Optimal Fiscal Rules for Green Growth in an Endogenous Growth Model

by

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Abstract

To explore the effects of a tighter environmental policy on long-run sustained growth, this paper uses a one-sector dynamic model of endogenous growth, based on the joint accumulation of private capital and abatement knowledge capital. It derives the optimal level of government investment and taxes for ‘green growth’ (or ecologically sustainable development). The result indicates that the long-run sustained balanced-growth path and the effects of some tightening environmental and fiscal policy depend heavily upon the magnitude of some key parameters of the economy such as production elasticity of abatement knowledge and pollution-conversion parameter. Moreover, unlike the general results suggested by the literature on growth and taxation, the rate of environmental investment that maximizes the long-run growth rate should be greater than its own ‘pure’ output elasticity by the amount of the premium for environmental sustainability. Thus, for green growth, the overall abatement knowledge-intensity of production, \( h/K \), should be directed by this rule.

**Key Words:** optimal investment and taxation; environmental quality; private capital; abatement knowledge; endogenous growth; capital income taxation.
I. Introduction

How will we meet the difficult global challenges before us, while simultaneously improving people’s lives and conserving the environment? To do this, it is crucial to understand the interactions among economic activities, technological progress, and ecological processes over time.

Environmental policies towards sustainable development may be easier if technological progress in abatement knowledge responds to economic incentives. If so, in what way can environmental investment and taxation contribute to the productivity of private factors of production and to sustainable economic growth?

This paper analyzes government policy measures for ‘green growth’ (defined here as ecologically sustainable growth) within an endogenous growth model with pollution and endogenous accumulation of abatement knowledge. In this model, the economy produces a single final-good and it also accumulates productive assets, which include new abatement knowledge capital, by devoting some fractions of output to investments. Environmental quality, modelled as a stock of renewable resource, acts not only as a public consumption good but also as a productive public input into production. Pollution is inevitable by production activities, but it can be reduced by increasing the stock of pollution abatement technical knowledge (e.g., clean technology) and/or by employing environmental regulatory instruments on production activities (e.g., pollution standard, pollution permit, or emissions tax).

Recent advances in endogenous growth theories have opened up the possibilities of analyzing the level and growth effects of various policy changes in the short-run and long-run. Until recently, environmental models of endogenous growth have

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1 We assume a one-sector production technology for a single output that can be used for private capital (inclusive of physical and human capital), abatement knowledge capital, and consumables.
2 In our case, pollution abatement technology is a public good that is an input to production. It can be viewed as knowledge about ‘clean’ production methods.
3 Environmental policy may hurt short-run growth but enhance long-run growth. To make explicit the trade-off between short-run costs and long-run benefits of environmental policy in welfare analysis, we need to for the entire dynamic adjustment path and convergence rate towards a new balanced growth path.
concentrated on the long-term growth rate. These environmental endogenous growth models mainly serve to examine the condition under which sustainable growth is feasible and optimal, and how environmental policy affects economic growth.

Most endogenous growth models abstract from stock-flow dynamics by modelling environmental quality or abatement effort as flow variables, thereby restricting the analysis to comparative steady states impacts. However, Bovenberg and Smulders (1995, 1996), Smulders and Gradus (1996), and Fullerton and Kim (2008) model environmental quality and/or abatement technology as a stock variable so that the model can analyze the adjustment path, including the comparative transitional dynamics towards a new balanced-growth path.

In another way, Bovenberg and Smulders (1995, 1996) also extend Chamley (1993), Mulligan and Sala-i-Martin (1993), and Cabble and Santos (1993), who analyze the transition in endogenous growth models with the usual two types of capital. Bovenberg and Smulders (1995, 1996) include three types of assets: private capital (physical and human capital), pollution abatement knowledge, and natural capital (i.e., environmental quality). They argue that a tightening of environmental policy may boost growth, at least in the long-run. Government intervention in the form of taxes on pollution or subsidies to investment in pollution abatement can internalize the difference between social costs of pollution and the social benefits. In other words, policy can reduce the distortionary gap between the social rate of return on investment in environmental quality (natural capital) and the return on private capital (e.g., physical or human capital).

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4 Most endogenous growth models have non-man-made (or non-reproducible) inputs take the form of ‘raw’ labor, but, in these environmental models, the inputs are supplied by nature. Models with nature as an input include Xepapadeas (1993), Tahvonen and Kuuluvainen (1993), Gradus and Smulders (1993), Van Ewijk and Van Wijnbergen (1995), Elbash and Roe (1996), Bovenberg and Smulders (1995, 1996), and Bovenberg and de Mooij (1997).

5 This implies that transitional dynamics of the man-made variables are absent if the model includes only one type of asset, namely physical capital. In this case, after a policy shock, the economy immediately jumps towards a balanced growth path on which all man-made commodities grow at the same rate while natural variables (e.g., environmental quality) remain constant. However, a second state variable allows for introducing transitional dynamics.

6 In the Bovenberg and Smulders (1995, 1996) model, public environmental R&D activities for pollution abatement knowledge are assumed to be financed through lump-sum taxation, and other prior taxes are not explicitly considered.
While this prior literature focuses on optimal corrective environmental policies for internalizing environmental externalities in a sustainable growth framework, the analysis below also allows for private and public abatement activities financed by distortionary factor income taxation, not lump-sum taxation, in a decentralized competitive market economy.\(^7\) As discussed in Rebelo (1991), Rebelo and Stokey (1995), and Jones and Manuelli (1997), taxes on income overall or income from capital lower the rate of saving and growth, while taxes on income from labor or on consumption directly do not.\(^8\)

To explore the effects of a tighter environmental policy on long-run sustained growth and welfare, this paper uses a dynamic model of endogenous growth, based on the joint accumulation of private capital and abatement knowledge capital. The model developed in this paper departs from the Bovenberg and Smulders (1995, 1996) model in three important ways. First, we allow for distortionary taxes on private capital income for the purpose of financing the on-going public investment in abatement knowledge capital. Second, we generalize the output elasticity of pollution that crucially affects the government financing method regarding a tighter environmental policy. Third, as a simplification, their two-sector model is reduced to one sector.\(^9\)

The paper is organized as follows. Section II sets up the model which incorporates pollution and endogenous evolution of abatement knowledge and environmental quality, and then describes the balance-growth equilibrium path in the market economy. Section III discusses a tighter environmental policy in the steady states of the endogenous growth model with a graphical solution procedure. Section IV derives the optimal tax rules in the endogenous growth framework with public and private incentives to abate

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\(^7\) Higher government spending and distortionary taxation lead to lower long-run per capita income (in the conventional neoclassical growth theory) or to lower long-run growth rate (in endogenous growth models). In particular, pre-existing taxes on private capital income may distort both inter-temporal decisions and inter-asset decisions. Since non-market environmental quality investment is public rather than private, the social and private rates of returns differ. In this case, the dynamic fiscal policy effects of shifting the tax burden from private capital towards polluting inputs may be beneficial. Depending on the starting point, substitution of environmental taxes for pre-existing distortionary income taxes may yield a 'double dividend' (i.e., not only improved environmental quality and but also enhanced welfare through a less distortionary way of raising revenue). The double dividend hypothesis was first voiced by Pearce (1991), and this literature was surveyed by Goulder (1995).

\(^8\) In the latter case, these taxes are has level, but not growth effects.

\(^9\) The qualitative results do not change if we consider a two sector-model in which one sector produces final goods and the other produces new abatement knowledge capital.
pollution, and this section also explores the response to capital income taxes, pollution taxes, and public investment in abatement knowledge. Finally, section V contains some concluding remarks.

II. The Model

This section presents a simple endogenous growth model in which pollution, environmental quality and accumulation of pollution abatement knowledge are endogenous. This model is used to explore the interaction between environmental policy and economic growth. It is an extension of the models explored Bovenberg and Smulders (1995, 1996).

The economy produces a single final good. Individual household utility depends on consumption of the final good and on the quality of the environment. Environmental quality is a stock that acts as a public input into production and as a nonrival consumption good. The economy has three assets. The first is private capital (including physical and human capital), and the second is public abatement knowledge capital (i.e., a nonrival environmental R&D good). Either of these first two types of asset can be accumulated, by devoting some fraction of output to either type of investment. The third type of asset is environmental quality (i.e., natural capital), and can be augmented by reductions in pollution, either by reducing production or by investing in the second asset (public abatement knowledge capital).

1. Model assumptions

As in Tahvonen and Kuuluvainen (1991), the natural environment is an asset in the economy. Growth and depletion of this renewable resource is modeled according to the following accumulation equation:

\[ \dot{N} = E(N) - P, \quad \partial E / \partial N > (\leq) 0, \quad \partial^2 E / \partial N^2 < 0, \]
where \( N \) denotes environmental quality (or the stock of natural capital), \( P \) is pollution, and where the dot over any variable (hereafter) represents the change over time. The first term \( E(N) \) represents natural or ecological growth through regeneration processes, and it can be viewed as the difference between natural resource reproduction and resource use for maintenance. \(^{10}\) Natural capital accumulation features diminishing returns, \( \frac{\partial^2 E}{\partial N^2} < 0 \), which implies that larger \( N \) makes it more difficult to regenerate the complete stock.

The second term \( P \) indicates the deterioration of environmental quality through the extractive use (or harvest) of natural resources in production. As illustrated in Fig.1, in a sustainable steady-state path, nature can absorb a maximal amount of pollution without deteriorating, that is \( P = E(N) \), so that \( E(N) \) represents the absorption capacity of the environment.

The production side of economic activity is described by the following production function and the uses side equation:

\[
(2a) \quad Y = A(N) F(K, Z) = C + \dot{K} + q_h \dot{h}, \quad \text{where}
\]

\[
(2b) \quad Z = h P^e.
\]

This sector produces a flow of final output that can be consumed (\( C \)), invested in accumulating private capital (\( \dot{K} \)), or devoted to generating new public knowledge (\( \dot{h} \)) about pollution abatement techniques.\(^{11}\) This is the overall resource constraint of the economy. Here, \( q_h \) denotes the shadow price of knowledge relative to private capital (i.e., price at which the output of the R & D sector is sold). Inputs in the production process are private capital (\( K \)), effective pollution or ‘harvested’ environmental resources (\( Z \)), and nonrival services from the natural environment (captured by \( N \)). Private capital (\( K \)) includes both physical and human capital. The effective pollution (\( Z \)) depends on the

\(^{10}\) Sustainable development can be defined by \( \dot{N} = 0 \) and requires that pollution \( P \) is constant in the long run and does not exceed the maximum absorption capacity.

\(^{11}\) This paper, basically, follows the notation of Bovenberg and Smulders (1995, 1996). They use \( Y, \ Z, \) and \( C \) for flow variables, and \( K, \ N, \) and \( h \) for stock variables (private capital, natural capital, and abatement-knowledge capital). Here, without loss of generality, we also ignore depreciation.
stock of available public abatement knowledge \((h)\) and the economy-wide level of pollution by extractive use of the environment \((P)\), as described by eq. (2b). The exponent term \(\varepsilon\) in the effective pollution function denotes the pollution-conversion parameter of extractive use of the environment. Environmental quality, \(N\), is a stock that enters production sector since a higher environmental quality renders the economy more productive. Output also depends on \(A(N)\), which measures the positive "production externality" associated with the environment as a natural input. \(F(\ )\) exhibits constant returns with respect to the two rival inputs. The production function is non-decreasing in all arguments and exhibits diminishing returns with respect to each factor alone, and all inputs are essential.

2. The Market Economy

Under a decentralized economy with perfect competition, identical and spatially homogeneous individual firms maximize the firm’s value by its choices of investment in private capital \(K\) and pollution \(P\) (ignoring the environmental production externality of the way \(P\) affects \(N\)). In equilibrium, all symmetric firms have the same pollution emission (e.g., measured as a concentration level, say, \(\mu g/ft^3\)) which equals the economy-wide concentration level.

The value of representative firm amounts to the present value of all future dividends:

\[\text{value} = \sum_{t=1}^{\infty} \frac{D_t}{(1+r)^t}\]

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12 This ‘effective pollution’ input conveys the idea that pollution is a necessary input to production, with its own downward sloping marginal product, but that the same level of input \(Z\) can be achieved with less actual pollution \(P\) if the firm has access to more abatement knowledge \(h\).

13 Note that, in the ‘effective pollution’ function, pollution-conversion parameter \(\varepsilon\) depends mainly on energy sources- (or polluting materials-) specific attributes and country-specific endowment conditions, whereas abatement knowledge \(h\) features general productivity of cleaner technology. Moreover, in this model, we will not impose any prior restrictions on the parameter, \(\varepsilon\), that plays a crucial role in the size of environmental and fiscal policy. In the studies by Bovenberg and Smulders (1995, 1996) and Smulders (1997), they did not explicitly consider this possibility. In their model, pollution and publicly provided abatement knowledge is simply perfect substitutes in production.

14 Basically, endogenous growth requires non-diminishing returns to the economy’s reproducible resources altogether at the aggregate level.

15 The model ignores population growth and depreciation of private or public capital. All variables can be interpreted as expressed in per ‘raw’ labor terms.
is a stream of instantaneous dividends. Pollution in the final-goods sector can further enhanced by public abatement knowledge in the form of effective pollution $Z \equiv h P$. Pure profits can be used for dividends or investment but are reduced by normal capital costs:

$$\Pi = D + \dot{K} - r K.$$  

Here, $\tau_p$ and $r$ denote a pollution tax rate and the market interest rate (the required return on capital, gross of tax), respectively.

Firms equalize the marginal product of private inputs to their respective prices.\(^{16}\) Given the total amount of natural capital, $N$, and public abatement knowledge, $h$, the optimal allocation of these inputs at any moment in time is governed by:

$$\frac{\partial Y}{\partial K} = A(N) \frac{\partial F}{\partial K} = r$$

$$\frac{\partial Y}{\partial Z} h \varepsilon P^{\varepsilon - 1} = A(N) \frac{\partial F}{\partial Z} h \varepsilon P^{\varepsilon - 1} = \tau_p$$

Note that firms equate the marginal product of pollution, given the available abatement technical knowledge $h$, to the cost of pollution (i.e., pollution tax or the price of pollution permits).

An arbitrage condition, ignoring natural capital ($N$), reveals that investment in private capital ($K$), and abatement knowledge capital ($h$), given a certain amount of economy-wide pollution level ($P$), should be traded off against each other:

$$r = \frac{\partial Y}{\partial K} = \frac{1}{q_h} \frac{\partial Y}{\partial Z} P^{\varepsilon} + \frac{\dot{q}_h}{q_h}.$$  

The right-hand side of (4c) consists of the current return in production and a capital gain (i.e., changes in the relative price). This arbitrage condition implies that the social return

\(^{16}\) In the absence of adjustment costs, the maximization of the present value of future returns is equivalent to the maximization of profit in each period.
on investment in abatement knowledge capital should equal the gross-of-tax return on alternative investments \( r \).

The representative household utility is

(5a) \[ W = \int_{0}^{\infty} e^{-\theta t} U(c, N) dt , \quad U_{c} > 0, \quad U_{N} < 0, \quad U_{N} \geq 0 , \quad \text{where} \]

(5b) \[ U(c, N) = \left( \frac{c N^{\phi}}{1 - 1/\sigma_{c}} \right)^{1-1/\sigma_{c}}, \quad \sigma_{c} \neq 1; \quad \text{and} \quad = ln c + \phi ln N, \quad \sigma_{c} = 1. \]

Eq. (5b) is a specific instantaneous utility function. The parameter \( \sigma_{c} \) measures the intertemporal substitution elasticity. Other symbols with subscripts denote partial derivatives (e.g., \( U_{c} = \partial U / \partial c \)). Also, the parameter \( \theta \) represents the pure rate of time preference or utility discount rate, and \( \phi \) measures the environmental preference (or the "consumption externality" associated with the environment).\(^{17}\)

As mentioned before, for simple exposition, without loss of generality, we abstract from population growth and normalize the size of population to one, so that individual consumption equals aggregate consumption (\( c=C \)). Consumption and environmental amenities (measured by \( N \)) contribute to utility. Households own private capital (i.e., physical and human capital), receive income from factor rentals, and spend this income on accumulation of private capital and consumption.\(^{18}\) Private capital owners pay a source-based tax on capital income, \( \tau_{K} \). Hence, the budget constraint is given by:

(5c) \[ C + \dot{K} = (1 - \tau_{K}) r K + \Pi. \]

Total government expenditure equals total tax revenue in every period:

(6) \[ \tau_{K} r K + \tau_{p} P = q_{h} \dot{h} \quad \text{or} \quad \tau_{K} + \dot{\tau}_{p} = \dot{h}, \]

where \( \dot{\tau}_{p} (\equiv \tau_{p} P / rK) \) represents the ratio of pollution tax revenue to private capital income and \( \dot{h} (\equiv q_{h} \dot{h} / rK) \) represents the ratio of public investment in abatement

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\(^{17}\) Growth is balanced in the optimum only if the utility function is of a special form like this, as discussed in King et al. (1988). This specification implies that the elasticity of marginal utility is constant \( (U_{c} c / U_{c} = -1/\sigma_{c}) \) and that the share of amenities in utility is constant \( (U_{N} N / U_{c} c = \phi) \). Since environmental quality is a public good, and agents do not take into account the effects of their decisions on its supply, the environmental preference parameter \( \phi \) also measures the "consumption externality."
knowledge to private capital income. The government raises revenues by adopting a positive tax rate on private capital income or a positive pollution tax.\textsuperscript{19} The revenues from the taxation on private capital income and on pollution are used to finance government expenditure on public investment ($q_h \dot{h}$).

The representative household chooses its consumption path and the allocation of its private capital in order to maximize its life-time utility (5a) subject to the household intertemporal budget constraint (5c), taking tax rates as given. Ignoring environmental quality as a public good in the maximization problem, this optimization yields the modified Keynes-Ramsey rule (optimal savings rule) with the required growth rate $g$:

$$\theta - \frac{\dot{U}_c}{U_c} = (1 - \tau_K) r \quad \text{or} \quad g \equiv \frac{\dot{c}}{c} = \sigma_c \left[ (1 - \tau_K) r - \theta + \phi \left( 1 - \frac{1}{\sigma_c} \right) \frac{N}{N} \right].$$

This equation, representing the trade-off between consumption and investment, reveals that postponement of consumption must be rewarded by a net-of-tax rate of return that compensates for the pure rate of time preference and the change over time in the marginal value of consumption (including the change in amenities over time).\textsuperscript{20}

Define $M (\equiv K + q_h \dot{h})$ as the value of the aggregate man-made capital stock in terms of consumption goods. In our model, balanced, sustainable growth equilibrium path is characterized in steady state in which environmental quality and pollution remains constant and all other economic variables such as man-made assets (private capital and knowledge), output, and consumption grow at a common endogenous growth rate $g$:

\begin{align*}
\dot{N} &= \dot{P} = \dot{q}_h = 0. \\
\dot{M} &= \dot{Y} = \dot{K} = \dot{h} = \dot{c} = \tau_p = g.
\end{align*}

\textsuperscript{18} Profit or rent accrued to the firm due to the production externality may be attributed to some fixed (non-reproducible) factor that is left implicit in the model such as raw labor.

\textsuperscript{19} We know that since natural capital and polluting inputs becomes scarce over time (compared to man-made capital), the shadow price (in terms of the consumption good) of natural capital, $q_N$, increases at the growth rate $g$ on a balanced-growth path.

\textsuperscript{20} In a situation of balanced growth, where $\dot{N} = 0$, this equation boils down to: $\dot{c} / c = \sigma_c \left[ (1 - \tau_K) r - \theta \right]$. 
Further, for growth to be feasible and sustainable, the production function must meet the following necessary conditions: i) allocative variables \( (c/Y) \) are constant, ii) production function features constant returns with respect to all growing man-made inputs, and iii) the production elasticities of the various inputs remains constant over time and various substitution elasticities are smaller than or equal to unity.\(^{21}\)

Moreover, in the ecologically-sustainable balanced-growth model, the long-run growth rate \( g \) depends on preferences, technology, and environmental policy, and is thus endogenized.

### III. Internalizing the Environmental Externalities

Aggregate output \( (Y) \), aggregate man-made asset \( (M) \), and aggregate private consumption \( (C) \) are related in the following way on a balanced growth path (See Appendix.):

\[
\begin{align*}
(9a) & \quad Y / M = r \\
(9b) & \quad (\dot{K} + q_s \dot{h}) / M = g \\
(9c) & \quad C / M = r - g .
\end{align*}
\]

The market equilibrium in our economy can be characterized by (4) and (7). Given the arbitrage conditions (4a) and (4c) for private capital and abatement knowledge capital, the current rate of return on man-made assets in terms of aggregate output amounts to the market interest rate in (9a).

Denote \( \alpha \equiv (\partial Y / \partial Z) \cdot Z / Y \) as the aggregate output elasticity with respect to economy-wide effective pollution \( Z \) (and hence also to abatement knowledge \( h \)).\(^{22}\) Then, from the equality of the rate of return to the two man-made capital stocks in the steady-

\(^{21}\) For details, see Rebelo (1991) and Bovenberg and Smulders (1995).
\(^{22}\) Since \( K \) and \( h \) grows at the same rate, the output elasticities with respect to inputs are constant. Due to the constant returns to scale assumption of \( F() \), the aggregate output elasticity of private capital is \( 1 - \alpha \)
state version of (4c), we know that $r = (1 - \alpha) \frac{Y}{K} = \alpha \frac{Y}{(q_h h)} = \frac{Y}{M}$. This equality gives an expression for the gross rate of return to both, $r^M$, in terms of $N$:

\begin{equation}
(10) \quad r = r^M(N, P) = r^M(N, E(N)), \quad \frac{\partial r^M}{\partial N} > 0, \frac{\partial r^M}{\partial P} > 0, \frac{\partial E}{\partial N} > (>)0,
\end{equation}

which reflects that production has constant returns to man-made capital ($K$ and $h; M$) and thus the rate of return to this factor only depends on pollution ($P$) and environmental quality ($N$).

The steady-state version of the modified Keynes-Ramsey rule establishes the relationship between growth and the interest rate in the long-run optimal balanced-growth path. It expresses the (ecologically sustainable) long-run required rate of return on savings (net of tax), $r^S$, that maximizes utility in a way that does not depend on environmental quality:

\begin{equation}
(11) \quad r = r^S \equiv \theta + \frac{g}{\sigma_c}.
\end{equation}

Firms only internalize the effects of their decisions about pollution to the extent imposed by the pollution tax. If, on the initial balanced-growth path, the pollution tax is not set optimally in (6b), but rather set exogenously at too low a level (i.e., pollution tax is much less than for the shadow price of $N$ relative to that of $K$), then pollution is excessive from a social point of view. In this case, the growth in the market economy may also be excessive. To derive the optimal level of environmental quality ($N$), maximization of utility (5a) subject to (1) and (2) and substituting the steady-state version of (1) into this yields the steady-state rate of return on natural capital, $r^N$, in terms of $N$:

\begin{equation}
(12) \quad r = r^N(N) \equiv \frac{1}{q_N} \left[ \frac{\partial U}{\partial N} + F \frac{\partial A}{\partial N} \right] + \frac{\partial E}{\partial N} + q_N.
\end{equation}

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23 This means that $\tau_p < q_N$ (where $q_N$ is the shadow price of $N$ relative to that of $K$). Note that the (static) optimality of the level of pollution requires that the marginal benefit of pollution ($\tau_p$) equals its marginal cost ($q_N$), which is the deterioration of the quality of the environment $N$ in eq. (1).
where $a_N \equiv \left( \frac{\partial A}{\partial N} N/A \right)$ denotes the elasticity of total factor productivity with respect to natural capital. Social optimality requires that the marginal returns to all kinds of capital are equalized. Hence, environmental quality $N$ should earn the same rate of return $r$ as the man-made asset $M$. The return on environmental quality (i.e., natural capital) consists of i) its contribution to utility (consumption externality), ii) its contribution to total factor productivity (production externality), iii) its contribution to ecological processes (marginal absorption capacity), and iv) a scarcity rent (capital gain).

From (11) and (12), we know that optimal growth depends on the intertemporal parameter ($\sigma c$), the pure rate of time preference ($\theta$), the environmental preference parameter ($\phi$), the environmental production externality parameter ($a_N$), the production elasticity of abatement knowledge ($\alpha$), and the pollution-conversion parameter ($\varepsilon$), and the ecological parameters behind the $E(\ )$.

Pollution and environmental problems give rise to externalities that are generally ignored by economic agents in the decentralized market economy. In this case, we are interested in the question whether growth is excessive in the market economy and how growth is affected by a tighter environmental policy to internalize the externalities. In a world of endogenous growth, environmental policy has permanent effects on the productivity of the economy. In the absence of a tax on pollution or a subsidy to private abatement activities, pollution is too high relative to the socially-desirable level and natural capital would be underaccumulated. If so, the private market economy growth may be inefficiently excessive.
Fig. 1 can be used to point out the implications of the change in the long-run rate of return and the corresponding growth rate. The starting situation is a market economy where $\tau_P < q_N$. This initial equilibrium has suboptimally low environmental quality $N^{MKT}$, and is represented by point M. Optimal environmental quality is found by equating the rate of return on environmental capital $r^N$ to the rate of return on man-made assets $r^M$. However, point O represents the social optimum. Since $r$ may fall or increase, depending on the shape of the $r^M$ curve (generally inverted-U shape due to the property of $E(N)$ curve) and the initial value of $N$, we distinguish between two cases. For instance, on the ascending phase of the $r^M$ curve, more growth and pollution is desirable from a
social point of view. In this case, we may see a second effect by which endogenous technological change in abatement knowledge makes tight environmental policy less desirable.

It is also interesting to explore the relative magnitudes of pollution tax revenues \( \tau_p P \) and expenditure on abatement knowledge \( q_h \dot{h} \) associated with environmental policy. This yields the following ratios concerning the pollution tax revenue and abatement knowledge accumulation on the equilibrium path (See Appendix.):

\[
\begin{align*}
(13a) \quad \frac{\tau_p P}{q_h \dot{h}} &= \varepsilon r \\
(13b) \quad \frac{q_h \dot{h}}{\tau_p P} &= (1/\varepsilon)(g/r) \\
(13c) \quad \hat{h} (\equiv \frac{q_h \dot{h}}{rK}) &= \left( \frac{\alpha}{1-\alpha} \right) \left( \frac{g}{r} \varepsilon \right).
\end{align*}
\]

This reveals that the golden rule for the stock of knowledge, on the balanced-growth path, heavily depends on the physical pollution-conversion parameter \( \varepsilon \) as well as the growth rate \( g \) and the market interest rate \( r \). From the first-order conditions for firm’s maximization problem, (4a) and (4b), we also know that the ratio of pollution tax revenue to private capital income:

\[
(13d) \quad \hat{\tau}_p (\equiv \frac{\tau_p P}{rK}) = \varepsilon \frac{\alpha}{(1-\alpha)}.
\]

Even though the Inada condition implies that \( r > g \) in (13b), the relative size of pollution tax revenues and expenditure on abatement knowledge is ambiguous as long as the pollution-conversion parameter is less than unity. Hence, we know that if \( g > \varepsilon r \), then public R&D spending on abatement knowledge should exceed the pollution tax revenues, thereby requiring additional, possibly distortionary, taxes to finance this public expenditure:

\[
(13e) \quad \tau_K = \left( \frac{\alpha}{(1-\alpha)} \right) (g/r - \varepsilon).
\]

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24 In this case, environmental quality (N) raises the marginal productivity of private capital (MPK).
IV. Effects of Capital Income Taxation

As has been stated by Atkinson and Stiglitz (1980), it is interesting to consider how taxation affects the long-run rate of growth of the economy. Since Romer (1986) and Lucas (1988), many attempts have been made to explain the long-run rate of growth endogenously. In the context of the so-called ‘new’ growth theory, the model employed in this paper allows reproducible factors to feature constant returns to scale so that the long-run growth rate is determined ‘endogenously’ by preference, technology, and policy variables. However, no attempt has been made to consider our features of interaction among three types of assets with distortionary income taxation and environmental externalities together.

This section now considers the effects of a change in the capital income tax rate on the long-run rate of growth.

Where the government uses tax revenue to finance public investment as a productive input, we can explore the government fiscal policies in the long-run balanced-growth path using the arbitrage condition, (4c), the government budget constraint, (6), and the modified Keynes-Ramsey rule that maximizes social welfare, (7). First, we trace out the relationship between the growth rate, g, and the ratio of public investment in abatement knowledge to private capital income, \( \hat{h} \), for the production elasticity of abatement knowledge, \( \alpha \), and the pollution tax rate, \( \tau_p \).

In the model, the ecologically-sustained balanced-growth equilibrium path requires the constancy of pollution and environmental quality, where \( P = E(N) \) over time, and the public abatement knowledge stock grows over time at the common growth rate of g (i.e., \( \hat{h} = g \cdot h \)). Therefore, the steady-state version of the optimal savings-investment rule (that establishes the relationship between growth and the long-run required rate of

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return on savings that maximizes utility, and the marginal product of private capital) is given by:

\[ g(\tau_K, r) = \sigma_c [(1 - \tau_K) r - \theta], \quad \text{where} \quad r = \left[(1 - \alpha)A(N)E(N)^{\alpha} \hat{h}^{\alpha} \right]^{1/\alpha} g^{1/\alpha - 1} \]

and \( \tau_K = \hat{h} - \hat{r}_p \). Eliminating the gross rate of return on private capital (\( r \)), this yields the following expression for the rate of long-run steady-state growth (\( g \)) as a function of the ratio of public investment in abatement knowledge to private capital income (\( \hat{h} \)):

\[ \Psi(g) \equiv \left( \frac{g}{\sigma_c} + \theta \right) g^{\alpha/\sigma_c - 1} = \left[(1 - \tau_K)A(N)E(N)^{\alpha} \hat{h}^{\alpha} \right]^{1/\alpha} \]

where \( \psi(g) \) is an increasing monotonic function of \( g \). Evidently, the common growth rate (\( g \)) of private consumption, capital, and output depends positively on the ratio of public investment to private capital income (\( \hat{h} \)) and negatively on the capital income tax rate (\( \tau_K \)).

Substituting the steady-state government budget constraint (6) into (15) allows us to obtain the (implicit) closed-form solution for the rate of growth (\( g \)) with respect to the ratio of public investment in abatement knowledge to private capital income (\( \hat{h} \)):

\[ \Psi(g) = (1 + \hat{r}_p - \hat{h}) \left[(1 - \alpha)A(N)E(N)^{\alpha} \hat{h}^{\alpha} \right]^{1/\alpha}; \quad \Psi'(g) > 0, \quad \hat{r}_p = \varepsilon \alpha / (1 - \alpha). \]

The second component on the right-hand side reflects the increased ‘productivity effect’ whereas the first component reflects the deteriorated ‘tax-base effect’. For a given tax rate, the increase in \( \hat{h} \) would be conductive to growth. However, this increase also requires a rise in the tax rate which, in turn, reduces the net-of-tax return to private capital. At the low levels of \( \hat{h} \) in (16), the ‘productivity effect’ dominates ‘tax-base effect’, and the net-of-tax return to capital rises. This would boost growth. On the contrary, at a sufficiently high level of \( \hat{h} \), the ‘tax-base effect’ overwhelms the
‘productivity effect,’ and the net-of-tax return to capital is depressed. This would decrease private investment and would deter growth.

On the (ecologically-sustainable) balanced-growth path, Fig. 2 traces out the theoretical relationship between growth and public investment in abatement knowledge capital. The growth rate initially rises with the ratio of public investment to private capital income, reaches a maximum $g_{\text{max}}$ and then falls towards zero. Up to $\tau_{K_{\text{max}}}$, taxation would encourage the long-run growth.

Fig. 2. Long-run Growth and Environmental R & D Investment
Knowing that $\Psi'(g) > 0$, and using the first-order condition that maximizes (16) with respect to the ratio of public investment in abatement knowledge to private capital income $\hat{h} (> 0)$, subject to the government budget constraint (6) (i.e., $\partial \Psi / \partial \hat{h} = [\alpha(1 + \hat{r}_p) - \hat{h}][(1 - \alpha)\hat{h}]^{-1}\left[(1 - \alpha)A(N)E(N)^{\alpha}N\hat{h}^{1-\alpha}\right]^{-1} = 0$), we can derive the following results regarding the long-run growth-maximizing fiscal policy rule:

\begin{align*}
\text{(17a)} & \quad \hat{h}^{\max} = \alpha (1 + \hat{r}_p), \quad \text{where} \quad \hat{r}_p = \varepsilon \alpha / (1 - \alpha) \\
\text{(17b)} & \quad \tau_k^{\max} = \alpha (1 - \varepsilon) \\
\text{(17c)} & \quad (\tau_p \text{ } P / \tau_k \text{ } K)^{\max} = (1/(1-\alpha))(\varepsilon / (1-\varepsilon)),
\end{align*}

which are similar to the general results suggested by the literature on growth and taxation.\textsuperscript{27} But, our results in (17) depart from the previous results due to the existence of environmental externalities. Unlike for other public investments (e.g., public infrastructure), the rate of environmental investment that maximizes the long-run growth rate should be greater than its own ‘pure’ output elasticity by the amount of the premium for environmental sustainability. Thus, the overall abatement knowledge-intensity of production, $h / K$, should be directed by this rule.

Moreover, other than in the special case ($\varepsilon = 1$) of the Bovenberg and Smulders (1995, 1996), the optimal mix of pollution tax revenues and capital income tax revenues in (17c) may be required as long as the pollution-conversion parameter is less than unity.\textsuperscript{28}

Therefore, it is demonstrated that the long-run sustained balanced-growth path and the effects of some tightening environmental and fiscal policy depends heavily upon the magnitude of some key parameters of the economy such as production elasticity of abatement knowledge ($\alpha$) and pollution-conversion parameter ($\varepsilon$).

\textsuperscript{27} See Barro (1990), Barro and Sala-i-Martin (1992), Glomm and Ravikumar (1994), Futagami and Mino (1995), and Aschauer (1997).

\textsuperscript{28} In the case of Bovenberg and Smulders (1995, 1996), revenues from pollution taxes are more than sufficient to finance environmental R & D subsidies. So, no additional taxes are needed. In their models, the pollution tax revenues measures exactly the return to abatement knowledge.
V. Conclusions

Using a one-sector endogenous growth model, this paper derives the optimal level of government investment and taxes for ‘green growth’. It combines the so-called ‘new’ growth theory and environmental issues towards sustainable development. It also incorporates abatement knowledge as a productive asset and also includes the natural environment as a renewable resource. The natural environment acts both as a public consumption good and as a public investment good. Accordingly, the economy has three types of assets: private capital, abatement knowledge, and renewable resources. These three assets are accumulated (or decumulated) by the endogenous flows of private savings, public investment, and pollution, respectively. In the model, we have two opposing forces to affect the long-run rate of economic growth and welfare. A lower level of pollution and harvested resources makes other reproducible factors less productive but, at the same time, improves the environmental quality that enhances productivity and welfare. Further, growth causes pollution, but growth generates resources for abatement knowledge that may give rise to optimum.

This paper contributes to the literature on growth, taxation, and the environment in three major ways. First, as long as the pollution-conversion parameter is less than unity, public R&D spending on abatement knowledge should exceed the pollution tax revenues on the optimum path, thereby requiring additional, possibly distortionary, taxes to finance this public expenditure.

Second, we show that the introduction of abatement knowledge and pollution-conversion factor has nontrivial implications for optimal environmental and fiscal policy. The long-run sustained balanced-growth path and the effects of some tightening environmental and fiscal policy depends heavily upon the magnitude of some key parameters of the economy such as production elasticity of abatement knowledge and pollution-conversion parameter.

Third, unlike the general results suggested by the literature on growth and taxation, the rate of environmental investment that maximizes the long-run growth rate should be
greater than its own ‘pure’ output elasticity by the amount of the premium for environmental sustainability. Thus, the overall abatement knowledge-intensity of production, $h/K$, should be directed by this rule.

References


Appendix: Optimality conditions with environmental quality

Using the maximization of welfare (5) subject to the ecological constraint (1) and resource constraint (goods market constraint) (2), we can derive the optimality conditions for the two control and three state variables $c$, $P$, $K$, $h$, and $N$ in this model. Denoting by $\lambda_K$, $\lambda_h$ and $\lambda_N$ the (current value) costate variables associated to the accumulation of private capital ($K$), abatement knowledge capital ($h$) and natural capital ($N$), respectively, we express the corresponding Hamiltonian of this optimization as:

\[
H \equiv U(c, N) + \lambda_c [A(N)F(K, Z(h, P)) - c - q_c J] + \lambda_p J + \lambda_e [E(N) - P],
\]

where $J \equiv \dot{h}$. In the main text, we use the costate variable $\lambda_K$ as numeraire. Thus, $q_h \equiv \lambda_h / \lambda_K$ denotes the shadow price of abatement knowledge relative to private capital, and $q_N \equiv \lambda_N / \lambda_K$ denotes the shadow price of environmental quality relative to private capital. The first-order conditions are:

\[
\text{(A.2) for } c: \quad \frac{\partial U}{\partial c} = \lambda \quad \text{or} \quad U = \lambda K
\]

\[
\text{(A.3) for } P: \quad A(N) \frac{\partial F}{\partial Z} h e^\rho e^{-1} = \lambda_N \Rightarrow A(N) \frac{\partial F}{\partial Z} h e^\rho e^{-1} = q_N
\]

\[
\text{(A.4) for } K: \quad A(N) \frac{\partial F}{\partial K} = \theta \lambda - \lambda K \Rightarrow A(N) \frac{\partial F}{\partial K} = \theta - \lambda K (= r)
\]

\[
\text{(A.5) for } h: \quad q_h A(N) \frac{\partial F}{\partial Z} h e^\rho e^{-1} = \theta \lambda_h - \lambda_h \Rightarrow q_h A(N) \frac{\partial F}{\partial Z} h e^\rho e^{-1} = \theta \lambda_h - \lambda_h
\]

\[
\text{(A.6) for } N: \quad \frac{\partial U}{\partial N} + \lambda_N F \frac{\partial A}{\partial N} + \lambda_N \frac{\partial E}{\partial N} = \theta \lambda_N - \lambda_N
\]

Differentiating (A.2) with respect to time and substituting into (A.4) yields the optimal savings rule (or modified Keynes-Ramsey rule) as in (7). In particular, the conditions for the optimal investment rule (or arbitrage conditions) for (4c), (10) and (12) follow from the canonical conditions (A.4), (A.5) and (A.6). This rule requires that the marginal returns to all kinds of capital are equalized. Hence, private capital ($K$),
knowledge capital \((h)\) and environmental quality \((N)\) should earn the same rate of return \(r\), which corresponds to \(\theta - \dot{\lambda}_s / \dot{\lambda}_e\). Using \(\lambda_N = q_N \lambda_K\), \(r = \theta - \dot{\lambda}_s / \dot{\lambda}_e\), and \(\lambda_K = \partial U / \partial c\), from (A.6), we can derive the social marginal product of environmental quality (or rate of return to natural capital, \(r^N(N)\)) as a function of \(N\) in (12). In this equation, the Hotelling rule for our model should be modified to be applicable to renewable resources that account for the ecological regeneration possibilities \((E(N))\) and include the external effects of environmental quality \((N)\) such as productivity and amenity effects. On the sustainable balanced-growth path, this equation can be further characterized by the environmental externality parameters in production and consumption, some technology parameters associated abatement and pollution, and the ecological parameters behind the ecological growth function, as shown by (12).

\[\text{\textsuperscript{29}}\] The Hotelling rule states that if natural resource is exhaustible, the rate of price increase should equal the rate of return in the absence of extraction costs. In this respect, our result can be interpreted as the generalized Hotelling rule.