Defense Expenditures and Allied Cooperation

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Abstract

Conflicting allied blocs often compete on the security issue. A conventional wisdom is that members of the latent bloc do not cooperates but members of the vital bloc cooperate within the bloc at the Nash solution. By developing a simple multi-country model of two conflicting blocs with the same size, we explore an interesting counterexample. Namely, a Nash outcome is likely that the latent bloc cooperates and the vital bloc does not cooperate. We show that the arms race effect may be dominant for the vital bloc, while the cooperative effect may be dominant for the latent bloc. In the case of Cold War game, the NATO countries did not organize a strong political body to seek more security benefits whereas the WTO countries organized a powerful political body under the leadership of USSR. Our simple model may explain this outcome by providing a numerical example.

Key words: security spending, conflicting blocs, arms race effect, cooperation effect
JEL classification numbers: H41, F13, D62

1.Introduction

It is widely recognized that each member of an allied bloc has an incentive to free ride on spending on the security activities of other allied members. In their classical paper, Olson and Zeckhauser (1966) applied the theory of private provision of public goods to allied countries and concluded that all allied countries gain when they determine the level of spending on security cooperatively. They highlighted the importance of allied cooperation in setting the spending on security activities. When the size of the alliance is large, the gains from cooperation would also be large. We call this the cooperation effect. See also McGuire (1974), Sandler (1977) and Kemp (1984) among others.

Conflicting blocs often compete on the security issue. When two conflicting blocs engage in security activities, an adversary's spending would rise in response to an increase in spending by the enemy alliance. We call this the arms race effect, which would apply to various areas of contentious public goods such as security activities, environmental issues, and defense spending. Bruce (1990) first pointed out that cooperation among allies in setting their defense spending is not necessarily welfare improving because of the arms race effect.

Namely, he considered a three-country model with two allied countries and an adversary and shows that all countries may be worse off when the allies cooperate on defense spending than when they do not. This is because defense spending by the adversary rises in response to a cooperative increase in defense spending by the enemy alliance. He pointed out that cooperation among allies in setting defense spending is not necessarily welfare-improving for them. Even if allied countries can cooperate, allied corporation will not become an equilibrium. This is an interesting result. In such a case the freeriding incentive is not a serious problem. "The whole notion of suboptimality of defense provision must be reconsidered when adversaries' reactions are included". (Sandler and Hartley (1995) p.42)

However, his analysis is restrictive in that one bloc has two countries and another bloc has one country. It should be stressed that an allied bloc usually has multiple countries. When the number of countries within the same bloc is large, gains from cooperation would also be large. Thus, if each bloc has a larger number of allied countries, we would expect that cooperative behavior becomes more desirable. By developing a simple multi-country model of arms races between two blocs, Ihori (2000) investigated to what extent such a conjecture would be plausible. When the number of countries in one bloc is larger than that in another bloc, the countries in the larger bloc might be better off by cooperating than by not cooperating even if there is a negative spillover from the adversarial smaller bloc. Thus, in a two-stage game cooperative behavior becomes a subgame perfect solution. These results suggest that the cooperation result may well be valid even if adversarial response of the opposing bloc is explicitly incorporated into the model of the two conflicting blocs. In this sense, bloc size divergence does matter in cooperation–noncooperation issues[1].

Nevertheless, we often observe non-cooperative behaviors of security provision within an alliance. If preferences are divergent between conflicting blocs, the arms race effect might dominate the cooperation effect. In this paper, we would like to focus on the heterogeneous preference between conflicting blocs. In reality, a latent bloc and a vital bloc would have different preferences with respect to the security issue. Consider, for example, the performance of NATO and WTO during the Cold War. For the NATO bloc, the security issue was vital since many member countries were seriously concerned with the potential threat from USSR. On the other hand, for the WTO bloc the security issue was not vital since most of member countries were mainly concerned with economic standards or resources for consumption.

If this is true, the natural conjecture is that the vital NATO countries behaved cooperatively, while the latent WTO countries behaved non-cooperatively during the Cold War. However, during the Cold War the NATO countries did not often organize a strong political body to seek more security whereas the WTO countries actually organized a strong political body to conduct security spending. We may say that NATO's decision making was non-cooperative, while WTO's decision making was cooperative within allies. Even if the NATO might have the vital interest in the security issue, it seems that they did not have an incentive to organize a strong political party, the counterpart of WTO. See also McGuire (1991).

Two explanations could hold for this seemingly paradoxical outcome. First, a plausible political explanation is that the degree of democracy is important to conduct joint decision making. From this viewpoint, it may not be easy for democratic countries to internalize the free-riding incentives. Thus, democratic NATO members preferred non-cooperative decision making, while non-democratic WTO members conducted cooperative decision making under the strong leadership

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1 It should be stressed that we do not consider the free-riding behavior within each group in the cooperative case.

of USSR.

In this paper, we would like to offer another explanation based on economics, using a simple game between two conflicting blocs. Namely, if the NATO bloc had a vital interest in the security issue, a non-cooperative outcome would be desirable, anticipating the reaction by WTO. On the other hand, if the WTO bloc had a vital interest in the non-security issue, a cooperative outcome would be desirable, anticipating the reaction by NATO.

It is useful to investigate the effects of divergent sizes and different preferences separately. Since a vital bloc often consists of a small number of members, a cooperative outcome could be attained from the divergent size effect. In the present paper we assume that the two conflicting blocs have different preferences over the security issue but the size of the blocs is the same. Thus, we do not consider differences in bloc size in this paper. We examine the plausibility of the conventional conjecture that a vital bloc cooperates whereas a latent bloc does not. By developing a simple multi-country model of two conflicting blocs with the same size, we explore an interesting counterexample against the conventional wisdom by showing that the latent bloc cooperates but the vital bloc does not cooperate at the Nash solution.

2. Analytical Framework

Consider a simple competition model of Cold War in which two blocs compete for security. Assume that there are $n + n$ countries and two opposing allies, α and β, in the world. Each allied bloc consists of n allied countries. Country i's utility function is given by

(1) $$U^i = u_i(c_i, G), \qquad (i=\alpha, \beta)$$

where U^i is its utility, c_i is its consumption of private goods, and G is regarded as international public goods mainly for bloc α and as public bads for bloc β. An increase in G benefits bloc α but hurts bloc β. In order to win the Cold War game, both blocs may spend resources to either raise G or reduce G. Thus, one bloc's public good is another's public bad, and each bloc can take action to shift the total security influence toward its own preferred level.

We formulate that the actual level of G is determined by

(2) $$G = G\left(\sum_{i\in\alpha} g_i, \sum_{i\in\beta} g_i\right) = \sum_{i\in\alpha} g_i - \sum_{i\in\beta} g_i$$

where g_i is the amount of security spending provided by country i. Following the seminal studies of Tullock (1980) and Becker and Mulligan (1998), the outcome of political conflict/contest between blocs α and β is summarized by a modified version of contest success function, eq. (2). The conflict/contest involved is presumably complicated, but a key factor used to determine the "output" of the conflict/contest is the "input" expended by the players. Function (2) is a reduced-form end result of what may be a very complicated process of Cold War game. In this reduced-form end result, the size of G directly depends on the amount spent by both blocs to gain security influence. We formulate that the outcome of the Cold War conflict/contest is a function of the difference between the security expenditures of blocs. More pressure by bloc α increases the size of G, whereas more pressure by bloc β decreases it. In order to simplify our analysis, we assume that it is the net of the pressures applied by the blocs that determines the actual security influence, G. Function (2) exhibits the property of homogeneity of degree one such that the same proportional increase or decrease in g_α and g_β raises the conflict/contest outcome by the same magnitude.

If we denote by A the initial level of vested security for countries, $2A$ means the total amount of "pie." An increase in g_α at the given g_β results in an increased distribution of pie, $2A$, in favor of bloc α but against bloc β, and vice versa. Put in another way, a given amount of $2A$ is allocated according to the net pressure G. If G > 0, bloc α can get more than A, and vice versa.

For simplicity, we assume that the utility function is specified in a Cobb–Douglas form for each bloc.

(1-1) $$U^{\alpha i} = c_{\alpha i}^{1-\theta}(A+G)^{\theta}$$

(1-2) $$U^{\beta i} = c_{\beta i}^{1-\mu}(A-G)^{\mu}$$

where superscript (subscript) α or β denotes the bloc that a member of allies belongs to. Variable θ is assumed to be relatively small for bloc α, whereas μ is assumed to be relatively large for bloc β. Namely we assume 1 > $\mu > \theta$ > 0. G is not vital for bloc α, but it is vital for bloc β. We call bloc α the latent bloc and bloc β the vital bloc with respect to G. Condition A > 0 is incorporated into eq. (1-2) so that $A-G$ > 0. It is true that the Cobb–Douglas functional form is very restrictive. However, in order to obtain concrete results and provide a counterexample against the conventional conjecture, this formulation is useful as a first step of this research.

Country i's budget constraint can be given by

(3) $c_i + g_i = Y_i$,

For simplicity we assume that $Y_\alpha = Y_\beta = Y$.

In the example of Cold War game, suppose bloc α is the WTO bloc and β the NATO bloc. We may say that WTO is the latent bloc, while NATO is the vital bloc with respect to the security issue. G may capture the size of security benefits of WTO. In order to exclude the effects of differences in bloc size, we assume that both blocs have the same number of members. By doing so, we focus on the implications of differences in preferences.

The structure of the game is as follows:

Stage I: A country of each bloc determines whether to cooperate or not within the bloc.

Stage II: The country determines its spending on security activities, and a Nash equilibrium is obtained.

Since all the countries in each bloc are identical, they behave in the same way. We do not consider the free-riding behavior within each bloc in the cooperative case. Therefore, each country may commit itself to the cooperation decision in the cooperative case. Thus, in stage I the allied countries uniformly decide whether to cooperate or not within the bloc. When cooperation is chosen, each bloc determines a representative country which will then decide the per-country contribution from inside the bloc in stage II. In case of noncooperation, each member determines its own contribution noncooperatively in stage II. We do not consider cooperation between the conflicting two blocs.

Cornes and Rubbelke (2012) used a formulation similar to G as eq. (2) and investigate the contentious public characteristics. They presented conditions under which the existence of a unique noncooperative equilibrium is retained and analyzed its normative and comparative static properties. They showed an interesting twist on the proposition of neutrality: resource growth may be entirely dissipated by conflict over a public characteristic.

3. Second Stage

3.1 Noncooperative Case

G is determined as a Nash equilibrium of a "game" between two blocs. First, we investigate the noncooperative case where each country determines its own spending on security activities in stage II, treating the rest of the allied countries' spending on security activities as given. In other words, allied countries within the same bloc do not make any cooperative decisions with respect to spending on allied security activities but behave at Nash conjectures. As discussed in detail by Bergstrom et al. (1986), Bernheim (1986), and Andreoni (1988), a nonnegativity constraint on providing public goods may well be binding as a solution if the number of rival countries becomes large. In order to present the results in the simplest way and in their strongest form, we consider only the case where the nonnegativity constraints are nonbinding in equilibrium.

We now derive a reaction function of country i of bloc α. Country i maximizes eq. (1-1) subject to its budget constraint

$$c_{\alpha i} + G_\alpha + A = Y_\alpha + (n-1)g_{\alpha j} - ng_\beta + A \tag{4}$$

taking the security activities of other countries $g_{\alpha j}, g_\beta$ as given. Here, $g_{\alpha j}$ denotes the spending on security activities by country j ($\neq$ i) of bloc α, and g_β denotes the spending on security activities by any identical country of bloc β.

From the first-order condition, we have

$$g_{\alpha i} + (n-1)g_{\alpha j} - ng_\beta + A = \theta[Y_\alpha + (n-1)g_{\alpha j} - ng_\beta + A]$$

Since all the countries of bloc α are identical, we have $g_{\alpha i} = g_{\alpha j} = g_\alpha$ at any solution. Substituting $g_{\alpha i} = g_{\alpha j} = g_\alpha$ into the above equation, we finally obtain

$$g_\alpha = \frac{1}{n-(n-1)\theta}[\theta Y_\alpha - (1-\theta)A + (1-\theta)ng_\beta] \tag{5}$$

which is a reduced reaction function of each country belonging to bloc α.

Variable g_α is an increasing function of the country's own income and the spending on security activities by the rival countries. An increase in θ raises g_α, which is intuitively plausible. Eq. (5) also includes the arms race response of g_α to g_β,

$$dg_\alpha / dg_\beta = \frac{(1-\theta)n}{n-(n-1)\theta},$$

which is positive and decreasing with θ from 1 at θ = 0 to 0 at θ = 1. In other words, if θ is small, an increase in g_β induces a large increase in g_α. The intuition is as follows. When θ is small, a change in the real income of bloc α would not affect the demand for G to a great extent, and hence, a decrease in real income due to an increase in g_β induces little decrease in the demand for *G*. On the contrary, it reduces private consumption to a great extent, resulting in a large increase in g_α.

Similarly, the reaction function of country i of bloc β can be given as

$$(6) \qquad g_\beta = \frac{1}{n-(n-1)\mu}[\mu Y_\beta - (1-\mu)A + (1-\mu)ng_\alpha]$$

An increase in μ raises g_β, which is intuitively plausible.

Henceforth, we call country α (or β) the representative country of bloc α (or β). In Figure 1, curve Y represents country α's reaction curve, and curve X represents country β's reaction curve. Both curves are upward sloping. Spending on security activities is a strategic complement reflecting the arms race between rival blocs. An intersection of both curves, N, represents the noncooperative Nash equilibrium point.

Figure 1 insert here

From eqs. (5) and (6), the security activities for both countries can be respectively given as

$$(7\text{-}1) \qquad g_\alpha = \frac{[n-(n-1)\mu][\theta Y_\alpha - (1-\theta)A] + (1-\theta)n[\mu Y_\beta - (1-\mu)A]}{[n-(n-1)\theta][n-(n-1)\mu] - (1-\theta)(1-\mu)n^2}$$

$$(7\text{-}2) \qquad g_\beta = \frac{[n-(n-1)\theta][\mu Y_\beta - (1-\mu)A] + (1-\mu)n[\theta Y_\alpha - (1-\theta)A]}{[n-(n-1)\theta][n-(n-1)\mu] - (1-\theta)(1-\mu)n^2}$$

When the marginal valuation of the issue for the rival bloc μ increases, the spending on security activities g_α also increases. Since an increase in the marginal valuation of the rival bloc β raises its political spending, g_β, this raises threats to the countries of the other bloc α, giving rise to the arms race effect. Thus, both g_α and g_β increase with θ and μ. G also increases with θ but decreases with μ.

Thus, we have

$$G + A = \frac{\theta}{n(\theta + \mu - 2\theta\mu) + \mu\theta}\{[2n(1-\mu) + \mu]A + \mu n[Y_\alpha - Y_\beta]\},$$

$$c_\alpha = \frac{1-\theta}{n(\theta+\mu-2\theta\mu)+\mu\theta}\left\{[2n(1-\mu)+\mu]A+\mu n[Y_\alpha - Y_\beta]\right\},$$

(8)

$$U^\alpha = \frac{(1-\theta)^{1-\theta}\theta^\theta}{n(\theta+\mu-2\theta\mu)+\mu\theta}\left([2n(1-\mu)+\mu]A+\mu n[Y_\alpha - Y_\beta]\right),$$

$$U^\beta = \frac{(1-\mu)^{1-\mu}\mu^\mu}{n(\theta+\mu-2\theta\mu)+\mu\theta}\left([2n(1-\theta)+\theta]A+\theta n[Y_\alpha - Y_\beta]\right)$$

As shown in eqs. (8), the marginal rate of substitution of $G+A$ with respect to c_α equals 1, which is the marginal cost of providing security spending. This is the well-known result of noncooperative solutions on public goods within a bloc. The same applies to country β. Eqs. (8) also suggest that the welfare of each country decreases with a rival's marginal valuation of the issue, whereas it may well increase with the country's own marginal valuation of the issue. This is intuitively plausible.

The actual level of national security, G, increases with the number of allied countries of bloc α; this is consistent with McGuire (1974). It is also interesting to note that the real variables including G, c, and U are independent of Y if $Y_\alpha = Y_\beta = Y$. More precisely, the net income, $n(Y_\alpha - Y_\beta)$, matters in the provision of contentious public goods. As shown in Cornes and Rubbelke (2012), a net increase in the resources available to the economy $dY_\alpha = dY_\beta > 0$ may have no real consequences in the provision of contentious public goods. This is the so-called super neutrality result, which holds in a general functional form of the utility function. It should also be noted that a redistribution between two conflicting blocs does have a real impact. A giving bloc loses, whereas a receiving bloc gains. On the contrary, a redistribution within a bloc does not have real effects. The conventional neutrality result, pointed by Shibata (1971) and Warr (1983), holds within each bloc.

3.2 Allied Cooperation

We now consider a cooperative case where the allied countries cooperate within their bloc. Note that there is still no cooperation (or negotiation) between the two conflicting blocs.

Consider the joint optimization problem of representative country α. Adding eq. (3) up to n and considering $g_{\alpha i} = g_{\alpha j} = g_{\alpha}$, the country's consolidated budget constraint may be written as

(9) $$nc_{\alpha} + G_{\alpha} + A = nY_{\alpha} - ng_{\beta} + A$$

Thus, country α jointly maximizes eq. (1-1) subject to the above consolidated budget constraint eq. (9), taking g_{β} as given. From the first-order condition, we have

$$ng_{\alpha} - ng_{\beta} + A = \theta(nY_{\alpha} - ng_{\beta} + A)$$

Thus, the reaction function of country α can be given as

(10) $$g_{\alpha} = \theta Y_{\alpha} - \frac{1-\theta}{n}A + (1-\theta)g_{\beta}$$

Eq. (10) implies that $\frac{\partial g_{\alpha}}{\partial g_{\beta}} = 1-\theta > 0$, which decreases with θ. This property is qualitatively the same as in the noncooperative case. When θ is low and the latent bloc α does not recognize the benefit of G to a great extent, it is desirable for bloc α not to change $A + G$ to a great extent. In order to reduce $A + G$ to a small extent, bloc α raises its spending on security activities to a great extent when bloc β increases its spending on security activities.

Note that if $n = 1$, eq. (10) reduces to eq. (5). When $n > 1$, dg_{α} / dg_{β}, the slope of the reaction function, $(1-\theta)$, is less than $(1-\theta)\,\mathrm{n}/\{n-(n-1)\theta\}$ in the noncooperative case. Thus, when the adversarial bloc β raises the level of security pressure, rival bloc α reacts by spending less in the cooperative case than in the noncooperative case. This is because in the cooperative case, a member of bloc α incorporates the positive reaction of other allied members when g_{β} increases, resulting in a smaller increase in g_{α} than in the noncooperative case, where it does not consider the positive reaction of other allied members. We also need to note that g_{α} is higher in the cooperative case than in the noncooperative case at the same level of g_{β}, since a cooperative behavior can internalize the free-riding motive. If $\theta = 1$, eq. (10) again reduces to eq. (5). On the contrary, if θ is small, the gap between cooperative g_{α} and noncooperative g_{α} becomes large.

Similarly, the reaction function of country β in the cooperative case can be given as

(11) $$g_\beta = \mu Y_\beta - \frac{1-\mu}{n}A + (1-\mu)g_\alpha$$

Again, if $n = 1$, eq. (11) reduces to eq. (6). If μ is large, the gap between cooperative g_β and noncooperative g_β is small.

There are three cases where cooperation is chosen in at least one bloc. First, let us investigate the cooperative case where all the countries of bloc α as well as bloc β cooperate. In Figure 1, curve S represents the reaction curve of country α when all the countries of bloc α cooperate, and curve T represents the reaction curve of country β when all the countries of bloc β cooperate. The intersection of curves S and T, denoted by point C, corresponds to the cooperative case where both blocs α and β cooperate respectively. The cooperative equilibrium levels of g_α, g_β are respectively given as

(12-1) $$g_\alpha = \frac{\theta Y_\alpha + \mu(1-\theta)Y_\beta - (1-\theta)A(2-\mu)/n}{\mu+\theta-\mu\theta}$$

(12-2) $$g_\beta = \frac{\mu Y_\beta + \theta(1-\mu)Y_\alpha - (1-\mu)A(2-\theta)/n}{\mu+\theta-\mu\theta}$$

An increase in θ or μ raises the security activities of both blocs, g_α, g_β, and G increases with θ but decreases with μ. An increase in μ lowers the welfare of bloc α by reducing G, and an increase in θ lowers the welfare of bloc β by raising G. Thus, we have

$$G + A = \frac{\theta}{\mu+\theta-\mu\theta}\{[2-\mu]A + \mu n[Y_\alpha - Y_\beta]\},$$

$$c_\alpha = \frac{1-\theta}{(\mu+\theta-\mu\theta)n}\{[2-\mu]A + \mu n[Y_\alpha - Y_\beta]\},$$

$$U^\alpha = \frac{(1-\theta)^{1-\theta}\theta^\theta}{\mu+\theta-\mu\theta}\frac{1}{n^{1-\theta}}\{[2-\mu]A + \mu n[Y_\alpha - Y_\beta]\},$$

$$U^\beta = \frac{(1-\mu)^{1-\mu}\mu^\mu}{\mu+\theta-\mu\theta}\frac{1}{n^{1-\mu}}\{[2-\theta]A + \theta n[Y_\beta - Y_\alpha]\}$$

(13)

As shown in eqs. (13), the total marginal rate of substitution of $G + A$ with respect to c_α equals 1, which is the marginal cost of providing political pressure. This condition is nothing but the Samuelson rule on public goods within the bloc, which is the well-known result of cooperative solution. Although the Samuelson rule holds for each bloc, it does not apply to the overall world. Since there is no cooperation between two conflicting blocs, the cooperative solution within the bloc here cannot attain the first-best result. The second-best theory suggests that the second-best utility is not necessarily higher at the cooperative solution than at the noncooperative solution.

We may also consider the partial cooperative case where the countries of bloc α do not cooperate whereas the countries of bloc β cooperate within each bloc. In this case, country α's reaction curve can be given as eq. (5) while country β's reaction curve can be given as eq. (11). We may also consider the partial cooperative case where the countries of bloc α cooperate while the countries of bloc β do not cooperate. We need to note that the net income $n(Y_\alpha - Y_\beta)$ matters in the provision of contentious public goods. Thus, the neutrality and super-neutrality results hold in the three cooperative cases as well.

4. First Stage

Table 1 indicates the hypothetical payoffs of four cases in the second stage of the game: (1) either bloc α or bloc β does not cooperate at point N, (2) bloc α cooperates while bloc β does not cooperate at point P, (3) bloc α does not cooperate while bloc β cooperates at point Q, and (4) both blocs α and β cooperate at point C.

Table 1 insert here

Now, we can investigate the Nash equilibrium by comparing four possible payoffs at the second stage: the noncooperative payoffs where no countries cooperate and the three cooperative payoffs where at least some allied cooperation occurs in blocs α and/or β.

In order to internalize the positive spillover effect between members within the same bloc, the countries of the same bloc, say α, should choose a representative, or have an agreement to determine security activities cooperatively. By doing so, the bloc's spending on security activities is stimulated and benefits all the countries of the bloc. This is the cooperation effect. However, the members of the rival bloc β react by raising their security activities, which would hurt the countries of bloc α. We call this the negative spillover of the arms race effect. If the negative spillover due to the arms race effect outweighs the positive spillover due to the cooperation effect, such cooperation hurts bloc α. This possibility was first pointed by Bruce (1990) in the study of national defense. Ihori (2000) showed that the cooperation effect might well dominate the arms race effect when the number of allied members is larger than the number of rival members.

By excluding the differences in bloc size, the present paper focuses only on the differences in preferences between the two blocs; θ is smaller than μ. Let us first investigate the optimal strategy of bloc α in stage I. Suppose bloc β cooperates. As shown in Table 1, it is desirable for bloc α to cooperate if and only if

$$D = n^{1-\theta}(\mu+\theta) - \mu\theta n^{1-\theta} - (\theta + \mu n - \theta n\mu)$$
$$= \mu n(n^{-\theta}-1)(1-\theta) + \theta(n^{1-\theta}-1) < 0$$

Since $n^{-1} < n^{-\theta} < 1$, this sign could be negative when μ is relatively large. D and E here are calculated using their definitions where θ replaced with μ, vice versa, in order to evaluate utilities of group β. Table 2 (i) suggests that D becomes negative if μ > 0.5 for n = 3. In other words, if μ is relatively large, bloc α gains by cooperating within the bloc.

Table 2 insert here

Suppose now that bloc β does not cooperate. Then, it is desirable for bloc α to cooperate if and only if

$$E = \mu n^{1-\theta} + \theta n^{2-\theta} - \theta n^{2-\theta}\mu - n(\theta + \mu - 2\theta\mu) - \mu\theta$$
$$= \mu n(n^{-\theta}-1) + \theta n(n^{1-\theta}-1) + \theta\mu(2n - 1 - n^{2-\theta}) < 0$$

This sign could be negative when μ is relatively large, since $2n < n^{2-\theta}+1$ for a small θ. Table 2 (ii) shows that E becomes negative if $\mu \geq 0.9$ for n = 3. Table 2 (iii) further shows that E becomes negative if $\mu \geq 0.8$ for n = 6. In other words, if μ is relatively large, it is always desirable for bloc α to cooperate within the bloc. Hence, cooperation is the dominant strategy for bloc α when μ is relatively large.

Table 3 insert here

Table 3 compares the payoffs of bloc β in the cooperative case and noncooperative case. D and E here are calculated using their definitions where θ replaced with μ, vice versa, in order to evaluate utilities of group β. Table 3 (i) shows that D is positive if $\theta \leq 0.5$ for n = 3. In other words, if θ is relatively small, bloc β gains by not cooperating within the bloc. Table 3 (ii) shows that E is positive if $\theta \leq 0.7$ for $n = 3$. Table 3 (iii) further shows that E becomes positive if $\theta \leq 0.7$ for $n = 6$ as well. In other words, if θ is relatively small, it is desirable for bloc β not to cooperate within the bloc. Therefore, noncooperation is the dominant strategy for bloc β when θ is relatively small. Hence, a Nash outcome is likely that the latent bloc cooperates and the vital bloc does not cooperate.

The intuition is as follows. When μ is relatively large, the effect of an increase in g_α on g_β is small. In such a case, an increase in the security activities of bloc α due to cooperation within the bloc would not stimulate security activities in the rival bloc β to a great extent. This is because the gap between cooperative g_β and noncooperative g_β is small when μ is relatively large. Hence, the arms race effect is small and bloc α gains by cooperating. Furthermore, when θ is relatively small, the negative spillover from an increase in the security activities of bloc β does not hurt bloc α to a great extent. In such a case, bloc α does not lose to a great extent from an arms race reaction by rival bloc β. In other words, if θ is relatively small, the cooperation effect dominates the arms race effect for bloc α. Qualitatively, the opposite mechanism applies to bloc β if μ is relatively large. That is, the arms race effect may dominate the cooperation effect for bloc β.

Table 4 insert here

The above mechanism is affected by the rate of substitution of the two goods. Table 4 shows the remainder of utility levels of bloc α in the non-cooperative case after deducting that in the cooperative case when bloc β cooperates, using the CES utility function $U^{\alpha i} = [(A+G)^{\sigma/(1-\sigma)} + (1-a)c_{\alpha i}^{\sigma/(1-\sigma)}]^{(1-\sigma)/\sigma}$, which should be compared with Table 2 (i). As shown, our results strengthen when the rate of substitution σ>1 and weaken when σ<1 because an increase or a decrease in σ works like that in θ in the Cobb-Douglas utility case when θ>0.5. The same mechanism operates for μ as well, thus our results that a Nash equilibrium of the latent bloc and the cooperative equilibrium in the vital bloc is more plausible when σ>1.

5. Some Remarks

In the real economy, it is often observed that a small number of countries form a powerful and well-organized allied bloc to cooperate within the bloc, whereas a large number of countries do not always build a powerful allied bloc. It is true that bloc size divergence has an important role since it affects the size of the cooperation effect. In addition to this, a natural conjecture is that if the former bloc has a vital interest in the issue, a cooperative strategy would be desirable although it stimulates security activities of the rival bloc, and it would be desirable for the latter bloc not to cooperate since it has a latent interest in the issue. One could argue that if the issue is vital, cooperation becomes desirable.

Our analytical result suggests that the above conjecture is not necessarily valid if the divergent bloc size effect is controlled for. We have shown that the arms race effect is large for the vital bloc but not for the latent bloc. Hence, the arms race effect may well dominate for the vital bloc whereas the cooperation effect dominates for the latent bloc. The cooperative behavior of the vital bloc might stimulate the offsetting security activities of the latent rival bloc to a great extent, hurting the vital bloc very much. The opposite mechanism applies to the latent bloc. Hence, a Nash outcome may well be that a member of the latent bloc cooperates and a member of the vital bloc does not cooperate within each bloc. In reality, the conflicting blocs on Cold War might have such features.

We have examined the plausibility of the conventional conjecture that a vital group cooperates whereas a latent group does not. By developing a simple multi-agent model of two conflicting blocs, we explore an interesting counterexample; a plausible Nash outcome is that the latent group cooperates but the vital group does not cooperate.

We may apply this analytical result to the case of an intergenerational redistribution issue such as a public pension reform. If the population is stationary, the group size of the elderly and the working people would be almost the same, and hence, we may ignore the differences in group size on this issue. The elderly and the working generations may have different preferences on pension reform. We may assume that the elderly has a vital interest on the issue but the younger generation does not. In reality, it seems that the elderly does not often organize a strong political body to seek more pension benefits whereas working people usually organize a strong political body such as a labor union, which would resist paying more contributions to support pension benefits. Even if elderly people are vital in this case, they might not have an incentive to organize a strong political party, the counterpart of a labor union.

Another example might be the recent situation in Eurozone. Zimmermann

(2015) categorized 10 Euro area countries into two blocs: the northern of Finland, Luxemburg, Germany, Austria and Netherland, and the southern of Spain, Belgium, France, Italy and Greece. The numbers of the two blocs are almost the same. Since funding from the northern countries to stabilizing facilities such as ESFS and ESM is a kind of subsidies from the northern countries to the southern countries, our analytical framework can be applied here.

The bailout plan for the European financial crisis was strongly needed by the southern countries such as Greece. However, they could not form a strong alliance against the conditions contended by the northern member coalition. On the contrary, northern members united in demanding the fiscal reform of the southern members although their benefits were relatively weak. It seems that the northern countries have less vital than southern countries with respect to the sustainability of Eurozone and hence the amount of subsidies. Nevertheless, it seems that they behave cooperatively, while the southern countries behave non-cooperatively.

6. Conclusion

This paper has investigated the cooperation and arms race effects of noncooperative and cooperative spending on the security activities of the allied members of two rival blocs with the same bloc size. We have shown that if the two rival blocs' preferences on the security issue are different, a Nash outcome is likely that the latent bloc cooperates and the vital bloc does not cooperate within each bloc. Intuition is as follows.

As for the vital bloc, the noncooperative supply of security activities is close to the cooperative level so that the gains from cooperative behavior may not be large, and hence, the noncooperative (free-riding) behavior of each member does not hurt the vital bloc to a great extent compared with the cooperative choice. Moreover, it could benefit the other allied members by depressing the arms race effect. On the contrary, as for the latent bloc, the noncooperative (free-riding) behavior hurts each member to a great extent since the gap between the cooperative and noncooperative levels of security activities is large. The vital bloc may lose much by cooperation since it would induce considerable arms race activities from the latent countries. We have shown that the arms race effect may be dominant for the vital bloc and that the cooperative effect may be dominant for the latent bloc.

In the case of Cold War game, it was often observed that the NATO countries did not organize a strong political body to seek more security benefits whereas the WTO countries organized a powerful political body under the leadership of USSR.

Although the Cobb–Douglas formulation is rather restrictive, our simple model may explain theses seemingly paradoxical outcomes by providing a numerical example. One limitation is that this model does not endogenize political bloc membership. This is a topic of interest for future research.

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Table 1

		Bloc β	
		N	C
Bloc α	N	$\dfrac{2n(1-\mu)+\mu}{\Delta}, \quad \dfrac{2n(1-\theta)+\theta}{\Delta^*}$	$\dfrac{2-\mu}{\Phi}, \quad \dfrac{2n(1-\theta)+\theta}{n^{1-\theta}\Phi^*}$
	C	$\dfrac{2n(1-\mu)+\mu}{n^{1-\theta}\Lambda}, \quad \dfrac{2-\theta}{\Lambda^*}$	$\dfrac{2-\mu}{n^{1-\theta}\Gamma}, \quad \dfrac{2-\theta}{n^{1-\mu}\Gamma^*}$

Notes: N means noncooperation, and C means cooperation.

$$\Delta \equiv \frac{n(\theta+\mu-2\theta\mu)+\theta\mu}{(1-\theta)^{1-\theta}\theta^{\theta}A}$$

$$\Delta^* \equiv \frac{n(\theta+\mu-2\theta\mu)+\theta\mu}{(1-\mu)^{1-\mu}\mu^{\mu}A}$$

$$\Phi \equiv \frac{\theta+\mu n-n\theta\mu}{(1-\theta)^{1-\theta}\theta^{\theta}A}$$

$$\Phi^* \equiv \frac{\theta+\mu n-n\theta\mu}{(1-\mu)^{1-\mu}\mu^{\mu}A}$$

$$\Lambda \equiv \frac{n\theta+\mu-n\theta\mu}{(1-\theta)^{1-\theta}\theta^{\theta}A}$$

$$\Lambda^* \equiv \frac{n\theta+\mu-n\theta\mu}{(1-\mu)^{1-\mu}\mu^{\mu}A}$$

$$\Gamma \equiv \frac{\theta+\mu-\theta\mu}{(1-\theta)^{1-\theta}\theta^{\theta}A}$$

$$\Gamma^* \equiv \frac{\theta+\mu-\theta\mu}{(1-\mu)^{1-\mu}\mu^{\mu}A}$$

Table 2 (i). Value of D for n = 3

	0.8	0.7	0.6	0.5	0.4	0.3 (θ)
0.9	-0.12	-0.16	-0.19	-0.20	-0.20	-0.18
0.8		-0.11	-0.13	-0.14	-0.14	-0.12
0.7			-0.07	-0.08	-0.07	-0.07
0.6				-0.01	-0.01	-0.01
0.5					0.05	0.05
0.4						0.11
(μ)						

Table 2 (ii). Value of E for n = 3

	0.8	0.7	0.6	0.5	0.4	0.3 (θ)
0.9	-0.08	-0.11	-0.12	-0.13	-0.13	-0.11
0.8		-0.00	0.00	0.01	0.01	0.01
0.7			0.12	0.14	0.15	0.14
(μ)						

Table 2 (iii). Value of E for n = 6

	0.8	0.7	0.6	0.5	0.4	0.3 (θ)
0.9	-0.31	-0.40	-0.48	-0.51	-0.50	-0.44
0.8		-0.03	-0.01	0.03	0.07	0.11
0.7			0.46	0.57	0.64	0.66
(μ)						

Table 3 (i). Value of D for n = 3

	0.8	0.7	0.6	0.5	0.4	0.3 (θ)
0.9	-0.05	-0.03	-0.01	0.01	0.03	0.05
0.8		-0.05	-0.01	0.02	0.06	0.09
0.7			-0.02	0.03	0.08	0.13
0.6				0.04	0.10	0.16
(μ)						

Table 3 (ii). Value of E for n = 3

	0.8	0.7	0.6	0.5	0.4	0.3 (θ)
0.9	-0.00	0.04	0.08	0.11	0.15	0.19
0.8		0.07	0.14	0.22	0.29	0.37
0.7			0.20	0.31	0.41	0.51
0.6				0.37	0.50	0.62
0.5					0.55	0.69
0.4						0.70
(μ)						

Table 3 (iii). Value of E for n = 6

	0.8	0.7	0.6	0.5	0.4	0.3 (θ)
0.9	-0.03	0.11	0.24	0.38	0.51	0.65
0.8		0.22	0.49	0.75	1.01	1.28
0.7			0.72	1.10	1.48	1.86
0.6				1.41	1.88	2.35
(μ)						

Table 4 (i). $U^{ai}(N,C) - U^{ai}(C,C)$ **for σ= 1.5**

	0.8	0.7	0.6	0.5	0.4	0.3 (θ)
0.9	-0.53	-0.65	-0.63	-0.54	-0.41	-0.29
0.8		-0.62	-0.62	-0.54	-0.42	-0.30
0.7			-0.57	-0.51	-0.41	-0.29
0.6				-0.39	-0.32	-0.24
0.5					-0.04	-0.02
0.4						0.62
(μ)						

Table 4 (ii). $U^{ai}(N,C) - U^{ai}(C,C)$ **for σ= 0.5**

	0.8	0.7	0.6	0.5	0.4	0.3 (θ)
0.9	0.36	0.32	0.30	0.27	0.26	0.24
0.8		0.51	0.47	0.44	0.42	0.39
0.7			0.65	0.61	0.58	0.55
0.6				0.79	0.76	0.73
0.5					0.97	0.93
0.4						1.19
(μ)						

Figure 1: Four Nash Equilibria