.56	0.1 (20)	0.2	0.5
$\Sigma_2=0.1$	0.87	0.015(3)	0.4226	0.5927
$\Sigma_3=0.015$	0.6676	0.06(12)	0.2583	0.5243
$\Sigma_4=0.06$	0.8109	0.025(5)	0.3630	0.5679
$\Sigma_5=0.025$	0.7146	0.045(9)	0.2876	0.5365
$\Sigma_6=0.045$	0.7782	0.035(7)	0.3345	0.5560
$\Sigma_7=0.035$	0.7501	0.04(8)	0.3126	0.5469
$\Sigma_8=0.04$	0.7649	0.035(7)	0.3239	0.5516
$\delta=0.1$				
$\Sigma_1=0$	0.8528	0.02(4)	0.3673	0.6322
$\Sigma_2=0.02$	0.8801	0.015(3)	0.3988	0.6453
$\Sigma_3=0.015$	0.8741	0.015(3)	0.3914	0.6423
$\delta=0.2$				
$\Sigma_1=0$	0.9466	0.005(1)	0.4521	0.7300
$\Sigma_2=0.005$	0.9491	0.005(1)	0.4575	0.7323

Table 2 The determination of control right allocation, compensation contract, effort level and social welfare with respect to different β

$\beta=0.4 (\delta=0.15)$	ϕ	Σ	α	e
$\Sigma_1=0$	0.5398	0.1(20)	0.375	0.5333
$\Sigma_2=0.1$	0.8792	0.015(3)	0.6858	0.6369
$\Sigma_3=0.015$	0.6692	0.06(12)	0.4588	0.5613
$\Sigma_4=0.06$	0.8224	0.025(5)	0.6041	0.6097
$\Sigma_5=0.025$	0.7207	0.045(9)	0.5	0.575
$\Sigma_6=0.045$	0.7884	0.03(6)	0.5649	0.5966
$\Sigma_7=0.03$	0.7411	0.04(8)	0.5180	0.5810
$\Sigma_8=0.04$	0.7744	0.035(7)	0.5503	0.5918

$\Sigma_9=0.035$	0.7588	0.035(7)	0.5347	0.5866
$\beta=0.5$				
$\Sigma_1=0$	0.9089	0.01(2)	0.4144	0.6831
$\Sigma_2=0.01$	0.9173	0.01(2)	0.4273	0.6885
$\beta=0.6$				
$\Sigma_1=0$	0.9798	0(0)	0.3219	0.7734

In these two tables all δ 's are smaller than δ^H and the ultimate efforts are all smaller than the social efficient level 0.83. Therefore the entrepreneur will actually take no interim action once he is given the remaining control rights according to assumption 1 although he will be forced to take some interim actions for those rights that are taken back by the investors. The introduction of the prospect-type utility into the entrepreneur's objective function is beneficial to the social welfare so long as his δ is still less than δ^H . The concern with gain or loss of his ownership interest will now help relax the financing constraint and improve the entrepreneur's effort level toward the efficient level.

If δ is greater than δ^H none of the interim actions should be taken even in the second best sense, and all control rights should be passed on to the entrepreneur. However the entrepreneur's recognized payoff $W + (1 + \alpha)\beta(R - W) + \delta(\beta(R - W - r) + \lambda\beta r)$ is greater than R when δ is greater than δ^H . Therefore the entrepreneur might instead take the interim action k if his recognized payoff is so high such that $\theta_k [W + (1 + \alpha)\beta(R - W) + \delta(\beta(R - W - r) + \lambda\beta r)] > \gamma_k$ even if $\theta_k R < \gamma_k$. In these circumstances the entrepreneur's effort level exceeds the social efficient level; and the higher δ , the greater extent of his effort's over-exertion. Moreover the entrepreneur might work on the interim actions that are not supposed to take, further impair the social welfare.⁹

5. Conclusion

It is common and natural to assume that a hired manager, dealing with resources which belong to others will be less careful in their use than an owner. The conventional approach on this corporate governance problem emphasizes the

⁹ For example in table 1 all interim actions should not be taken and all control rights are allocated to the entrepreneur when $\delta=0.4$. If the entrepreneur actually takes no action, then the social welfare is 0.0355. But the entrepreneur takes the first action ($k=1$) since his expected recognized payoff from taking this action is greater than his cost. The social welfare reduces to 0.0347. If δ increase to 0.5, the social welfare decreases from 0.0355 to 0.0116 if the entrepreneur takes no actions. However now there are four interim actions ($k=1\sim 4$) that he would take instead. So the social welfare decreases further to 0.0048.

managerial contract design or the allocation of control rights. To square with observed facts, we pinpoint the distinction of ownership versus contract in inspiring managers' due diligence and safeguarding the interests of investors. This study draws upon a special feature of human nature, i.e., loss aversion affiliated with ownership, to buttress managers' working incentive and remedy the deficiency of contract design or control right allocation.

This study concludes that one's aversion toward ownership loss can mitigate the inefficiency of moral hazard. The entrepreneur's effort is enhanced because his envisioned marginal payoff of effort now includes not only the contract compensation, but also his feel of changes in his ownership. This particular mental concern or psychological aversion toward any loss in what he has owned will help unwind the financing constraint. Meanwhile the social welfare will be enhanced due to an improvement in entrepreneur's effort. As to the control right, it plays a substitution role for cash flow right to investors. Investors, when being transferred more control rights, are willing to offer larger compensation to the entrepreneur; thereby uplifting the entrepreneur's effort level as well as the social welfare. Besides, this effort-enhancing benefit from the allocation of control right to investors is found to be smaller when the entrepreneur has higher initial ownership or places a greater emphasis on his gain and loss. Therefore under these circumstances fewer control rights **need to be** transferred to investors (or more control rights should be retained by the entrepreneur).

However this beneficial effect brought by the entrepreneur's concern with gain or loss of his ownership may turn into a negative one once the entrepreneur over-evaluates his prospect-type utility in such an extent that his effort level exceeds the social efficient level. It is noticeable that this over-exertion of effort can only occur with the inclusion of the prospect-type utility. The entrepreneur's effort is always smaller than the efficient one under other contract designs without considering this ownership ramification. Moreover the entrepreneur may undertake some actions that are not supposed to be done when he is given the control right. It will be very likely to occur when the entrepreneur over-emphasizes his prospect-type payoff in such an extent as to bring about the "over-effort" syndrome and aggravate the social welfare.

Appendix 1

Proof of eq.4 and 5: We derive $(\alpha(\Sigma), e(\Sigma))$ that simultaneously satisfy eq.3 and "IR" constraint. From eq.3 we get $\frac{2ce}{a} = D' + \alpha\beta(R-W)$,

where $D' \equiv W + (1+\delta)\beta(R-W) + \delta(\lambda-1)\beta r$. α can then be written as $\alpha = \frac{(2ce/a) - D'}{\beta(R-W)}$. This

equation of α is substituted into the “*IR binding*” constraint $(ae + \Sigma)(1 - (1 + \alpha)\beta)(R - W) = (1 - \beta)r$ to obtain a quadratic equation of effort level e

$$2ac e^2 + (2c\Sigma - a^2D)e + a[(1 - \beta)r - \Sigma D] = 0,$$

where $D = D' + (1 - \beta)(R - W) = R + \delta[\beta(R - W - r) + \lambda\beta r] \equiv R + \delta P$ is the maximal marginal benefit of effort for the entrepreneur. According to the above quadratic equation, $e(\Sigma)$ is solved as eq.A1,

$$e(\Sigma) = \frac{a^2 D - 2c\Sigma \pm \sqrt{(2c\Sigma + a^2D)^2 - 8a^2c(1 - \beta)r}}{4ca}. \quad (\text{A1})$$

Besides $\alpha(\Sigma)$ is solved by substituting (A1) into the “*IR binding*” constraint,

$$\alpha(\Sigma) = \frac{1 - \beta}{\beta} \left[1 - \frac{4cr}{(R - W)[a^2D + 2c\Sigma \pm \sqrt{(2c\Sigma + a^2D)^2 - 8a^2c(1 - \beta)r}]} \right]. \quad (\text{A2})$$

In the benchmark case $\delta = 0$ and $\Sigma = 0$, the effort level is smaller than the efficient one $e^E = aR/2c$ (see the proof of lemma 1 below). Thus the board will definitely select the larger value solution to induce higher effort in the face of moral hazard (which is closer to the efficient one). When δ is so large such that the larger e solution may become greater than the efficient one, the board still pick the larger e solution because the stock bonus α of the smaller value solution would become negative, which is unreasonable. Q.E.D

Proof of lemma 1: By setting $\delta = 0$ and $\Sigma = 0$ in eq.A1 and A2, the resulting (α, e) is shown as

$$\alpha = \frac{1 - \beta}{\beta} \left[1 - \frac{4cr}{a(R - W)[aR \pm \sqrt{a^2R^2 - 8c(1 - \beta)r}]}, \right] \quad (\text{A3})$$

$$e = \frac{aR \pm \sqrt{a^2R^2 - 8c(1 - \beta)r}}{4c}. \quad (\text{A4})$$

From eq.A3 and A4, (α, e) have real solutions if and only if $a^2R^2 \geq 8c(1 - \beta)r$ or equivalently $\beta \geq 1 - \frac{a^2R^2}{8cr}$. Besides, the two α solutions are all smaller than the fully diluted level $\frac{1 - \beta}{\beta}$. To assure that α of the larger value solution is nonnegative, we

require that $4cr \leq a^2R(R - W) + a(R - W)\sqrt{a^2R^2 - 8c(1 - \beta)r}$ be met. This inequality is satisfied if $4cr < a^2R(R - W)$ holds. Otherwise we derive the necessary condition as $2cr < a^2(R - W)[\beta(R - W) + W]$. Assumption 3 combines these two inequalities to guarantee that the larger α solution is nonnegative because the relative size of

$$\frac{a^2R(R - W)}{4} \quad \text{and} \quad \frac{a^2(R - W)[\beta(R - W) + W]}{2} \quad \text{is indefinite.}$$

If assumption 2 or 3 is not satisfied, the constraints can be loosened by increasing Σ or δ . From eq.A1 and A2 the necessary condition for having real $(\alpha(\Sigma), e(\Sigma))$ solution, i.e. $\beta \geq 1 - \frac{(a^2 D + 2c\Sigma)^2}{8a^2 cr}$, is less stringent than $\beta \geq 1 - \frac{a^2 R^2}{8cr}$ for all positive Σ or δ . Moreover it can be seen easily that the necessary condition for α of the larger value solution to be nonnegative, $4cr \leq (a^2 D + 2c\Sigma)(R - W) + (R - W)\sqrt{(a^2 D + 2c\Sigma)^2 - 8a^2 c(1 - \beta)r}$, is also less strict than $4cr \leq a^2 R(R - W) + a(R - W)\sqrt{a^2 R^2 - 8c(1 - \beta)r}$. As a result, the allocation of control rights to investors or the introduction of prospect-type utility into the entrepreneur's objective function will help relax the financing constraints. Q.E.D

Appendix 2

Proof of proposition 2: We derive the high critical value δ^H first. As $e(\Sigma)$ is an increasing function of Σ , its minimum value is $\frac{aD + \sqrt{a^2 D^2 - 8c(1 - \beta)r}}{4c}$ by setting $\Sigma = 0$ in eq.A1. If this value is larger than $aR/2c$, then $e(\Sigma)$ is larger than the efficient effort level for all $\Sigma \geq 0$. We compare this minimum value with $aR/2c$ to derive the critical value δ^H :

$$aD + \sqrt{a^2 D^2 - 8c(1 - \beta)r} > 2aR.$$

The sufficient condition for this inequality to hold is $\delta > R/P$, where $P \equiv \beta(R - W - r) + \lambda\beta r$ is the prospect-type payoff. The necessary condition for this inequality to hold is $\delta > \frac{2c(1 - \beta)r}{a^2 R \times P} \equiv \delta^H$ (δ^H is smaller than R/P since at the

efficient level it must be that $\frac{a^2 R}{2c} \times R > (1 - \beta)r$). Next we derive the low critical value δ^L . Because the success probability $p(e) + \Sigma$ should be smaller than (equal to) one, the maximum value of Σ is $1 - ae$. By substituting $\Sigma = 1 - ae$ into eq.A1 the maximum value of $e(\Sigma)$ is:

$$e(\Sigma = 1 - ae) = \frac{a(D - (1 - \beta)r)}{2c}.$$

If this maximum value is smaller than $aR/2c$, then $e(\Sigma)$ is smaller than $aR/2c$ for all possible $\Sigma \geq 0$. So we compare this maximum value with $aR/2c$ to derive the low critical value δ^L :

$$D - (1 - \beta)r < R.$$

The necessary condition for this inequality to hold is $\delta < \frac{(1-\beta)r}{p} \equiv \delta^L$.

Proof of the constraints for $p(e) \leq 1$ and $p(e) + \Sigma \leq 1$: We know from the above proof that $e(\Sigma = 1 - ae) = \frac{a(D - (1-\beta)r)}{2c}$. So Σ must be smaller than $1 - \frac{a^2(D - (1-\beta)r)}{2c}$ to satisfy $p(e) + \Sigma < 1$. Besides the condition for $p(e) < 1$ is $D - (1-\beta)r < 2c/a^2$ or $\delta < \frac{(\frac{2c}{a^2} - R) + (1-\beta)r}{p} \equiv \hat{\delta}$. In this paper we only consider those $\delta \leq \hat{\delta}$. Moreover this $\hat{\delta}$ is bigger than δ^H so long as $R > (1-\beta)r$.

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