The Role of the Third Currency as an International Currency *

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Abstract

The role of an international currency is one of the topics that have widely been discussed in literature. One noteworthy feature is that the role of an international currency differs across the regions. In particular, the U.S. dollar is almost the sole international currency in Asia, while the use of the U.S. dollar is limited in Europe Why does the degree of circulation of the third currency vary from place to place? This paper focuses on the welfare implications of international circulation of the third currency. In the paper, we develop a two-country search model by introducing a third currency into the model.

A two country search model is a useful framework to analyze the role of international currencies. However, few previous studies analyzed the role of the third currency in the search model. In this paper, we show that when local currencies are used as an international currency, the third currency does not improve welfare levels in a strong sense. In contrast, when local currencies are not used as an international currency, introducing the third currency improves welfare, when the total amount of money stock is not large.

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1 Introduction

The role of an international currency is one of the topics that have widely been discussed in literature. One noteworthy feature is that the role of an international currency differs across the regions. In particular, the U.S. dollar is almost the sole international currency in Asia, while the use of the U.S. dollar is limited in Europe For example, Table 2 of Goldberg and Tille (2009) suggests that the shares of invoicing in the U.S. dollar are much higher in Asian countries than in the E.U. Why does the degree of circulation of the third currency vary from place to place? This paper sheds light on this question, focusing on the welfare implications of international circulation of the third currency.

Concerning the role of the third currency, Krugman (1980) pointed out that the third currency serves as a vehicle currency, since it saves transaction costs in currency exchange market, when transaction cost is decreasing in the volume of currency exchange. Rey (2001) incorporated this property of a thick market externality into a three-country, three-currency general equilibrium model, and explained the process of dollarization from the interaction between international trade flows and transaction costs in currency exchange. Instead of the thick market externality, Devereux and Shi (2013) incorporated fixed costs for setting up trading posts of currency exchange to endogenize transaction costs. However, none of them analyzed the role of the third currency in a search model which gives a deeper microfoundation to the choice of international currencies.

In the paper, we develop a two-country search model by introducing a third currency into a search model. A two country search model is a useful framework to analyze the role of international currencies. As a seminal paper, Matsuyama et al. (1993) analyzed the emergence of an international currency. Wright and Trejos (2001) extended the paper by incorporating Nash bargaining between sellers and buyers to endogenize prices and exchange rates. Liu and Shi (2010) and Zhang (2014) also studied internationalization of home currencies. These papers, however, did not consider the third currency. Our paper departs from these studies in the sense that we develop a two-country, three currency model by introducing the third currency into a two country, two currency search model.

Kannan (2009) examined the welfare gain from circulating an international currency by developing a three country, three currency search model. The paper focused on the case where an existing local currency becomes an international currency, but did not consider the case where the third currency circulates as a vehicle currency. In contrast, this paper incorporates the third currency into a modified version of Wright and Trejos (2001) to focus on the effects of introducing the third currency into an economy with or without an international currency. In particular, we explain under what environments the introduction of the third currency improves welfare.

Comparing the welfare levels before and after the introduction of the third currency, we show that when local currencies are used as an international currency, the third currency cannot improve welfare levels. However, we also show that when local currencies are not used as an international currency, introducing the third currency may improve welfare levels.

Section 2 constructs a two country, two currency model, and then examine the two equilibria: the equilibrium where there exists no international currency and the equilibrium where two home currencies circulate in both countries. Section 3 introduces the third currency into the two equilibria, and examine how the two equilibria will change when the economy has the third currency. Section 4 examines the welfare changes by introducing the third currency, based on the models in each equilibrium. Section 5 concludes.

2 Two-Country, Two-Currency Model

The economy is populated by a continuum of infinitely lived agents with unit mass. In the economy, there are two countries: Country A and Country B. Let $n = 1/2$ denote the size of the population of Country A. Then, the size of the population of Country B is $1 - n = 1/2$. Let γ be the relative frequency with which each agent meets a foreign agent, i.e. the degree of economic integration, and satisfy $\gamma < 1$, which implies that each agent meets no agent with some probability.

We assume that goods are divisible while money is indivisible for tractability. Then, agents cannot hold goods and money at the same time. The agents are divided into two types, depending on each inventory. One is those who hold money, and the other is those who do not hold money but can produce goods.

Currency 1 is issued in Country A, and Currency 2 is issued in Country B. Let M_1 and M_2 denote the money stocks of Currency 1 per capita in Country A and Currency 2 per capita in Country B, respectively. We assume $M_1 = M_2 = M$. Let m_1^A, m_2^A, m_1^B, and m_2^B denote the money holdings of Currency 1 per capita and Currency 2 per capita in Country A and in Country B, respectively.

Each agent randomly meets another agent every period, and decides whether to trade or not, so that he maximizes his expected lifetime utility from consuming goods. Let β denote the discount factor.

We assume no barter trade. Then, each trade entails money to purchase goods. This implies that an agent who does not hold money always accepts money. When an agent without money meets another agent with money, he negotiates the quantity of goods to sell, and then he produces the goods and exchanges them for one unit of money.

Let q_i^k denote the quantity of goods to produce when an agent without money from Country k, where $k = A, B$, trades with another agent with Currency i, where $i = 1, 2$. This implies that the price of one unit of goods is $1/q_i^k$. The cost of producing q_i^k is denoted as $c(q_i^k)$, while the utility from consuming q_i^k is denoted as $u(q_i^k)$. We assume $u(q_i^k) = u_0 + \alpha q_i^k$ and $c(q_i^k) = cq_i^k$ for simplicity. When an agent consumes his production goods, he gains the utility of $\bar{u}$.

Let V_0^A, V_1^A, and V_2^A denote the value functions when an agent from Country A holds goods, Currency 1, and Currency 2, respectively. The value functions of an agent from Country B are similarly denoted.

In the following sections, we formulate Nash bargaining problem between a seller and a buyer, and analyze the equilibrium conditions of symmetric equilibrium by utilizing a two-country, two-currency search model. We assume the bargaining power of buyers θ is 100%, i.e. $\theta = 1$ for simplicity. The bargaining takes place only once, and when the negotiation breaks down, the seller keeps his inventory until the next period without physical cost. Since the economic structures of Country A and Country B are symmetric, we only describe the case of Country A.

2.1 Symmetric Equilibrium with No International Currency

This section analyzes the equilibrium conditions of the symmetric equilibrium with no international currency in which agents in Country A do not accept Currency 2 and agents in Country B do not accept Currency 1. In this equilibrium, currency holdings are given as follows.

$$m_1^A = M, m_1^B = 0, m_2^A = 0, m_2^B = M. \tag{1}$$

The Nash bargaining problem between sellers and buyers are given as follows.

$$max_{q_i^k}(u_0 + \alpha q_i^k + \beta V_0^k - \beta V_i^k)^\theta \{-\alpha q_i^k + \beta V_i^k - (\beta V_0^k + \bar{u})\}^{1-\theta},\ i = 1, 2, \text{ and } k = A, B. \tag{2}$$

The first order condition is given as follows.

$$\theta\{-\alpha q_i^k + \beta V_i^k - (\beta V_0^k + \bar{u})\} = (1-\theta)(u_0 + \alpha q_i^k + \beta V_0^k - \beta V_i^k). \tag{3}$$

Substituting $\theta = 1$, the following equation is obtained.

$$-\alpha q_i^k + \beta V_i^k = \beta V_0^k + \bar{u}. \tag{4}$$

The value function of agents from Country A with no money is given as follows.

$$V_0^A = \frac{1}{2}m_1^A(-\alpha q_1^A + \beta V_1^A) + \frac{\gamma}{2}m_1^B(-\alpha q_1^A + \beta V_1^A) + (1 - \frac{1}{2}m_1^A - \frac{\gamma}{2}m_1^B)(\beta V_0^A + \bar{u}). \tag{5}$$

Substituting equation (4) into equation (5), the following equation holds.

$$V_0^A = \frac{\bar{u}}{1-\beta}. \tag{6}$$

Similarly, the value functions of agents from Country A with money are given as follows.

$$\begin{aligned} V_1^A &= \frac{1}{2}(1-m_1^A)(u_0 + \alpha q_1^A + \beta V_0^A) + \{1 - \frac{1}{2}(1-m_1^A)\}\beta V_1^A, \\ V_2^A &= \frac{\gamma}{2}(1-m_2^B)(u_0 + \alpha q_2^B + \beta V_0^A) + \{1 - \frac{\gamma}{2}(1-m_2^B)\}\beta V_2^A. \end{aligned} \tag{7}$$

Equations (1), (4), (6) and (7) give the value functions and the quantities of production goods.

$$\begin{aligned} V_1^A &= \frac{\frac{1}{2}(1-M)(u_0-\bar{u})}{1-\beta}, \\ V_2^A &= \frac{\frac{\gamma}{2}(1-M)\{1-\beta+\beta\frac{1}{2}(1-M)\}(u_0-\bar{u})}{\{1-\beta+\beta\frac{\gamma}{2}(1-M)\}(1-\beta)} \\ q_1^A &= \frac{\beta\frac{1}{2}(1-M)(u_0-\bar{u})-\bar{u}}{\alpha(1-\beta)}, \\ q_2^A &= \frac{\beta\frac{\gamma}{2}(1-M)\{1-\beta+\beta\frac{1}{2}(1-M)\}(u_0-\bar{u})}{\alpha\{1-\beta+\beta\frac{\gamma}{2}(1-M)\}(1-\beta)} - \frac{\bar{u}}{\alpha(1-\beta)}. \end{aligned} \tag{8}$$

Equation (8) implies that an increase in M raises prices $1/q_1^A$ and $1/q_2^B$, and that a higher degree of economic integration γ does not affect prices and hence the exchange rate $(1/q_2^A)/(1/q_1^A) = 1$.

The equilibrium conditions are given as $u_0 + \alpha q_i^k > \alpha q_i^k$, $V_1^A > V_0^A$, and $V_0^A \geq V_2^A$. Although the first condition always holds, the second and the third ones are not necessarily satisfied. From equations (6) and (8), the second and the third

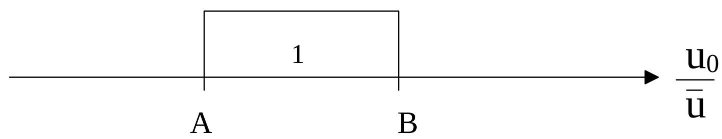

Figure 1: Region of the symmetric equilibrium with two local currencies

conditions are rewritten as follows.

$$\frac{u_0}{\bar{u}} > \frac{1}{\frac{1}{2}(1-M)} + 1 \equiv A,$$
$$\frac{u_0}{\bar{u}} \leq \frac{1-\beta+\beta\frac{\gamma}{2}(1-M)}{\frac{\gamma}{2}(1-M)\{1-\beta+\beta\frac{1}{2}(1-M)\}} + 1 \equiv B. \tag{9}$$

Hence, the equilibrium conditions are given as follows.

$$\frac{1}{\frac{1}{2}(1-M)} + 1 < \frac{u_0}{\bar{u}} \leq \frac{1-\beta+\beta\frac{\gamma}{2}(1-M)}{\frac{\gamma}{2}(1-M)\{1-\beta+\beta\frac{1}{2}(1-M)\}} + 1. \tag{10}$$

This equation implies that if $\theta = 1$, a symmetric equilibrium never exists. Hence, a symmetric equilibrium exists if u_0 is relatively large to $\bar{u}$ and if $\gamma < 1$. If the first equation does not hold, money never circulates due to $V_1^A \leq V_0^A$. Thus, we assume in the following sections that the first equation of equation (10) always holds, i.e. u_0 is relatively large to $\bar{u}$. Under this assumption, the symmetric equilibrium with no international currency always exists. The equilibrium region is summarized as in Figure 1. Note that if $V_0^A \geq V_A^1$ and $V_0^A \geq V_A^2$ hold, the economy becomes autarky.

2.2 Symmetric Equilibrium with Two International Currencies

This section analyzes the equilibrium conditions of the symmetric equilibrium with two international currencies in which all agents accept both Currency 1 and Currency 2. In this equilibrium, currency holdings are given as follows.

$$m_1^A + m_1^B = M, m_2^A + m_2^B = M. \tag{11}$$

Since the two countries are identical, the following conditions are satisfied.

$$m_1^A = m_1^B, m_2^A = m_2^B. \tag{12}$$

From equation (12), equation (11) are rewritten as follows.

$$m_1^A = m_1^B = \frac{1}{2}M, m_2^A = m_2^B = \frac{1}{2}M. \tag{13}$$

The Nash bargaining problem between sellers and buyers are given as follows.

$$max_{q_i^k}(u_0 + \alpha q_i^k + \beta V_0^k - \beta V_i^k)^\theta \{-\alpha q_i^k + \beta V_i^k - (\beta V_0^k + \bar{u})\}^{1-\theta},\ i = 1, 2, \text{ and } k = A, B. \tag{14}$$

The first order condition is given as follows.

$$\theta\{-\alpha q_i^k + \beta V_i^k - (\beta V_0^k + \bar{u})\} = (1 - \theta)(u_0 + \alpha q_i^k + \beta V_0^k - \beta V_i^k). \tag{15}$$

Substituting $\theta = 1$, the following equation is obtained.

$$-\alpha q_i^k + \beta V_i^k = \beta V_0^k + \bar{u}. \tag{16}$$

The value function of agents from Country A with no money is given as follows.

$$V_0^A = \frac{1}{2}m_1^A(-\alpha q_1^A + \beta V_1^A) + \frac{1}{2}m_2^A(-\alpha q_2^A + \beta V_2^A) + \frac{\gamma}{2}m_1^B(-\alpha q_1^A + \beta V_1^A)$$
$$+\frac{\gamma}{2}m_2^B(-\alpha q_2^A + \beta V_2^A) + (1 - \frac{1}{2}m_1^A - \frac{1}{2}m_2^A - \frac{\gamma}{2}m_1^B - \frac{\gamma}{2}m_2^B)(\beta V_0^A + \bar{u}). \quad (17)$$

Substituting equation (16) into equation (17), the following equation holds.

$$V_0^A = \frac{\bar{u}}{1-\beta}. \quad (18)$$

Similarly, the value functions of agents from Country A with money are given as follows.

$$V_1^A = \frac{1}{2}(1 - m_1^A - m_2^A)(u_0 + \alpha q_1^A + \beta V_0^A) + \frac{\gamma}{2}(1 - m_1^B - m_2^B)(u_0 + \alpha q_1^B + \beta V_0^A)$$
$$+\{1 - \frac{1}{2}(1 - m_1^A - m_2^A) - \frac{\gamma}{2}(1 - m_1^B - m_2^B)\}\beta V_1^A,$$
$$V_2^A = \frac{1}{2}(1 - m_1^A - m_2^A)(u_0 + \alpha q_2^A + \beta V_0^A) + \frac{\gamma}{2}(1 - m_1^B - m_2^B)(u_0 + \alpha q_2^B + \beta V_0^A)$$
$$+\{1 - \frac{1}{2}(1 - m_1^A - m_2^A) - \frac{\gamma}{2}(1 - m_1^B - m_2^B)\}\beta V_2^A. \quad (19)$$

In the equilibrium, $V_1^A = V_2^A$ and $V_1^B = V_2^B$ hold. Since the two countries are identical, $V_1^A = V_1^B$ and $V_2^A = V_2^B$ hold. Equations (13), (16), (18) and (19) give the value functions and the quantities of production goods.

$$V_1^A = \frac{\frac{1+\gamma}{2}(1-M)(u_0 - \bar{u})}{1-\beta},$$
$$q_1^A = \frac{\beta\frac{1+\gamma}{2}(1-M)(u_0 - \bar{u}) - \bar{u}}{\alpha(1-\beta)}. \quad (20)$$

Equation (20) implies that an increase in M raises prices $1/q_i^k$ ($i = 1, 2$, $k = A, B$), and that a higher degree of economic integration γ decreases $1/q_i^k$, but does not

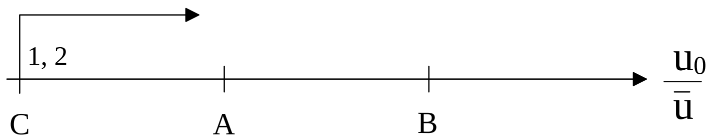

Figure 2: Region of the symmetric equilibrium with two international currencies

affect the exchange rate $(1/q_2^A)/(1/q_1^A) = 1$.

The equilibrium conditions are given as $u_0 + \alpha q_i^k > \alpha q_i^k$ and $V_1^A = V_2^A > V_0^A$. Although the first condition always holds, the second one is not necessarily satisfied. From equations (18) and (20), the second condition is rewritten as follows.

$$\frac{u_0}{\bar{u}} > \frac{1}{\frac{1+\gamma}{2}(1-M)} + 1 \equiv C. \tag{21}$$

Since the right hand side of equation (21) is less than the left hand side of equation (10), the equilibrium always exists as long as the first equation of equation (10) holds. The equilibrium region is summarized as in Figure 2. Note that if $V_0^A \geq V_A^1 = V_A^2$ holds, i.e. $\bar{u}$ is relatively large to u_0, the economy becomes autarky.

3 Two-Country, Three-Currency Model

This section develops a two-country, three-currency model by introducing the third currency into the two-country, two-currency model. Since the basic structure is the same as in section 2, the rest of this section describes only the settings specific to a two-country, three-currency model.

In this model, in addition to Currency 1 and Currency 2, there exists the other currency issued by a third country named Currency 3 or the third currency. Let M_1, M_2, and M_3 denote the money stocks of Currency 1 per capita in Country

A, Currency 2 per capita in Country B, and Currency 3 per capita, respectively. We assume $M_1 = M_2 = M_0$. Let m_1^A, m_2^A, m_3^A, m_1^B, m_2^B, m_3^B, and denote the money holdings of Currency 1 per capita, Currency 2 per capita, and Currency 3 per capita in Country A and in Country B, respectively.

Let V_1^A, V_2^A, and V_3^A denote the value functions when an agent from Country A holds Currency 1, Currency 2, and Currency 3, respectively. The value functions of an agent from Country B are similarly denoted. Let q_i^k denote the quantity of goods to produce when an agent without money from Country k, where $k = A, B$, trades with another agent with Currency i, where $i = 1, 2, 3$. This implies that the price of one unit of goods is $1/q_i^k$.

3.1 Symmetric Equilibrium with the Third Currency as the Only International Currency

This section analyzes the equilibrium conditions of the symmetric equilibrium with one international currency in which agents in Country A do not accept Currency 2, agents in Country B do not accept Currency 1, and all agents accept the third currency. Since the two countries are identical, $m_3^A = m_3^B$ is satisfied. In this equilibrium, currency holdings are given as follows.

$$m_1^A = M_0, m_1^B = 0, m_2^A = 0, m_2^B = M_0, m_3^A = \frac{M_3}{2}, m_3^B = \frac{M_3}{2}. \tag{22}$$

The Nash bargaining problem between sellers and buyers are given as follows.

$$max_{q_i^k}(u_0 + \alpha q_i^k + \beta V_0^k - \beta V_i^k)^\theta \{-\alpha q_i^k + \beta V_i^k - (\beta V_0^k + \bar{u})\}^{1-\theta},\ i = 1, 2, 3, \text{ and } k = A, B. \tag{23}$$

The first order condition is given as follows.

$$\theta\{-\alpha q_i^k + \beta V_i^k - (\beta V_0^k + \bar{u})\} = (1-\theta)(u_0 + \alpha q_i^k + \beta V_0^k - \beta V_i^k). \tag{24}$$

Substituting $\theta = 1$, the following equation is obtained.

$$-\alpha q_i^k + \beta V_i^k = \beta V_0^k + \bar{u}. \tag{25}$$

The value function of agents from Country A with no money is given as follows.

$$\begin{aligned} V_0^A &= \frac{1}{2}m_1^A(-\alpha q_1^A + \beta V_1^A) + \frac{\gamma}{2}m_1^B(-\alpha q_1^A + \beta V_1^A) + \frac{1}{2}m_3^A(-\alpha q_3^A + \beta V_3^A) \\ &+ \frac{\gamma}{2}m_3^B(-\alpha q_3^A + \beta V_3^A) + (1 - \frac{1}{2}m_1^A - \frac{\gamma}{2}m_1^B - \frac{1}{2}m_3^A - \frac{\gamma}{2}m_3^B)(\beta V_0^A + \bar{u}). \end{aligned} \tag{26}$$

Substituting equation (25) into equation (26), the following equation holds.

$$V_0^A = \frac{\bar{u}}{1-\beta}. \tag{27}$$

Similarly, the value functions of agents from Country A with money are given as follows.

$$\begin{aligned} V_1^A &= \frac{1}{2}(1 - m_1^A - m_3^A)(u_0 + \alpha q_1^A + \beta V_0^A) + \{1 - \frac{1}{2}(1 - m_1^A - m_3^A)\}\beta V_1^A, \\ V_2^A &= \frac{\gamma}{2}(1 - m_2^B - m_3^B)(u_0 + \alpha q_2^B + \beta V_0^A) + \{1 - \frac{\gamma}{2}(1 - m_2^B - m_3^B)\}\beta V_2^A, \\ V_3^A &= \frac{1}{2}(1 - m_1^A - m_3^A)(u_0 + \alpha q_3^A + \beta V_0^A) + \frac{\gamma}{2}(1 - m_2^B - m_3^B)(u_0 + \alpha q_3^B + \beta V_0^A) \\ &+ \{1 - \frac{1}{2}(1 - m_1^A - m_3^A) - \frac{\gamma}{2}(1 - m_2^B - m_3^B)\}\beta V_3^A. \end{aligned} \tag{28}$$

Equations (22), (25), (27) and (28) give the value functions and the quantities of production goods.

$$
\begin{aligned}
V_1^A &= \frac{\frac{1}{2}(1-M_0-\frac{M_3}{2})(u_0-\bar{u})}{1-\beta}, \\
V_2^A &= \frac{\frac{\gamma}{2}(1-M_0-\frac{M_3}{2})\{1-\beta+\beta\frac{1}{2}(1-M_0-\frac{M_3}{2})\}(u_0-\bar{u})}{\{1-\beta+\beta\frac{\gamma}{2}(1-M_0-\frac{M_3}{2})\}(1-\beta)}, \\
V_3^A &= \frac{\frac{1+\gamma}{2}(1-M_0-\frac{M_3}{2})(u_0-\bar{u})}{1-\beta}, \\
q_1^A &= \frac{\beta\frac{1}{2}(1-M_0-\frac{M_3}{2})(u_0-\bar{u})-\bar{u}}{\alpha(1-\beta)}, \\
q_2^A &= \frac{\beta\frac{\gamma}{2}(1-M_0-\frac{M_3}{2})\{1-\beta+\beta\frac{1}{2}(1-M_0-\frac{M_3}{2})\}(u_0-\bar{u})}{\alpha\{1-\beta+\beta\frac{\gamma}{2}(1-M_0-\frac{M_3}{2})\}(1-\beta)} - \frac{\bar{u}}{\alpha(1-\beta)}, \\
q_3^A &= \frac{\beta\frac{1+\gamma}{2}(1-M_0-\frac{M_3}{2})(u_0-\bar{u})-\bar{u}}{\alpha(1-\beta)}.
\end{aligned} \tag{29}
$$

Equation (29) implies that an increase in M_0 or M_3 raises prices $1/q_1^A$, $1/q_2^B$, $1/q_3^A$, and $1/q_3^B$. It also implies that a higher degree of economic integration γ reduces $1/q_3^A$ and $1/q_3^B$, and lowers $(1/q_3^A)/(1/q_1^A)$ and $(1/q_3^B)/(1/q_2^B)$, although γ does not affect prices $1/q_1^A$ and $1/q_2^B$ and hence the exchange rate $(1/q_2^A)/(1/q_1^A) = 1$.

The equilibrium conditions are given as $u_0 + \alpha q_i^k > \alpha q_i^k$, $V_1^A > V_0^A$, $V_0^A \geq V_2^A$, and $V_3^A > V_0^A$. Although the first condition always holds, the other ones are not necessarily satisfied. From equations (27) and (29), they are rewritten as follows.

$$
\begin{aligned}
\frac{u_0}{\bar{u}} &> \frac{1}{\frac{1}{2}(1-M_0-\frac{M_3}{2})} + 1 \equiv A', \\
\frac{u_0}{\bar{u}} &\leq \frac{1-\beta+\beta\frac{\gamma}{2}(1-M_0-\frac{M_3}{2})}{\frac{\gamma}{2}(1-M_0-\frac{M_3}{2})\{1-\beta+\beta\frac{1}{2}(1-M_0-\frac{M_3}{2})\}} + 1 \equiv B', \\
\frac{u_0}{\bar{u}} &> \frac{1}{\frac{1+\gamma}{2}(1-M_0-\frac{M_3}{2})} + 1 \equiv C'.
\end{aligned} \tag{30}
$$

Since the right hand side of the last equation of equation (30) is less than the right hand side of the first equation of equation (30), the equilibrium always exists as long as the first equation of equation (30) holds. Hence, the equilibrium conditions are rewritten as follows.

$$\frac{1}{\frac{1}{2}(1-M_0-\frac{M_3}{2})}+1<\frac{u_0}{\bar{u}}\leq\frac{1-\beta+\beta\frac{\gamma}{2}(1-M_0-\frac{M_3}{2})}{\frac{\gamma}{2}(1-M_0-\frac{M_3}{2})\{1-\beta+\beta\frac{1}{2}(1-M_0-\frac{M_3}{2})\}}+1. \tag{31}$$

When $\gamma=1$, a symmetric equilibrium does not exist, since the left hand side of the first equation equation (31) equals to the right of the second equation of equation (31). Hence, this equation implies that a symmetric equilibrium exists if u_0 is large relative to $\bar{u}$ and if $\gamma<1$. Note that if $V_0^A\geq V_A^1$, $V_0^A\geq V_A^2$, and $V_0^A\geq V_A^3$ hold, i.e. $\bar{u}$ is relatively large to u_0, the economy becomes autarky. The equilibrium region is summarized as in Figure 3.

If the first equation of equation (31) does not hold, each local currency does not circulate in each home country, since $V_1^A\leq V_0^A$. Hence, we assume in the following sections that the left hand side of the first equation of equation (31) always holds. Under this assumption, an equilibrium in which each local currency circulates in each home country always exists. Since the left hand side of the first equation of equation (31) is larger than that of equation (10) and the right hand side of the second equation of equation (31) is smaller than that of equation (10), the condition under which each local currency circulates in each home country in an equilibrium with the third currency is stricter than in an equilibrium without the third currency.

Comparing figure 1 and figure 3, given that the total amount of money is the same, the equilibrium regions of the equilibrium with no international currency

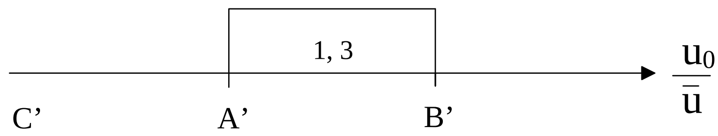

Figure 3: Region of the symmetric equilibrium with the third currency as the only international currency

analyzed in section 2.1 and the equilibrium with the third currency as the only international currency analyzed in section 3.1 coincide with each other.

3.2 Symmetric Equilibrium with Three International Currencies

This section analyzes the equilibrium conditions of the symmetric equilibrium with three international currencies in which agents accept any currency, i.e. $M_1 = M_2 = M_0$、$n = 1 - n = 1/2$. In this equilibrium, currency holdings are given as follows.

$$m_1^A + m_1^B = M_0, m_2^A + m_2^B = M_0, m_3^A + m_3^B = M_3. \tag{32}$$

Since the two countries are identical, the following conditions are satisfied.

$$m_1^A = m_1^B, m_2^A = m_2^B, m_3^A = m_3^B. \tag{33}$$

From equation (33), equation (32) are rewritten as follows.

$$m_1^A = m_1^B = \frac{1}{2}M_0, m_2^A = m_2^B = \frac{1}{2}M_0, m_3^A = m_3^B = \frac{M_3}{2}. \tag{34}$$

The Nash bargaining problem between sellers and buyers are given as follows.

$$max_{q_i^k}(u_0 + \alpha q_i^k + \beta V_0^k - \beta V_i^k)^\theta \{-\alpha q_i^k + \beta V_i^k - (\beta V_0^k + \bar{u})\}^{1-\theta},\ i = 1, 2, 3, \text{ and } k = A, B. \tag{35}$$

The first order condition is given as follows.

$$\theta\{-\alpha q_i^k + \beta V_i^k - (\beta V_0^k + \bar{u})\} = (1-\theta)(u_0 + \alpha q_i^k + \beta V_0^k - \beta V_i^k). \tag{36}$$

Substituting $\theta = 1$, the following equation is obtained.

$$-\alpha q_i^k + \beta V_i^k = \beta V_0^k + \bar{u}. \tag{37}$$

The value function of agents from Country A with no money is given as follows.

$$\begin{aligned}
V_0^A &= \frac{1}{2}m_1^A(-\alpha q_1^A + \beta V_1^A) + \frac{1}{2}m_2^A(-\alpha q_2^A + \beta V_2^A) + \frac{1}{2}m_3^A(-\alpha q_3^A + \beta V_3^A) \\
&+ \frac{\gamma}{2}m_1^B(-\alpha q_1^A + \beta V_1^A) + \frac{\gamma}{2}m_2^B(-\alpha q_2^A + \beta V_2^A) + \frac{\gamma}{2}m_3^B(-\alpha q_3^A + \beta V_3^A) \\
&+ (1 - \frac{1}{2}m_1^A - \frac{1}{2}m_2^A - \frac{1}{2}m_3^A - \frac{\gamma}{2}m_1^B - \frac{\gamma}{2}m_2^B - \frac{\gamma}{2}m_3^B)(\beta V_0^A + \bar{u}).
\end{aligned} \tag{38}$$

Substituting equation (37) into equation (38), the following equation holds.

$$V_0^A = \frac{\bar{u}}{1-\beta}. \tag{39}$$

Similarly, the value functions of agents from Country A with money are given as follows.

$$
\begin{aligned}
V_1^A &= \frac{1}{2}(1-m_1^A-m_2^A-m_3^A)(u_0+\alpha q_1^A+\beta V_0^A)+\frac{\gamma}{2}(1-m_1^B-m_2^B-m_3^B)(u_0+\alpha q_1^B+\beta V_0^A)\\
&+\{1-\frac{1}{2}(1-m_1^A-m_2^A-m_3^A)-\frac{\gamma}{2}(1-m_1^B-m_2^B-m_3^B)\}\beta V_1^A,\\
V_2^A &= \frac{1}{2}(1-m_1^A-m_2^A-m_3^A)(u_0+\alpha q_2^A+\beta V_0^A)+\frac{\gamma}{2}(1-m_1^B-m_2^B-m_3^B)(u_0+\alpha q_2^B+\beta V_0^A)\\
&+\{1-\frac{1}{2}(1-m_1^A-m_2^A-m_3^A)-\frac{\gamma}{2}(1-m_1^B-m_2^B-m_3^B)\}\beta V_2^A,\\
V_3^A &= \frac{1}{2}(1-m_1^A-m_2^A-m_3^A)(u_0+\alpha q_3^A+\beta V_0^A)+\frac{\gamma}{2}(1-m_1^B-m_2^B-m_3^B)(u_0+\alpha q_3^B+\beta V_0^A)\\
&+\{1-\frac{1}{2}(1-m_1^A-m_2^A-m_3^A)-\frac{\gamma}{2}(1-m_1^B-m_2^B-m_3^B)\}\beta V_3^A. \qquad (40)
\end{aligned}
$$

In the equilibrium, $V_1^A=V_2^A=V_3^A$ and $V_1^B=V_2^B=V_3^B$ hold. Since the two countries are identical, $V_1^A=V_1^B$, $V_2^A=V_2^B$, and $V_3^A=V_3^B$ hold. Equations (34), (37), (39) and (40) give the value functions and the quantities of production goods.

$$
\begin{aligned}
V_1^A &= \frac{\frac{1+\gamma}{2}(1-M_0-\frac{M_3}{2})(u_0-\bar{u})}{1-\beta},\\
q_1^A &= \frac{\beta\frac{1+\gamma}{2}(1-M_0-\frac{M_3}{2})(u_0-\bar{u})-\bar{u}}{\alpha(1-\beta)}. \qquad (41)
\end{aligned}
$$

Equation (41) implies that an increase in M_0 or M_3 raises prices $1/q_i^k$ ($i=1,2,3$, $k=A,B$), and that a higher degree of economic integration γ decreases $1/q_i^k$, but does not affect the exchange rate $(1/q_i^A)/(1/q_j^A)$ ($j=1,2,3, i\neq j$).

The equilibrium conditions are given as $u_0+\alpha q_i^k>\alpha q_i^k$ and $V_1^A=V_2^A=V_3^A>V_0^A$. Although the first condition always holds, the second one is not necessarily satisfied. From equations (39) and (41), the second condition is rewritten as follows.

$$
\frac{u_0}{\bar{u}}>\frac{1}{\frac{1+\gamma}{2}(1-M_0-\frac{M_3}{2})}+1. \qquad (42)
$$

Figure 4: Region of the symmetric equilibrium with three international currencies

Since the right hand side of equation (42) is larger than the right hand side of equation (21) and less than the left hand side of equation (31), the equilibrium always exists as long as equation (21) and the first equation of equation (31) hold. The equilibrium region is summarized as in Figure 4. Note that if $V_0^A \geq V_A^1 = V_A^2 = V_A^3$ holds, i.e. $\bar{u}$ is relatively large to u_0, the economy becomes autarky.

Comparing figure 3 and figure 4, local currencies become international currencies, the values of holding those currencies increase and equal to the value of holding the third currency. Moreover, comparing figure 2 and figure 4, given that the total amount of money is the same, the equilibrium regions of the equilibrium with two international currencies analyzed in section 2.2 and the equilibrium with three international currencies analyzed in section 3.2 coincide with each other.

4 Welfare Change Caused by International Circulation of the Third Currency

This section analyzes the welfare changes caused by introducing the third currency into the equilibrium with two local currencies in section 2.1 and the equilibrium with two international currencies in section 2.2, respectively. The analysis reveals whether international circulation of the third currency, $\frac{M_3}{2}$, improves the welfare levels, depending on the money stock of each home currency, M_0, in the case where an international currency does not exist and in the case where an international currency already exists.

In the following analysis, we assume that the level of money stock before the introduction of the third currency, M, maximizes the welfare levels of each country. Once the third currency is introduced into the economy, the ex-post total money stock $M_0 + \frac{M_3}{2}$ exceeds the ex-ante money stock M and its level does not necessarily maximize welfare. In such case, however, withdrawing home currencies may increase welfare. Then, we make the following assumption that the ex-post total money stock $M_0 + \frac{M_3}{2}$ exceeds the ex-ante money stock M, but home currencies can be partly withdrawn.

$$\text{Assumption 1}: \ M \leq M_0 + \frac{M_3}{2} \leq M + \frac{M_3}{2}, \text{ i.e. } M - \frac{M_3}{2} \leq M_0 \leq M. \quad (43)$$

Under the assumption, $A \leq A'$ and $B \leq B'$ hold.

From equations (9), (21), (31), and (42), if $A' \leq \frac{u_0}{\bar{u}} \leq B$, 4 equilibrium conditions examined in the previous sections are always satisfied. Hence, we also make the following assumption to compare welfare levels among four equilibria.

$$\text{Assumption 2}: \ A' \leq \frac{u_0}{\bar{u}} \leq B,$$
$$\text{i.e. } \gamma(1-M)\{1-\beta+\beta\frac{1}{2}(1-M)\} \leq (1-M_0-\frac{M_3}{2})\{1-\beta+\beta\frac{\gamma}{2}(1-M)\}. \quad (44)$$

4.1 The Case with Two International Currencies

The welfare level in the case with two international currencies in a two country, two currency model analyzed in section 2 is given as follows.

$$m_1^A V_1^A + m_2^A V_2^A = \frac{(1+\gamma)M(1-M)(u_0-\bar{u})}{2(1-\beta)}. \tag{45}$$

On the other hand, the welfare level in the case with three international currencies in a two country, three currency model analyzed in section 3 is given as follows.

$$m_1^A V_1^A + m_2^A V_2^A + m_3^A V_3^A = \frac{(1+\gamma)(M_0+\frac{M_3}{2})(1-M_0-\frac{M_3}{2})(u_0-\bar{u})}{2(1-\beta)}. \tag{46}$$

Subtracting the right hand side of equation (45) from that of equation (46) gives $\frac{(1+\gamma)(M_0+\frac{M_3}{2}-M)(1-M-M_0-\frac{M_3}{2})(u_0-\bar{u})}{2(1-\beta)}$. Since $M_0+\frac{M_3}{2}-M \geq 0$ holds under Assumption 1, introduction of the third currency improves welfare if $1-M-M_0-\frac{M_3}{2} \geq 0$, while it declines welfare if $1-M-M_0-\frac{M_3}{2} \leq 0$. For example, when $M=\frac{1}{2}$, the first equation of Assumption 1 becomes $M_0+\frac{M_3}{2} \geq \frac{1}{2}$, and then, welfare improves in a weak sense, only in the case $M_0+\frac{M_3}{2}=\frac{1}{2}$. From equation (45) and equation (46), the optimal money stocks in the two currency model and three currency model are $\frac{1}{2}$. Hence, the welfare levels improve in a weak sense, only when the ex-post total money stock is the same as the ex-ante money stock $M=\frac{1}{2}$.

4.2 The Case with Two Local Currencies

The welfare level in the case with two local currencies in a two country, two currency model analyzed in section 2 is given as follows.

$$m_1^A V_1^A = \frac{M(1-M)(u_0-\bar{u})}{2(1-\beta)}. \tag{47}$$

On the other hand, the welfare level in the case where only the third currency is an international currency in a two country, three currency model analyzed in section 3 is given as follows.

$$m_1^A V_1^A + m_2^A V_2^A + m_3^A V_3^A = \frac{(M_0+\frac{M_3}{2})(1-M_0-\frac{M_3}{2})(u_0-\bar{u})}{2(1-\beta)} + \frac{\gamma\frac{M_3}{2}(1-M_0-\frac{M_3}{2})(u_0-\bar{u})}{2(1-\beta)}. \tag{48}$$

Comparing equation (47) and equation (48), given that the total amount of money stock is the same in both models, i.e. $M = M_0 + \frac{M_3}{2}$, three currency model gives higher welfare levels. Hence, unlike in the case with two international currencies, unless the total amount of money stock is sufficiently large, introduction of the third currency improves welfare in the case with two local currencies.

Figure 5 depicts the welfare levels of the two models in the case where the total amount of money stock M or $M_0 + \frac{M_3}{2}$ ranges from 0.25 to 0.8. We assume that $\gamma = 0.8$, $M_3 = 0.1, 0.2$, and $2(1-\beta) = u_0 - \bar{u}$. The figure shows that international circulation of the third currency improves welfare, unless the total amount of money stock is sufficiently large. For example, if the total amount of money stock is in the range of $[0.25, 0.62]$ in the case of $M_3 = 0.1$, and if it is in the range of $[0.25, 0.66]$ in the case of $M_3 = 0.2$, introduction of the third currency as an international

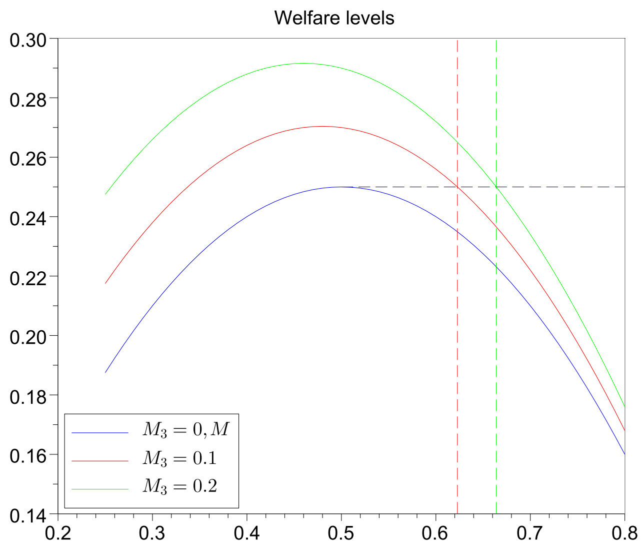

Figure 5: Welfare levels in Two Currency Model and Three Currency Model

currency improves welfare. Since the ex-ante money stock is 0.5, local currencies need not be withdrawn in the case of $M_3 = 0.1$, while they should be redeemed in the case of $M_3 = 0.2$. The figure also shows that as the third currency increases, the total amount of money stock which maximizes welfare levels decrease. This reflects economies of scale arising from the international circulation of the third currency, which does not appear in the model with two international currencies analyzed in section 4.1.

The optimal money stock in a two currency model is 1/2 from equation (46). On the other hand, the optimal total money stock in a two currency model is 1/2 from equation (48), since there exist one stable equilibrium where $M_0 = 0$ and $M_3 = 1/2$ and one unstable equilibrium where $M_0 = 1 + \frac{1}{\gamma}$ and $M_3 = -\frac{1}{\gamma}$ (See Appendix A). Hence, the welfare levels improve during transition periods to the stable equilibrium, but in the long run, they improve in a weak sense, only when the ex-post total money stock is the same as the ex-ante money stock $M = \frac{1}{2}$ as in section 4.1. It follows that home currencies should be replaced by the third currency in the long run.

4.3 Conditions to improve welfare levels

Introduction of the third currency has two effects arising from an increase in the total amount of money stock and an emergence of a new international currency. Since this paper studies the latter effect, we focus on the case where the ex-ante money stock maximizes the welfare level of each country, i.e. $M^* = \frac{1}{2}$ from equation (45) and equation (47), and analyze the conditions to improve the welfare level. In this case, the first equation of Assumption 1 becomes $\frac{1}{2} \leq M_0 + \frac{M_3}{2}$.

As showed in section 4.1, the third currency never improves welfare levels in a strong sense in the case where there already exists an international currency. On the other hand, in the case where there does not exist an international currency, equation (48) implies that the third currency may improve welfare levels in the short run.

Comparing equation (47) and equation (48) under Assumption 1 and Assumption 2, the conditions to improve welfare levels from the third currency are given as follows.

$$\frac{1}{4} < (M_0 + \frac{M_3}{2})(1 - M_0 - \frac{M_3}{2}) + \gamma \frac{M_3}{2}(1 - M_0 - \frac{M_3}{2}). \tag{49}$$

Since Assumption 2 does not hold when $\gamma = 1$, a home currency does not circulate in each home country, when γ is large. On the other hand, equation (49) does not hold when $\gamma = 0$, the third currency does not improve welfare, when γ is small. For example, in the case of $M = M_0 = 0.5$, $\beta = 0.99$, and $M_3 = 0.1$, γ should satisfy $0.1111 < \gamma \leq 0.2590$. This implies that unless home currencies are withdrawn, the third currency can improve welfare only when the degree of economic integration is extremely low.

Note that if the supply of the third currency is small, the third currency can improve welfare in the short run, even when γ is high. For example, welfare increases if $0.0101 < \gamma \leq 0.2849$ in the case of $M_3 = 0.01$, and if $0.0010 < \gamma \leq 0.2875$ in the case of $M_3 = 0.001$. Moreover, if large amount of home currencies are withdrawn, welfare levels may improve. For example, welfare increases if $0 < \gamma \leq 0.2878$ in the case of $M_0 = 0.45$, and if $0.0909 < \gamma \leq 0.3165$ in the case of $M_0 = 0.4$.

It follows that welfare improves in the short run, when the total amount of money stock is controlled. This effect arises from economies of scale in the international circulation of the third currency. In the long run, however, the optimal amount of the total money stock converges to $\frac{1}{2}$. Hence, home currencies should be replaced with the third currency to improve welfare in the long run.

5 Conclusion

This paper studies the effects of international circulation of the third currency on welfare levels by developing a two-country, three-currency search model. We show that when local currencies are used as an international currency, the third currency does not improve welfare levels in a strong sense. In contrast, when local currencies are not used as an international currency, introducing the third currency improves welfare, when the total amount of money stock is not large.

Appendix A

Given the money stock of the third currency M_3, differentiating equation (48) with M_0 gives the optimal amount of a home currency as follows.

$$\frac{M_3}{2} = -\frac{M_0^*}{2+\gamma} + \frac{1}{2+\gamma}. \tag{50}$$

On the other hand, given the money stock of a home currency M_0, differentiating equation (48) with M_3 gives the optimal amount of the third currency as follows.

$$\frac{M_3^*}{2} = \frac{1}{2} - \frac{2+\gamma}{2(1+\gamma)}M_0. \tag{51}$$

From equation (50) and equation (51), there exist one unstable equilibrium and one stable equilibrium. In one equilibrium, the optimal money stocks of a home currency and the third currency are given by $M_0 = 1+\frac{1}{\gamma}$ and $M_3 = -\frac{1}{\gamma}$, respectively. Since the absolute value of the slope of equation (51) is larger than that of equation (50), this equilibrium is unstable. In the other equilibrium, the optimal money stocks of a home currency and the third currency are given by $M_0 = 0$ and $M_3 = \frac{1}{2}$. In any case, the optimal money stocks of a home currency and the third currency are given by $M_0 = 0$ and $M_3 = \frac{1}{2}$ in the long run.